MATHEMATICS

Grade 6

Teacher Guide

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Foreword

Education and development are closely related endeavours. This is the main reason why it is said that education is the key instrument in Ethiopia's development. The fast and globalised world we now live in requires new knowledge, skills, attitudes and values on the part of each individual. It is with this objective that the curriculum, which is a reflection of a country's education system, must be responsive to changing conditions.

It is more than fifteen years since Ethiopia launched and implemented the *Education and Training Policy*. Since then our country has made remarkable progress in terms of access, equity and relevance. Vigorous efforts also have been made, and continue to be made, to improve the quality of education.

To continue this progress, the Ministry of Education has developed a Framework for Curriculum Development. The Framework covers all pre-primary, primary, general secondary and preparatory subjects and grades. It aims to reinforce the basic tenets and principles outlined in the *Education and Training Policy*, and provides guidance on the preparation of all subsequent curriculum materials – including this teacher guide and the student textbooks that come with it – to be based on active-learning methods and a competency-based approach.

Publication of a new Framework and revised textbooks and teacher guides are not the sole solution to improving the quality of education in any country. Continued improvement calls for the efforts of all stakeholders. The teacher's role must become more flexible ranging from lecturer to motivator, guide and facilitator. To assist this, teachers have been given, and will continue to receive, training on the strategies suggested in the Framework and in this teacher guide.

Teachers are urged read this guide carefully and to support their students by putting into action the strategies and activities suggested in it. The guide includes possible answers for the review questions at the end of each unit in the student textbook, but these answers should not bar the students from looking for alternative answers. What is required is that the students are able to come up with, and explain knowledgeably, their own possible answers to the questions in the textbook.

Introduction

Mathematics is one of the school disciplines that focus on the enhancement of student's mathematical power and proficiency that lead to purposeful and worthwhile mathematical work. As a science of patterns and relationships, mathematics relies on **logic**, **reasoning, problem solving** and **creativity**. It is characterized by a cycle of learning that includes representation, manipulation and validation.

Nowadays learning mathematics is becoming helpful in almost every kind of human endeavor. It serves as a basic precise language for the other field of studies such as **science** and **technology.** All sciences use the language of mathematics to describe objects and events, to characterize relationships between variables, and to argue logically. It can be said that learning mathematics is essential in everyday life.

Mathematics involves certain interrelated learning elements such as:-

- Comprehension of mathematical terms, concepts, operations and relationships.
- Skill in carrying our procedures flexibly, accurately, efficiently and appropriately.
- Ability to formulate, represent and solve mathematical problems.
- Logical thought, reflection, explanation and justification.

The need to develop continuous assessment implementation teacher guide arise from the following basic assumptions:-

- Effective mathematics instruction requires periodic and constant flow of information about students learning progress or learning deficiencies.
- Repeated and regular assessment of students provides better picture of the instructional process for mathematics teacher.
- A system of continuous assessment in mathematics teacher helps to measure a wide range of mathematical skills (such as problem solving and critical thinking) that cannot easily be assesses by time-limit terminal examinations.

- Implementation of continuous assessment improves the motivation of students to work hard and helps to get involved in learning mathematics.
- The other support systems such as teacher's resource materials (Syllabus, text books, teachers guides) and refreshment courses should be in place to effectively implement.
- Finally it is possible to implement a system of continuous assessment in mathematics in spite of the increased effort time and energy it demands form both teachers and students.

Organization of this teachers Guide

This teacher guide is organized unit by unit. It contains the following major themes:

i. **Introduction**: **-** includes the role and rational and special Characteristics of learning the subject matter, guidelines on how to use the teacher guide and the nature of continuous assessment.

ii. **Competencies of each unit**: **-** drawn from mathematics syllabus of grade 6.

iii. Suggested teaching aids.

iv. Sub-unit competencies of each unit.

v. Sub-unit introduction of each unit.

vi. Teaching notes of each unit.

vii. Answers to Activities and Exercises.

viii. Continuous assessment.

ix. Answers to Miscellaneous Exercises for unit by unit.

x. Topics, period allotment and location chart.

xi. Syllabus

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The Concept of Active learning and Continuous Assessment

What is Active Learning?

Active learning: as the name suggests, is a process whereby students are actively engaged in the learning process, rather than “Passively” absorbing lectures. Students are rather encouraged to think, Solve problems, do activities carefully selected by the teacher, answer questions, formulate questions of their own, discuss, explain, debate or brain storm question , explore and discover, work cooperatively in group to solve problems and work out project.

Teachers' are strongly advised to discuss and work out difficult questions. As far as possible the class should not be teacher centered. Attention should be given to the following points in motivating students to participate in the lesson through activities, class work, home work, Group work and reading the text book independently.

- Give students a chance to express theorems, definitions, properties and rules in their own for each unit.
- Make students work the activities in class either individually, in pair or in small groups:
- Make the lesson lively be relating it with real examples from the students' environment.
- Use order to methods in teaching i.e from simple to complex methods in teaching.
- In order to evaluate students and find out individual weakness and help them, regular tests should be prepared carefully by referring to the unit out comes in the syllabus.
- Use different types of teaching aids based on each unit.

What is Assessment?

Assessment: is a process by which information is obtained relative to some known objective or goal. The teachers assess at the end of a lesson or unit or the end of a school year through testing. Generally assessment is defined as collecting information on the progress of students learning using varieties of procedures (Example checklist, formal tests, self-assessment, creative writing, portfolios).

Purposes of Assessment

Teachers have many purposes for assessment of students. Some of the main reasons are:

1. **Improving instructional materials**: **-** Teachers need information regarding how effective teaching procedures, activities, the text book and other materials are in teaching.
2. **Improving students learning**: Both teachers and students need to know how students are doing.
3. **Determining content mastery**: Teachers evaluate students to determine if and when they have mastered the subject matter.
4. **Teaching: Evaluation activities**, if appropriately planned and used, can be powerful learning activities.
5. **Grading Students**:**-** Parents, administrators, and sometimes employees need evidence of pupil progress.

Forms of Assessment

There are two forms of assessment. These are continuous assessment and summative assessment.

Continuous Assessment***:*** of learners' progress could be defined as a mechanism whereby the final grading of learners in the **cognitive**, **affective and psychomotor** domains of learning systematically take account of all their performances during a given period of schooling.
Continuous assessment is an assessment approach that involves the use of a variety of assessment so as the assess various components of learning:

- The thinking processes (cognitive skills),
- Behaviors, personality traits (affective characteristics) and
- Manual dexterity (psychomotor domain)

Summative Assessment

This is a summary assessment of the extent to which learners have mastered the intended objectives. It normally occurs at the end or the completion of a semester teaching.

The Need for Continuous Assessment

Continuous assessment as a method of evaluating the progress and achievement of students on a day – to – day basis is relevant to get a clear picture of every students' performance.

Most importantly, planning a continuous assessment system at school level is useful to gather adequate and reliable information about:-

- The present status of every students
- The students motivation to participate actively in the teaching – learning process;
- Students progress in his/her learning;
- Students learning difficulties for diagnosing problems and to take remedial measures;
- Students preferences, interests and attitudes; and
- The effectiveness of teaching methods, techniques, and learning material used by teachers.

Steps in the Continuous Assessment

The following are the major steps to follow in Continuous Assessment:-

Step i: Overview the unit out comes, contents, methods and tools of the unit.

Step ii: Produce a schedule of assessment for the unit.

Step iii. Determine the items for the suggested assessments of the unit.

Step iv: Construct questions for the types of assessments suggested for the unit based on the determined items.

Step v: Administer the suggested assessment tools constructed specifically on the bases of the schedule.

Step vi: Grade or mark what was done by students.

Step vii: Record the assessed results.

Teachers should have format (s) for recording the assessment results of students. The format (s) may be centrally or regionally designed or individually formulated by the teachers themselves. In any case, the recording format has to include at least, the names of students, grade level, subject type, and the marks allotted for each assessment task.

Step viii:- Report the recorded results.

Methods /Strategies of continuous Assessment

The methods of continuous assessment enables you to assess a wide range of learning competencies and behaviors using a variety of instruments some of which are:-

- Tests (quizzes)
- Classroom discussions, exercises, assignment or group works
- Project
- Observations
- Interview
- Group discussions
- Questionnaires

Different competencies may require different assessment techniques and instruments. For example, oral questions and interviews may serve to assess listening and speaking abilities.

Below are a descriptions of these methods of continuous assessment used in this assessment used in this assessment guide and their possible uses.

Tests

These usually consist of a range of questions covering almost all of the objectives of a unit. Students are required to respond to questions within a specified time, not more than half an hour. Tests could be phrased in different ways:

Close – ended (selection type such as true – false, multiple – choice, matching type) and open – ended (short – answer, essays, completion type).

Group Projects

A Project: is an exercise on a single objective or topic that requires investigation in with the time constraints more investigation in with the time constraints more relaxed than assignments. More over, projects require much more information than assignments and require the involvement of a group of learners working together.

Marking

Marking or grading: is the process of offering different types of symbols to academic progress or achievement of students. The marks given to students academic achievement are usually reported to the school administration in general and parents in particular. Designing a good marking scheme can help to be uniformly fair to all students.

The following are some suggestions on how to mark a semester's achievement.

1. One final semester examination 30%
2. Mid examination 20%
3. Tests 15%
4. Quizzed 10%
5. Home work 5%
6. Class activities, class work and presentation 10%
7. Project work, in groups or individually 10%

Recording and Reporting Students' progress and Achievement

Recording Students' achievement is an important aspect of continuous assessment. The reports on students' progress and performance may be miss- leading and incomprehensible unless records are properly kept.

The major records to be kept are teacher's records, student's cumulative report card and transcript.

a) **The teacher's record book:** is a permanent record book which every teacher must keep in his/her class. The teacher's record book is expected to contain a detailed scheme of work, an accurate diary or daily record of work and progress report.

b) **The student's cumulative record card:** this contains the most available information of students development through out the primary school course. The following main information should be including in the students cumulative record card.

- Personal information about the students
- Weekly or periodic report of academic performance.
- Report on his/her character.
- Report on the terminal tests
- Report on the summary of progress in all areas of the school curriculum.

c) The transcript:- This includes the results of continuous and Summative assessments add up to 100%. Below is a record format of transcript.

Students	Continuous Assessment									
	Weight	Class work, Class activities	Home Work	Quizzes	Project; Group Work	Test 1	Test 2	Mid Exam	Final Exam	Total
1		10%	5%	10%	10	5%	10%	20%	30%	100%
2										
3										
4										
.										
.										
N										

Reporting Makes educators more accountable to learners, parents, the education system and the border community.

N.B: This plan is more preferably during the beginning of the semester (year).

Topics, and period allotment

Unit	Sub-unit	Sub-units	Number of period Total
Unit 1 Basic concepts of sets	1.1 Introduction to sets	3	19
	1.2 Relations among sets	6	
	1.3 Operations on sets	10	
Unit 2 The divisibility of whole numbers	2.1 The Notion of divisibility	6	23
	2.2Multiples and divisors	17	
Unit 3 Fractions and decimals	3. The simplification of fractions	5	41
	3.2 The conversion of fractions, decimals and percentage	10	
	3.3 Comparing and ordering	5	
	3.4 Further on addition and subtraction of fractions and decimals	10	
	3.5 Further on multiplication and division of fractions and decimals	11	
Unit 4 Integers	4.1 Introduction to integers	5	18
	4.2 Comparing and ordering integers	5	
	4.3 Addition and subtraction of integers	8	
Unit 5 Linear equations, linear inequalities	5.1 Solution of simple linear equations and inequalities	7	25
	5.2 Coordinates	6	
	5.3 Proportionality	12	
Unit 6 Geometry and measurement	6.1 Angles	8	44
	6.2 Construction of triangles	12	
	6.3 Congruent triangles	12	
	6.4 Measurement	12	

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UNIT ONE

BASIC CONCEPTS OF SETS

Introduction

This unit deals with introductory ideas of sets. The unit gives emphasis to explaining what is meant by 'set' and 'element' describing relationship among sets, and representing relationship among sets by Venn diagram. Each topic is presented by giving descriptive examples and explanations.

The activities and exercise given in each sub-unit are designed to involve students and think critically about the lesson presented.

Unit outcomes

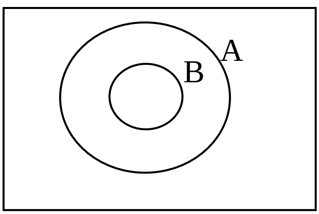
At the end of this unit, students will be able to:

- understand the concept of set.
- describe the relation between two sets.
- perform two operations (intersection and union) on sets.

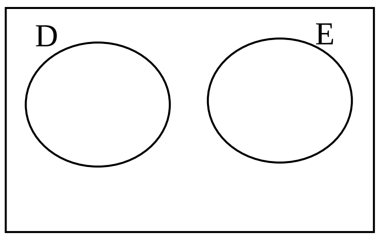
Suggested teaching Aids in unit 1

In addition to the students' textbook and the teachers' guide, you are expected to prepare and bring to the class different drawings of Venn diagrams that represent sets.

Charts containing the following are recommended.

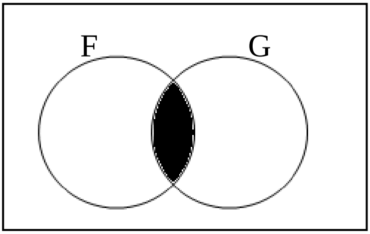
i) 

$B \subseteq A$

ii) 

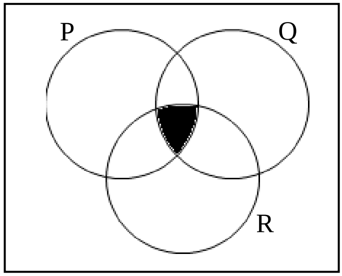
Disjoint sets

iii)



$F \cap G$

iv)



$P \cap Q \cap R$

1.1 Introduction to sets

Periods allotted: 3 periods

Competency

At the end of this subunit, the students will be able to:

- explain what is meant by "set" and "element".

Introduction

This sub unit is devoted to introducing the idea of set and explaining elements of a set. Real life examples which describe sets are discussed in more detail. Various activities and exercises are provided in order to involve students in the discussion of this sub unit.

Teaching Notes

Encourage students to give their own examples of sets (like the set of male students in the class). Guide students to come to an idea of empty set and its symbol by using examples (like the set of students in your class who are taller than 2 meters). Assist students to use the appropriate symbols and terms related to a set.

You may ask the following question: which of the following sets are not empty?

a) The set of natural numbers less than 1.
b) The set of two digit whole numbers that are divisible by 2.
c) The set of female teachers in Addis Ababa.

Answers to Group 1.2

1. e.g. The set of children.

    The set of birds.

    The set of hens.

    The set of cats.

2. e.g. The set of hens contains five elements.

Answers to Activity 1.1

a) {September, October, November, December, January, February, March, April, May, June, July, August}

b) {0,1,2,3,4,...,98}

c) the set has no members.

Answers to Activity 1.2

a) True

b) True

c) False

d) False

Answers to Activity 1.3

1. Not – empty. e.g May is a member of this set.
2. Not – empty. It is the set containing elements 0, 3 and 6.
3. Empty.

Assessment

You can give students a number of problems on introductory ideas of sets in the form of class work, home work, assignment, quiz or test in order to assess students' performance.

You may also give the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

1. Identity whether each of the following is true or false.

   a) $2 \in \{1, 2, 3\}$ b) $0 \in \{20, 30, 40, 50\}$

   c) 4 is not element of the set of factors of 40

   d) 20 is an element of the set of multiples of 2.

2. List the elements of the following sets.

   a) The set of common factors of 8 and 20

   b) The set of multiples of 9 between 10 and 30.

Answers to Additional Assessment

1. a) true b) false c) false d) true
2. a) 1, 2 and 4

   a) 18 and 27

Answers to Exercise 1.A

1. a) True b) False c) True d) True e) True f) True
2. a) $\{1, 2, 3, 4, 6, 12, 24\}$ c) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

   b) $\{4, 5, 6, 7, 8, 9, 10\}$ d) $\{24, 32\}$
3. a) The set of multiples of 4 which are less than 20.

   b) The set of vowels in English alphabet.

   c) The set of whole numbers which are less than 6.
4. a, c, e, f and g are empty.

Selected problems to slow learners

1. Identify whether each of the following statements is true or false.

   a) $2 \in \{0,2,4\}$ c) $\phi \in \{0,10,20\}$

   b) $3 \notin \{1, 2, 3\}$ d) $\frac{1}{2} \in \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}\right\}$

2. List the elements of the following sets.

   a) The set of factors of 9.

   b) The set of factors of 20.

   c) The set of multiples of 2 between 5 and 11.

3. Which of the following sets are empty?
   a) The set of factors of 5.
   b) The set of factors of 8 which are greater than 8.
   c) The set of birds weighing 1000kg .
4. How many elements are there in the set containing natural numbers less than 5?
5. How many elements are there in the set containing the alphabets of English language?
6. List the elements of the set containing all odd numbers which are less than 10
7. Let A = {1, 2, 3, 10, 100}. Is $o \in A$?
8. Is the set, containing all multiples of 3 less than 20, with limited number of elements?
9. List the elements of set K containing the last five letters of the English alphabet.
10. What is the number of elements in an empty set?

Selected problems to fast learners

1. List the elements of the following sets.
   a) The set of factors of 200.
   b) The set of multiples of 7 between 100 and 130.
   c) The set of countries on Africa whose name start with the letter N.
2. Give examples of sets which are empty.
3. Give examples of sets which are not empty.
4. How many elements are there in the set containing multiples of nine between 100 and 140?
5. How many elements are there in the set containing all whole numbers divisible by 7 and which lies between 20 and 50.
6. List the elements of the set containing all factors of 126.
7. "The set containing all unlucky numbers" does this define a set? Why?
8. Is the set containing all multiples of 3, with limited number of elements?

9. Give an example of a set containing only one element.
10. Let V = the set of whole numbers less than 12, and
    R = the set of multiples of 3 between 1 and 14. Then list all the elements that are common to both sets V and R.

1.2 Relations among sets

Periods allotted: 6 periods

Competency

At the end of this sub unit, the students will be able to:

- describe relationship among sets such as proper subset, subset, equal and equivalent sets.

Introduction

This sub unit deals with the idea of describing relationship among sets such as sub set, proper subset, equal and equivalent sets. Definitions and descriptive examples are included to elaborate relationship among sets in more detail.

Teaching Notes

Let students identify and practice the notion of subset, proper subset, equal and equivalent sets using several illustrative examples (the number of elements may not be greater than 3).

Example: let A = {a, b}, determine the proper subset and subsets of set A.

Solution

$\varnothing$, {a}, {b}, {a, b} are subsets of set A.

$\varnothing$, {a}, {b} are proper subsets of set A.

Motivate students to do problems to determine proper subsets, equal sets and equivalent sets.

You may ask the following questions:

1. Write the subset of the following sets.
   a) The set of all factors of 4 that are less than 4.
   b) The set of all multiples of 3 between 8 and 12.

2. Write the proper subset of the following sets.
   a) The set of one digit whole numbers between 7 and 10.
   b) The set of whole numbers between 15 and 19 that are divisible by 2.

Answers to Activity 1.4

1. Yes   2. Yes

Assessment

You can give problems on relations among sets in the form of class work, home work, assignment, quiz or test in order to assess the students' level of understanding. You are expected to check their work and the overall performance of students during discussion. You can also give the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

Find all subsets of the set P = {a, b, c, d}.

Answer to Additional assessment

∅, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {b, d}, {a, b,c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}.

Answers to Exercise 1.B

1. a) True b) False c) True d) False e) True f) False
2. A = C, D = H, F = J
3. Sets A, C, D, G and H are equivalent sets.
   Sets B, F and J are also equivalent sets.
4. a, b and d are empty sets
5. a) ∅, {a}, {c}, {e}, {a, c}, {a, e}, {c, e} and {a, c, e} are subsets of A. They are 8 in all.
   b) ∅, {a}, {c}, {e}, {a, c}, {a, e} and {c, e} are proper subsets of A. They are 7 in all.

Selected problems to slow learners

1. Find pairs of equal sets and equivalent sets from the sets given below
   A = {1, 2, 3, 4, 5}
   B = the set of whole numbers between 0 and 6
   D = {1, 3, 5}
   E = the set of odd whole numbers less than 6
2. Find all subsets and proper subsets of set A, where A = { 3, 4}.
3. Are the sets A = {a, b, c} and B = {1, 2, 3} equivalent?
4. Let A = {4, 5, 6} and B = {5, 4, 6}. Is $A \subseteq B$? Is $B \subseteq A$?
5. Write all subsets of set A, where set A contains all multiples of 2 less than 4.
6. Let P = the set of whole numbers less than 8, and Q = the set of even numbers between 3 and 11. Which of the following is true?
   a) P = Q c) $Q \subseteq p$ e) $Q \subseteq \phi$
   b) $P \subseteq Q$ d) $\phi \subseteq P$
7. Below are given few pairs of sets. Name which of them are equal.
   a) A = the set of all digits in 685
      B = the set of all digits in 856
   b) A = { 0, 1, 4, 9, 16}
      B = $\{0^2, 1^2, 2^2, 3^2, 4^2\}$
   c) A = the set of even integers between 5 and 9
      B = the set of factors of 7
8. Are two equal sets necessarily equivalent?

Selected problems to fast learners

1. Give examples of sets which are equivalent to set B, where
   B = { 11, 13, 15 }.
2. Find all subsets and proper subsets of set D, where D = { 2, 5, 8, 11}
3. Let A = the set of students in class X, and
   B = the set of roll numbers in class X. Is set A equivalent to set B?
4. Find a set a which can satisfy the relation A C B , where B = { a}.
5. Find the numbers of all subsets of the set containing the letter of the word “ BREAD”.

6. Which of the following is true?

   a) $\{0\} = \phi$ c) ϕ C$\{0\}$ e) ϕ C [}

   b) $\{\phi\} = \{0\}$ d) 0 C ϕ

7. If $A \subseteq B$ and $B \subseteq C$, is it true that $A \subseteq C$?
8. Below are given few pairs of sets. Name which of them are equal sets.
   a) A = the set containing the letters in the word "WOLF".
      B = the set containing the letters in the word "FOLLOW".
   b) A = the set of all vowels in the English alphabet.
      B = The set of all letters in the word " DIGIT".
9. If $A \subseteq B$ and $B \subseteq A$, then what can you say about sets A and B?

1.3 Operations on sets

Periods allotted: 10 periods

Competencies

At the end of this subunit, the students will be able to:

- determine the intersection of two given sets.
- determine the union of two given sets.
- use venn diagram to represent union and intersection of two sets.

Introduction

This subunit deals with operations on sets. For the purpose of presentation, the subunit is subdivided in to three topics: intersection of sets, union of sets, and the use of Venn diagram.

Teaching Notes

You are expected to ensure active participation of students by involving them in the activities and exercises provided in this sub unit.

1.3.1. The Intersection of Sets

Motivate students practice to determine intersection of two sets. To let students understand intersection of sets, you can let them do Activity 1.5. Help them do the examples in the student textbook and more questions of such a type.

Answers to Activity 1.5

1. 4 and 8 belong to both sets P and Q
2. A = {7, 14} and B = {1, 2, 4, 8}. Therefore, you see no element which belongs to both sets A and B.

Assessment

You may give problems on intersection of sets in the form of class work, home work, assignment, quiz or test in order to assess students' lever of understanding. You can also give the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

Let N=the set of natural numbers,

W=the set of whole numbers.

Determine $N \cap W$.

Answer to Additional Assessment

$N \cap W = N$

Answers to Exercise 1.C

1. a) $A \cap B = \{4, 8\}$

   b) Divisors of 10 = {1, 2, 5, 10}

   Divisors of 12 = {1, 2, 3, 4, 6, 12}

   $A \cap B = \{1, 2\}$

   c) Multiples of 3 less than 20 = {0, 3, 6, 9, 12, 15, 18}

   Multiples of 6 less than 20 = {0, 6, 12, 18}

   $A \cap B = \{0, 6, 12, 18\}$

   d) Even numbers less than 8 = {0, 2, 4, 6}

   Odd numbers less than 8 = {1, 3, 5, 7}

   $A \cap B = \varnothing$

2. In the case of (d), $A \cap B = \varnothing$
3. If A = B, then $A \cap B = A$ or $A \cap B = B$

1.3.2 The union of sets

Assist students practice to determine union of two sets. Let students do Activity 1.6 in order to understand union of sets.

Answers to Activity 1.6

1, 2, 3, 4 and 5 are elements which belong to either sets A or B.

Answers to Group Work 1.3

$P = A \cup B = \{2,4,6\}$

Subsets of P are:

$\phi, \{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}, \{2,4,6\}$

Assessment

You may give problems on union of sets in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also give the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

Let E= the set of even whole numbers,

D=the set of odd whole numbers,

M=the set of all multiples of 3.

Then determine

a) $E \cup D$

b) $(E \cap M) \cup (D \cap M$

Answers to Additional Assessment

a) The set of whole numbers

b) M

Answers to Exercise 1.D

1. a) $P \cup Q = \{2, 3, 4, 5, 6, 7\}$

   b) $P = \{11, 13, 15, 17, 19\}$, $Q = \{12, 14, 16, 18\}$

   $P \cup Q = \{11, 12, 13, 14, 15, 16, 17, 18, 19\}$

   c) $P \cup Q = \{a, b, c, d, e, 1, 2, 3, 4, 5\}$

   d) $P \cup Q$ = {Ayal, Alemu, Bekele, Chala, Derartu, Habtamu, Hagos, Mohammed}

e) $P = \{4, 9, 16, 25\}$, $Q = \{4, 9, 13, 16, 25\}$

$P \cup Q = \{4, 9, 13, 16, 25\}$

2. If $A = \varnothing$, then $A \cup B = B$
3. If $A = B$, then $A \cup B = A$ or $A \cup B = B$
4. a) $A \cup B = \{1, 3, 4, 5, 6, 7\}$

   b) $A \cup C = \{3, 4, 5, 8, 10, 12\}$

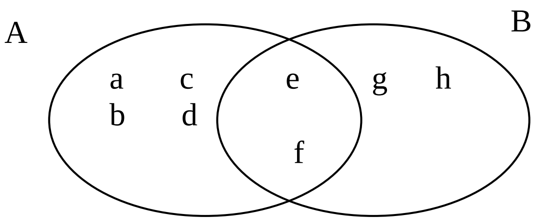
   c) $B \cup C = \{1, 3, 6, 7, 8, 10, 12\}$ d) $A \cap B = \{3\}$ (e) $\varnothing$

   f) $\varnothing$

1.3.3 Venn Diagram

Assist students to represent the intersection and union of two sets by using Venn diagram.

Help students to solve problems of intersection and union of sets from a given diagram. You may use examples like:



Find $A =$ ___________

$B =$ ___________

$A \cap B =$ ______________

$A \cup B =$ ______________

Figure 1.1

Ask students to determine the union and intersection of two sets and represent them by Venn diagrams.

Assist students to practice in solving simple word problems. You may use examples like:

In a certain school the members of mini media club are Jemal, Kebede, Almaz and Ujulu and the members of Anti – AIDS Club are Derartu, Jemal, Ujulu and Fatuma, then use Venn diagram to represent the situation.

Give simple word problems to solve.

Answers to Activity 1.7

$A \cap B = \{1, 3\}$

Answers to Group Work 1.4

d) $Q \cap R$

Assessment

You may give problems on Venn diagram as a form of class work, home work, assignment, quiz or test in order to assess students' progress. You can also give the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

Give an example of two sets which are disjoint.

Answer to additional assessment

Answer may vary.

e.g. A = The set of even numbers between 40 and 50.

B = The set of odd numbers between 40 and 50.

You can see that $A \cap B = \varnothing$

Answers to Exercise 1.E

1. a) $P = \{1,2,3,4,8, 9\}$ b) $Q = \{1,3,5,6,7,10\}$
   c) $P \cap Q = \{1,3\}$ d) $P \cup Q = \{1,2,3,4,5,6,7,8,9,10\}$ e) No
2. a) $A = \{4, 7, 8\}$ b) $B = \{4, 6, 7, 8, 9\}$ c) $A \cup B = B$
   d) $A \cap B = A$ e) Yes

3.

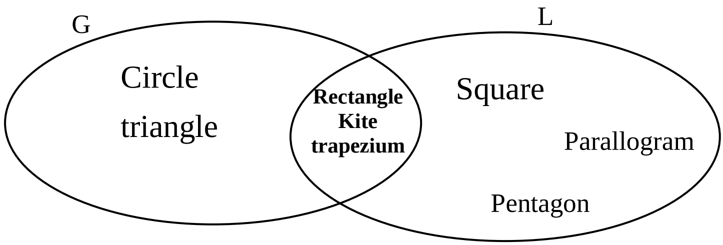


Figure 1.2

4.

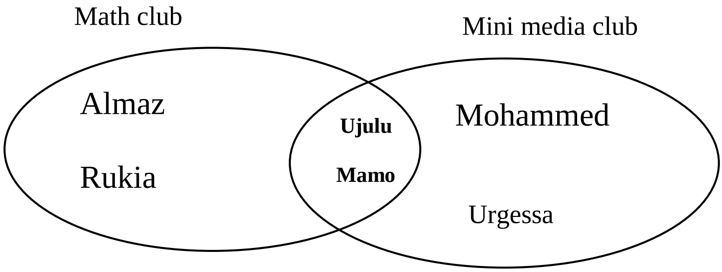


Figure 1.2

5.

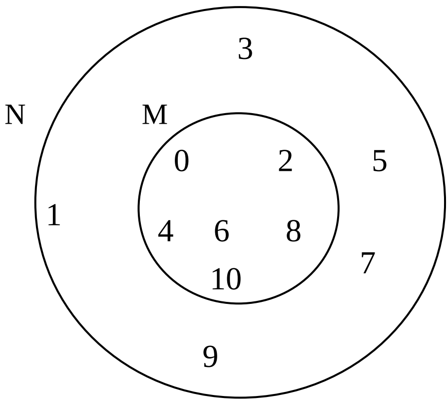


Figure 1.3

6. (a) 6 (b) 8 (c) 3 (d) 2

7.

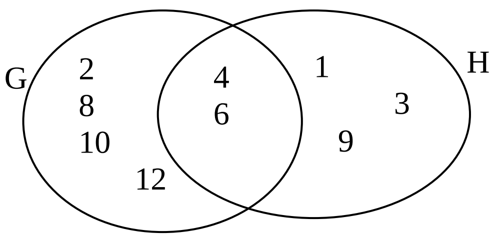


Figure 1.4

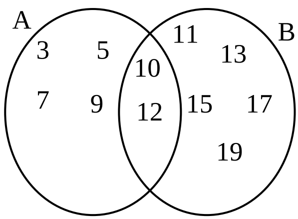
Assessment

In order to assess students' progress you can ask questions on representation of sets, elements of a set, subsets, proper subsets and operation on the sets in the form of group work, assignment, quiz or test. Check their work and keep record.

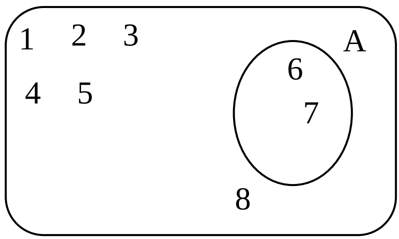
Diagnosing your students' level of understanding, you can give more exercises to help slow learners work hard, if any, and add further notes and more exercises for fast learners.

Selected problems to slow learners

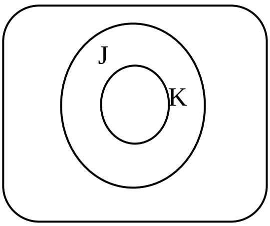
1. Based on the Venn diagram, list the element of each of the following sets.
   a) A
   b) B
   c) $A \cup B$
   d) $A \cap B$



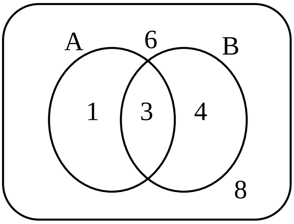
2. Draw a Venn diagram showing the relation between sets p and Q where P = { 1, 2, 3, 4} and Q = { 2, 4, 5}.
3. List all the elements of set A. Show in the Venn diagram given below which elements are not a member of set A?



4. Shade the region $J \cap K$.



5. Which of the following is false?

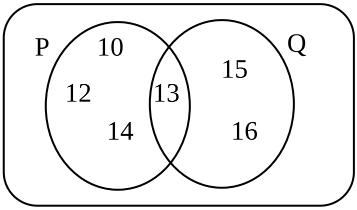


a) $1 \in A$ d) $3 \in A \cap B$

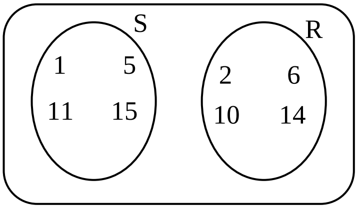
b) $3 \in A \cap B$ e) $6 \notin A$

c) $4 \notin A$

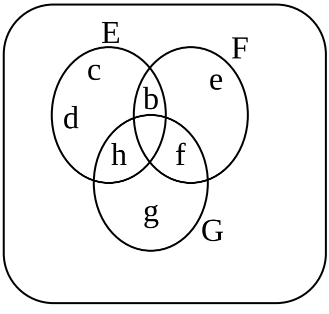
6. Write down the elements of $P \cup Q$ from the given Venn diagram shown at the right.



7. What is $S \cap R$ from the Venn diagram shown at the right.



8. What is $E \cup F \cup G$ from the Venn diagram shown at the right.

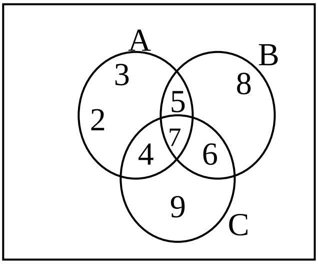


Selected problems to fast learners

1. Based on the Venn diagram list the elements of each of the following sets.

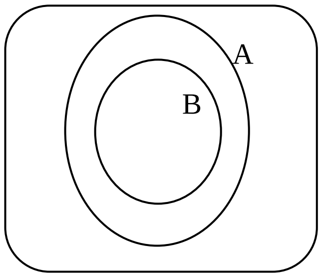
   a) $A \cap B$ c) $A \cap C$ e) $(A \cup B) \cup C$

   b) $B \cap C$ d) $A \cap (B \cap C)$



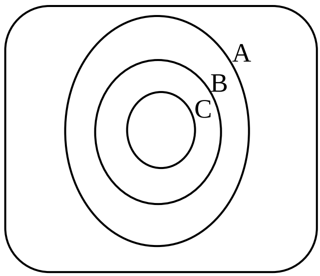
2. Give an example of sets which are disjoint. And also draw a Venn diagram for the disjoint sets.

3. Give examples of two sets A and B where the Venn diagram is like the one shown at the right



4. Draw a Venn diagram for two sets A and B where $A \cap B \neq \phi$.

5. Give examples of three sets A, B and C where the Venn diagram is like the one shown at the right.



6. Let A = { a, e, i, o, u}

   B = { 1, 2, 3, 4, 5}

   C = { 2, 4, 6, 8, 10}

   Then find $B \cap (A \cup C)$.

7. Draw a Venn diagram which shows the relation between the three sets A, B and C, where

   A = the set of multiples of 5 less than 11

   B = the set of odd numbers less than 11

   C = the set of multiples of 3 less than 11

Answers to review exercise

1. a) True b) False c) False d) False e) True

   f) True g) True h) True
2. a) Ø, {0}, {4}, {6}, {0, 4}, {0, 6}, {4, 6}, {0, 4, 6}

   b) Ø, {0}, {4}, {6}, {0, 4}, {0, 6}, {4, 6}
3. a) A= {1, 2, 3, 4, 5} c) AnB= {4, 5}

   b) B= {4, 5, 6, 7} d) AuB= {1, 2, 3, 4, 5, 6, 7}
4. A= {0, 7, 14, 21, 28,…..}, B={1, 2, 3, 5, 6, 10, 15, 30}

   a) Yes

   b) E.g. The set of multiples of 14.

   c) E.g. The set of divisors of 15.
5. P= {1, 3, 5, 7, 9, 11,……}

   Q= {0, 2, 4, 6, 8, 10,……}

   a) yes

   b) PnQ= Ø

   c) PuQ= the set of whole numbers.
6. a) A= {1, 2, 3, 4, 5} b) B= {3, 4, 7, 8, 9}

   c) C= {4, 5, 6, 7}

   d) AuB= {1, 2, 3, 4, 5, 7, 8, 9}

   e) AuC= {1, 2, 3, 4, 5, 6, 7}

   f) BuC= {3, 4, 5, 6, 7, 8, 9}

   g) AnB = {3, 4} h) BnC= {4, 7}

   i) AnC = {4, 5} j) Au (BuC) = {1, 2, 3, 4, 5, 6, 7, 8, 9}

   k) An (BnC) = {4} l) Au (BnC) = {1, 2, 3, 4, 5, 7}

   m) An (BuC) = {3, 4, 5}

7.

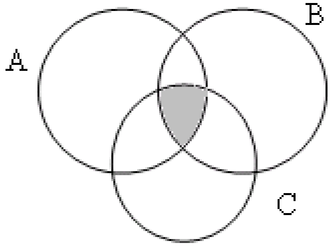


Figure 1.5

8. P={S,T,A,R}

   Total number of subsets of set P=16

9. a) $A\cup (B\cup C) = \{1,2,3,4,5,6,7,8,9\}$

   and $(A\cup B) \cup C=\{1,2,3,4,5,6,7,8,9\}$.

   Therefore, $A\cup (B\cup C)= (A\cup B) \cup C$

   b) $A\cap (B\cap C) =\phi$

   $(A\cap B) \cap C=\phi$

   Therefore, $A\cap (B\cap C)=(A\cap B)\cap C$

10. a) $A\cup B=\{x, w, v, r, u, y\}$

    b) $A\cap B=\{v, r\}$

    c) $A\cap (B\cup C)=\{v, r, u\}$

    d) $B\cap C=\{w, r\}$

    e) $A\cap B\cap C=\{r\}$

    f) $A\cup B\cup C=\{x, w, r, v, z, u, y\}$

    g) {t, s}

UNIT TWO

THE DIVISIBILITY OF WHOLE NUMBERS

Introduction

This unit begins by revising the test for divisibility discussed in earlier grade levels. Then it introduces prime and composite numbers. The unit also gives emphasis to methods of writing prime factorization of a given whole number. Each topic is presented by giving definitions and descriptive examples. The activities and exercises included in this unit are designed to involve and encourage students to think critically about the lessons presented.

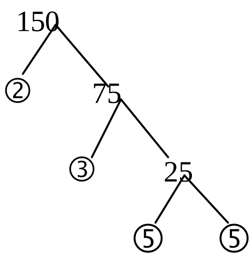
Unit outcomes

At the end of this unit, students will be able to:

- know the divisibility tests
- identify prime and composite numbers
- write prime factorization of a given whole number

Suggested Teaching Aids in Unit 2

You can present different charts containing prime numbers and composite numbers, and a model diagram for factor tree that demonstrate prime factorization of a whole number. You can also encourage students to prepare different charts by themselves.



$150=2\times3\times5^2$

2	18,24
3	9,12
2	3,4
2	3,2
3	3,1
	1,1

LCM(18,24)

$= 2^3\times3^2$

$= 72$

2.1 The notion of Divisibility

Periods allotted: 6 periods

Competency

At the end of this sub unit, students will be able to:

- identify whole numbers that are divisible by 2,3,4,5,6,8,9, and 10.

Introduction

This sub unit revises and discusses the tests for divisibility in more detail.

Teaching notes

You are expected to encourage students and ensure their active participation in the discussion of activities and exercises presented in this sub unit.

Tests for Divisibility

A natural number is divisible by:

Two if the last digit is even.

Example: 50,182 and 368 are divisible by 2.

Three if the sum of the digits is divisible by 3.

Example: 576 is not divisible by 3 because 5+7+6=18and 18 is divisible by 3.

425 is not divisible by 3 because 4+2+5=11 and 11 is not divisible by 3.

Four if the last two digits form a number which is divisible by 4.

Example:3728 is divisible by 4 because 28 is divisible by 4.

573 is not divisible by 4 because 73 is not divisible by 4.

Five if the last digit is 0 or 5.

Example: 420 and 4325 are divisible by 5 but 6362 is not divisible by 5.

Six if the number is divisible by both 2 and 3.

Example: 504 is divisible by 6 because:

i) it is divisible by 2 since the last digit, 4 is even.

ii) It is divisible by 3 because 5+0+4= 9 and 9 is divisible by 3.

Eight if it is even and the last three digits form a number which is divisible by 8.

Example; 41256 is divisible by 8 since 256 8 = 32

Nine if the sum of the digits is divisible by 9.

Example: 306 is divisible by 9 because 3+0+6=9 and 9 is divisible by itself.

1278 is also divisible by 9 because 1+2+7+8=18 and 18 is divisible by 9.

Ten if the last digit is 0.

Example: 10 and 10 are divisible by 10

Eleven if the difference between the sum of digits in the odd place and the sum of the digits in the even places is 0 or divisible by 11.

Example: 438,922 is divisible by 11 because 2+9+3=2+8+4= 14 and 14 – 14 = 0.

Twelve if it is divisible by 3 and 4.

Example: 108 is divisible by 12 since it is divisible by 3 and 4.

Fifteen if it is divisible by 3 and 5.

Example: 105 is divisible by 15 since it is divisible by 3 and 5.

Eighteen if it is divisible by 2 and 9.

Example: 324 is divisible by 18 since it is divisible by 2 and 9.

Twenty if it is divisible by 4 and 5

Example: 180 is divisible by 20 since it is divisible by 4 and 5.

Note: Any natural number is divisible by 1.

Answers to Activity 2.1

a) 2781 ÷ 3 = 927. Thus 2781 is divisible by 3.

b) 7020 ÷ 3= 2340. Thus 7020 is divisible by 3.

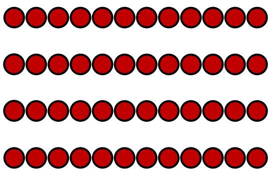
c)

$$\begin{array}{r|l} & \;\;352 \\ \hline 3 & 10,561 \\ & -9 \\ \hline & \;\;15 \\ & -15 \\ \hline & \;\;\;\;6 \\ & \;\;\;\;6 \\ \hline & \;\;\;\;1 \end{array}$$

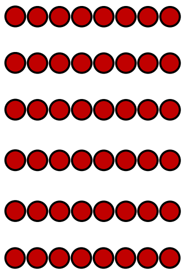
10, 561= 352 x3 +1

you can see that the remainder in this division is different from 0. Thus 10,561 is not divisible by 3.

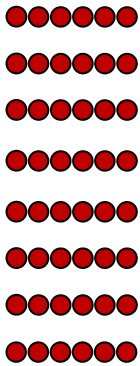
Answers to Activity 2.2

1. A number that is divisible by 10 has units' digit 0. Thus, it is also divisible by 5.
2. 

48÷ 4 = 12



48÷6 = 8



48÷ 8 = 6

3. When the sum of the digits of a number is divisible by 3 but not by 9. Eg. 60 or 21

Answers to Group Work 2.1

56

Assessment

You may give problems on divisibility of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess students level of understanding. You can also give students the following questions to answer as additional assessment.

Additional assessment

Identify each of the following statements as true or false.

a) 243,852 is divisible by 6.
b) 17040 is divisible by 5.
c) If a whole number is divisible by 17, then it is also divisible by 34.
d) If a whole number is divisible by 42, then it is also divisible by 21.

Answers to additional assessment

a) True b) True c) False d) True

Answers to Exercise 2.A

Number	Divisible by							
	2	3	4	5	6	8	9	10
178,620	✓	✓	✓	✓	✓			✓
6,348,025				✓				
179,600	✓	✓	✓	✓		✓		✓
163,245		✓		✓				
118,224	✓	✓	✓		✓	✓	✓	
712,800	✓	✓	✓	✓	✓	✓	✓	✓
125,046	✓	✓			✓		✓	

2. a) False b) True c) True d) True e) False f) False
g) True h) False i) True j) True

3. 0, 3, 6 and 9

Selected Problems to slow learners

1. Find a two digit number divisible by 2.
2. Find a three digit number divisible by 3.
3. Find a four digit number divisible by 10.
4. Find a five digit number divisible by both 5 and 10.
5. Is 64,926 divisible by 3?
6. Is 456,705 divisible by 5?

7. Is 63,496 divisible by 8?
8. Is 473, 603 divisible by 10?
9. Are the following divisible by 4?
   63 , 92, 104, 672
10 Are all whole numbers divisible by 2?
11. Give an example of a number which is divisible by both 2 and 4

Selected Problems to fast learners

1. Find a four digit number divisible by both 2 and 3.
2. Find a six digit number divisible by both 3 and 10.
3. Find a seven digit number divisible by both 2 and 5.
4. Is 816, 483 divisible by 9?
5. Is 87 ,456 divisible by 11?
6. Is 3, 141, 732 divisible by both 2 and 4?
7. Find in the places marked $\square$ in the number 9 $\square$ 72 such that the number is divisible by 9.
8. Give an example of a three digit number which is divisible by 12.
9. Give an example of a four digit number which is divisible by 18.

2.2 Multiples and Divisors

Periods allotted: 17 periods

Competencies

At the end of this sub unit, students will be able to:

- identify prime and composite numbers.
- write the prime factorization of a given whole number.
- explain the concept of common divisor, greatest common divisor (GCD) of two whole numbers.
- identify relatively prime numbers.
- determine the common and the least common multiple (LCM) of two whole numbers.
- explain the concept of common multiples of two given whole numbers.

- determine the least common multiple (LCM) of two or three natural numbers with one or two digit numerals.

Introduction

This sub unit discusses about multiples and divisors. Definitions of prime numbers, composite numbers, relatively prime numbers and the method of writing prime factorization of a whole number is presented with descriptive examples. Activities and exercises are included in order to involve students in solving problems in this sub unit.

Teaching notes

In order to present this sub unit, the sub unit is sub divided in to four topics. You are expected to encourage and involve students in the discussion of each topic in the sub unit.

2.2.1 Revision on Multiples and divisors

You may start the lesson by revising how to find multiples and divisors of a given whole number. Encourage students to find multiples and divisors of a given whole number.

Answers to Group Work 2.2

36

Answers to Activity 2.3

a) 1×6, 2×3 b) 1×16, 2×18, 4×4 c) 1×19

d) 1×36, 2×18, 3×12, 4×9, and 6×6

Assessment

You can give problems on multiples and divisors in the form of class work, home work, assignment, quiz or test in order to assess students' performance. You can also give the following problems to students to answer as additional assessment.

Additional Assessment

Find a) multiples of 19 which are found between 60 and 100.

b) divisors of 46.

Answers to additional assessment

a) 76 and 95.

b) 1,2,23 and 46.

Answers to Exercise 2.B

1. a) 0, 7, 14, 21, 28 and 35
   b) 0,9,18,27,36 and 45
   c) 0, 13, 26, 39, 52 and 65
   d) 0, 20, 40, 60, 80 and 100
   e) 0, 32, 64, 96, 128, 160

2. a) 1, 2, 3, 5, 6, 10, 15 and 30
   b) 1 and 37
   c) 1, 2, 4, 5, 8, 10, 20 and 40
   d) 1 and 19
   e) 1, 3, 5, 9, 15, and 45
   f) 1,2,5,10,25 and 50
   g) 1, 2, 4, 8, 16, 32 and 64

2.2.2 Prime and composite numbers and prime Factorization

You may start the lesson by revising how to find multiples and divisors of a given whole number.

After introducing what prime and composite numbers are, encourage students to identify some prime and composite numbers.

Assist students to express a given whole number as product of its prime factors (complete factorization). You may use the factor tree methods.

Give problems by asking to list all the divisors and some of the multiples of a given number.

Motivate students to differentiate prime numbers from composite numbers.

Answers to Activity 2.4

1. a) prime b) composite c) prime
2. b) $75 = 3 \times 5^2$
3. a) $27=3^3$ b) $28 = 2^2 \times 7$
   c) $52 = 2^2 \times 13$ d) $108 = 2^2 \times 3^3$

Assessment

You can give problems on identifying prime and composite numbers, and prime factorization of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding of the lesson. You can also give the following problems to students to answer as additional assessment.

Additional Assessment

Which of the following whole numbers are prime?

a) 970 b) 37 c) 102 d) 117

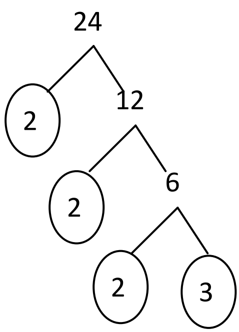
Answer to additional assessment

b) 37

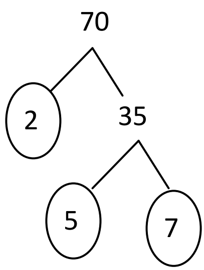
Answers to Exercise 2C

1. b) False b) True c) True d) True e) True
   f) True g) False

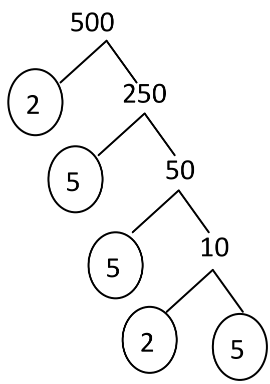
2. (a)



(b)



(c)



3. a) $100 = 2^2 \times 5^2$ b) $144 = 2^4 \times 3^2$

c) $150 = 2 \times 3 \times 5^2$ d) $225 = 3^2 \times 5^2$

e) $300 = 3 \times 2^2 \times 5^2$

4. a) 19 b) 29 c) 37

5. a) 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38 and 39

b) 83, 89 and 97

6. 2 and 3

7. Colour the region containing numbers 2, 3, 5, 7, 13, 17, 23,31,37,41,43 and 47

8. a) composite b) prime

c) composite d) composite

9. 5

10. n = 11

11. 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

2.2.3 Common Divisors

Discuss the concepts of common divisors, greatest common divisor of two whole numbers and relatively prime numbers. Help students to determine the common divisor and greatest common divisor and greatest common divisor (G.C.D) of two whole numbers.

You may use examples like: Determine the GCD of 20 and 30.

Divisors of 20 = {1, 2, 4, 5, 10, 20}

Divisors of 30 = {1, 2, 3, 5, 6, 10, 15, 30}

Common divisors of 20 and 30 = {1, 2, 5, 10}

GCD of 20 and 30 = 10

Answers to Group Work 2.3

a) 2,3 and 5 c) 2 and 3

b) 2,3 and 7 d) 6

Answers to Activity 2.5

8

Assessment

You can give problems on common divisors in the form of class work, home work, assignment, quiz or test in order to assess students performance. You can also give the following problems to students to answer as additional assessment.

Additional Assessment

Find the GCD of

a) 40, 60 and 80
b) 25, 60 and 75
c) 75, 100 and 150

Answers to additional assessment

a) 20 b) 5 c) 25

Answers to Exercise 2.D

1. a) Factors of 24 = 1,2,3,4,6,8,12 and 24.
   Factors of 30 = 1,2,3,5,6,15 and 30
   Common factors of 24 and 30 = 1, 2, 3 and 6
   b) Factors of 60 = 1,2,3,4,5,6,10,12,15,20,30 and 60. Factors of 80= 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80. Common factors of 60 and 80= 1, 2, 4, 5, 10 and 20.
   c) Factors of 32 = 1, 2, 4, 8, 16 and 32. Factors of 48 = 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.
   Common factors of 32 and 48 = 1, 2, 4, 8 and 16.
   d) Factor of 35 = 1, 5,7 and factors of 57 = 1,3,19 and 57. Common factors of 35 and 57 = 1. Thus 35 and 57 relatively prime.

2. a) 5 b) 16 c) 3 d) 10 e) 30
3. a and d
4. 15 5. 102 6. Answers may vary. Eg. 28 and 56
7. 8 and 9

2.2.4 Common multiples

After introducing the concept of common multiple, assist students to determine the least common multiple (LCM) of two or three whole numbers by considering numbers that have one or two digits.

Answers to Activity 2.6

i) Multiples of 5

= {0, 5, 10, 15, 20, 25, 30, 35, …}

Multiples of 6

= {0, 6, 12, 18, 24, 30, 36, …}

ii) Common multiples of 5 and 6

= {0, 30, 60, 90, …}

LCM = 30

Answers to Group Work 2.4

4 or 8 people

Answers to Exercise 2.E

1. a)

2	10,16
5	5,8
2	1,8
2	1,4
2	1,2
	1,1

LCM of 10 and 16 is $2^4 \times 5 = 80$

b)

2	12,18
3	6,9
2	2,3
3	1,3
	1, 1

LCM of 12 and 18 is $2^2 \times 3^2 = 36$

c)

2	24,60
2	12,30
3	6,15
2	2,5
5	1,5
	1, 1

LCM of 24 and 60 is $2^3 \times 3 \times 5 = 120$

d)

2	32, 40, 50
2	16, 20, 25
2	8, 10, 25
2	4, 5, 25
2	2, 5, 25
5	1, 5, 25
5	1, 1, 5
	1, 1, 1

LCM of 32, 40 and 50 = $2^5 \times 5^2 = 800$

e)

2	24, 30, 45
3	12, 15, 45
2	4, 5, 15
2	2, 5, 15
5	1, 5, 15
3	1, 1, 3
	1, 1, 1

LCM of 24, 30 and 45 = $2^3 \times 3^2 \times 5 = 360$

2. GCD = = 4

3. 18 × = (72) (12)

$\therefore$ x = 48

4. Yes

5. (x) (2x) = (9) (18)

(2xx) = 2(9) (9) $\therefore$ x = 9 and the two numbers are 9 and 18.

6. 4 and 9
7. After 2hrs (i.e. at 11: 00am)
8. When one number is a multiple of the other.
9. When their GCD is 1.
10. Answers may vary. eg. 2, 3 and 5

Assessment

You can give students additional exercise on divisibility, writing prime factorization, and finding GCD and LCM of two or three whole numbers as group work and let them present their work. As this is the end of the unit you can also give quiz or test encompassing every topic of the unit.

Selected problems to slow learners

1. Find the first seven multiples of 2.
2. Find all divisors of 20.
3. Write the prime factorization of 100.
4. Find all prime numbers between 10 and 20.
5. Find all composite numbers between 7 and 23.
6. Find all two digit prime numbers between 20 and 40.
7. Find all factors of 70.
8. Find common divisors of 20 and 30.
9. Are 15 and 16 relatively prime?
10. The product of two numbers is 16. Their LCM is 8. Find their GCD.
11. The GCD of a certain number and 7 is 1. If their LCM is 21, find the other number.

Selected problems to fast learners.

1. Find all prime numbers between 120 and 150.
2. Find LCM (250, 400).
3. Find GCF (90, 120).
4. Find the largest prime number less than 100.
5. Find all prime numbers between 180 and 200.
6. Find all factors of 400.

7. Find common divisors of 300 and 500.
8. Are 20 and 21 relatively prime?
9. The product of two numbers is 320 . Their LCM is 80. Find their GCD.
10. The GCD of a certain number and 120 is 10. If their LCM is 840, find the other number.

Answers to review exercise

1. a) True b) False c) True d) True e) True
   f) True g) False
2. a) 120 b) 250 c) 1 d) 12
3. a) 3,000 b) 6,000 c) 27,000 d) 1, 184,000
4. 5
5. 300
6. 105 and 56 have GCD of 7
7. B
8. a) v c) vii e) iv
   b) iii d) i f) ii g) vi

9. All prime numbers less than 10 are 2, 3, 5 and 7
   The required number is the product of all these prime numbers. Thus, the required number
   $= 2\times3\times5\times7$
   $= 210$
10. If a number is divisible by 6, then it is also divisible by 2 and 3
    - If a number is divisible by 10, then it is also divisible by 2 and 5
    - If a number is divisible by 8, then it is also divisible by 2 and 4
    - I f a number is divisible by 9, then it is also divisible by 3.
      Thus LCM (10, 8, 9, 76) is the smallest number that is divisible by 2, 3, 4, 5, 6 7, 8, 9 and 10

LCM (10, 8, 9, 7, 6) = 2520

11. The number assumed
    $= 7\times 11\times 13$
    $= 1001$
12. Size of carton he should order = 18 because 126 is divisible by 18. But 126 is not divisible by 8. If he were to order cartons which contain 8 eggs, he will be left over with 6 eggs.

UNIT THREE

FRACTIONS AND DECIMALS

Introduction

The main task of this unit is to extend and deepen the knowledge and capability of the students about fractions and decimals by systematically dealing with fractions and decimals representation of numbers, ordering and operating with fractions and decimals to enable students to use the newly gained knowledge, abilities and skills for solving simple problems represented by fractions and decimals. For this purpose, each topic in this unit is presented by giving definitions and descriptive examples.

The activities and exercises given in each sub unit are designed to encourage students' participation in the discussion of fractions and decimals.

Unit Out comes

At the end of this unit, students will be able to:

- Understand fractions and decimals and realize that there are two ways to represent the same numbers.
- Develop skill in ordering, adding, subtracting, multiplying and dividing fractions and decimals.
- Work with problems represented by fraction and decimals.

Suggested Teaching Aids in Unit 3

You can present different charts showing place value and shaded regions representing fractions. You can also encourage students to prepare different representative diagrams of fractions, and percentage. Apart from the use of the students' text book, you need to elaborate real life problems from your surroundings so that students can best appreciate how useful fractions and decimals are.

Place Value Chart

Trillions			Billions			Millions			Thousands			Ones			Tenths	Hundredths	Thousandths
Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Ten	Ones	One tenths	One hundredths	One thousands

The place Value of each digit in the number 379.468

3 7 9 . 4 6 8

Hundreds (100)

Tens(10)

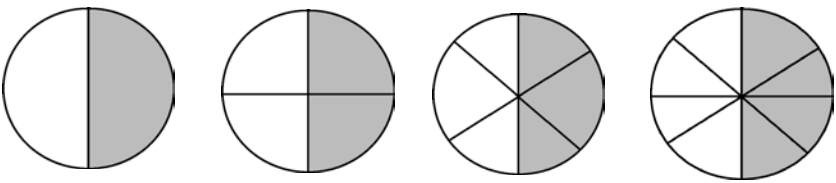
Ones (1)

Decimal point

Tenths $\left(\frac{1}{10}\right)$

Hundredths $\left(\frac{1}{100}\right)$

Thousandths $\left(\frac{1}{1000}\right)$



3.1 The simplification of Fractions

Periods allotted: 5 periods

Competency

At the end of this sub unit, students will be able to:

- reduce fractions to lowest terms.

Introduction

This sub unit begins by revising what students have learnt about fractions in their previous mathematics lessons, and then discusses the method of simplifying fractions.

Teaching notes

You can start by revising about fractions. Assist students to practice simplification of fractions by factorizing both the numerator and denominator of a given fraction. You may use the concept of GCD for reducing fractions to lowest terms.

Make sure students understand the simplification of fractions. Encourage students to do class Activity 3.1 by themselves. Give problems on reducing fractions to lowest term.

Answers to Activity 3.1

1. numerator =6, denominator = 55

2. proper fractions = $\frac{1}{10}$, and $\frac{12}{13}$

   Improper fractions = $\frac{7}{6}$, $\frac{22}{19}$, $\frac{8}{8}$ and $\frac{10}{1}$

   Mixed numbers = $3\frac{1}{2}$ and $33\ \frac{5}{6}$

3. $\frac{160}{9} = 17\ \frac{7}{9}$ because

4. $\frac{4}{7} = \frac{4 \times 4}{7 \times 4} = \frac{16}{28}$ Thus, x= 16

5. $9\ \frac{5}{8} = \frac{(9x8)+5}{8} = \frac{72+5}{8} = \frac{77}{99}$

$$\begin{array}{r|l} & \ \ 17 \\ \hline 9 & 160 \\ & \ \ 9 \\ \hline & \ \ 70 \\ & \ \ 63 \\ \hline & \ \ \ \ 7 \end{array}$$

6. $\frac{50 \div 50}{100 \div 50} = \frac{1}{2}$

Assessment

You can give problems on simplification of fractions in the form of class work, home work, assignment, quiz or test in order to assess the students' level of understanding of this sub unit. You can also give the following questions to fast learner or interested students to answer as additional assessment.

Simplify

a) $\frac{1000}{2500}$ b) $\frac{272}{384}$ c) $\frac{126}{189}$

Answers to additional assessment

a) $\frac{2}{5}$ b) $\frac{17}{24}$ c) $\frac{2}{3}$

Answers to Exercise 3.A

a) GCD (4, 16) = 4, thus $\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$

b) GCD (8, 12) = 4, Thus $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

c) GCD (45, 20) = 5, thus $\frac{45}{20} = \frac{45 \div 5}{20 \div 5} = \frac{9}{4}$

d) GCD (72, 48) = 24, thus $\frac{72}{48} = \frac{72 \div 24}{48 \div 24} = \frac{3}{2}$

e) GCD (128, 224) = 32, thus

$\frac{128}{224} = \frac{128 \div 32}{224 \div 32} = \frac{4}{7}$

f) GCD (28, 140) = 28, thus $\frac{28}{140} = \frac{28 \div 28}{140 \div 28} = \frac{1}{5}$

g) GCD (128, 384) = 128, thus $\frac{128}{384} = \frac{128 \div 128}{384 \div 128} = \frac{1}{3}$

h) GCD (2160,270) = 270 = 270, thus $\frac{2160}{270} = \frac{2160 \div 270}{270 \div 270} = \frac{8}{1}$ =8

Selected problems to slow learners

1. Write each of these fractions in simplest form.

   a) $\frac{4}{20}$ c) $\frac{5}{25}$ e) $\frac{50}{150}$

   b) $\frac{8}{32}$ d) $\frac{20}{60}$

2. Is $\frac{6}{7}$ equivalent to $\frac{30}{55}$?

3. Show that the given fractions are equivalent.

   a) $\frac{2}{6}$ and $\frac{1}{3}$ b) $\frac{33}{39}$ and $\frac{11}{13}$ c) $\frac{4}{8}$ and $\frac{1}{2}$

4. Find x if $\frac{1}{2} = \frac{x}{10}$

5. Reduce to the lowest term.

   a) $\frac{2}{10}$ b) $\frac{20}{30}$ c) $\frac{75}{100}$ d) $\frac{100}{200}$

6. Express 1 , 2, 3 and 4 as fractions whose denominator is 6.

7. Express 5, 7, 9 and 11 as fractions whose denominator is 8.

8. Write any two equivalent fractions to each of the following fractions.

   a) $\frac{1}{7}$ b) $\frac{2}{9}$ c) $\frac{4}{13}$ d) $\frac{5}{7}$

9. Find x if $\frac{1}{4} = \frac{25}{x}$

Selected problems to fast learners

1. Write each of these fractions in simplest form.

   a) $\frac{6}{216}$ c) $\frac{18}{909}$ e) $\frac{15}{3000}$

   b) $\frac{3}{243}$ d) $\frac{5}{750}$

2. Determine whether the fractions in each pair are equivalent.

a) $\frac{2}{3}$ and $\frac{8}{9}$ c) $\frac{5}{9}$ and $\frac{25}{45}$

b) $\frac{7}{25}$ and $\frac{6}{20}$ d) $\frac{5}{11}$ and $\frac{45}{99}$

3. Find x if $\frac{2}{5} = \frac{x}{540}$.

4. Reduce to the lowest term.

a) $\frac{6}{42}$ c) $\frac{1150}{3500}$ e) $\frac{1512}{1764}$

b) $\frac{150}{270}$ d) $\frac{7500}{12500}$

5. Express 5, 7, 12, and 14 as fractions whose denominator is 8.
6. Express 3, 8, 15 and 21 as factions whose denominator is 13.
7. Write any two equivalent fractions to each of the following fractions.

a) $\frac{3}{13}$ b) $\frac{7}{11}$ c) $\frac{18}{21}$ d) $\frac{52}{3}$

8. Find x if $\frac{15}{18} = \frac{20}{x}$.

3.2 The conversion of Fractions, decimals and percentage

Periods allotted:10 periods

Competencies

At the end of this sub unit, students will be able to:

- convert fractions to decimals and percentages.
- convert terminating decimals to fractions and percentages.
- convert percentages to fractions and decimals.

Introduction

In this sub unit students will be introduced to conversion of fractions, decimals and percentages. The presentation of this sub unit is sub divided in to three topics: Conversion of fractions to decimals and percentages, conversion of decimals to fractions and percentages, and conversion of percentages to fractions and decimals. You are expected to ensure active participation of students in the discussion of the sub unit.

Teaching notes

In order to begin, it is better to motivate students by highlighting the topics of this sub unit. Then you can continue to discuss the three topics.

3.2.1 Conversion of fractions to decimals and percentages

You may start the lesson by introducing the concept of rounding off decimals (terminating and repeating), then assist students to convert fractions in to decimals and percentages using long division and give their answer by percentages using long division and give their answer by rounding to two or three decimal places. Motivate students to do exercise on conversion of fractions to decimals and percentages. For example by considering the number of students in a class, check if students can determine the number of boys and girls as fraction, as decimal and as percentage.

Answers to Activity 3.2

1. a) True b) True c) True

   d) False, because $\frac{4}{50} = \frac{4\times 2}{50\times 2} = \frac{8}{100} = 0.08$

   e) True f) False, because $\frac{6}{100} = 0.06$

2. a) $\frac{2}{5} = \frac{2\times 3}{5\times 3} = \frac{6}{15}$ and $\frac{1}{3} = \frac{1\times 5}{3\times 5} = \frac{5}{15}$

   6 > 5 implies that $\frac{6}{15} > \frac{5}{15}$ Thus $\frac{2}{5} > \frac{1}{3}$

   b) $\frac{5}{6} = \frac{5\times 3}{6\times 3} = \frac{15}{18}$ and $\frac{7}{9} = \frac{7\times 2}{9\times 2} = \frac{14}{18}$ as GCD of 6 and 9 is 18. You know that 15 > 14 and $\frac{15}{18} > \frac{14}{18}$

   Thus $\frac{5}{6} > \frac{7}{9}$

   c) GCD of 8 and 7 is 56

   Now $\frac{3}{8} = \frac{3\times 7}{8\times 7} = \frac{21}{56}$ and $\frac{4}{7} = \frac{4\times 8}{7\times 8} = \frac{32}{56}$

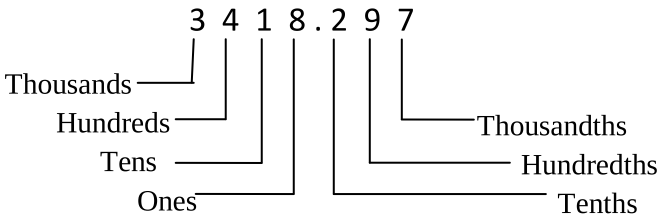
   21< 32 and $\frac{21}{56} < \frac{32}{56}$ Thus $\frac{3}{8} < \frac{4}{7}$

Answers to Group Work 3.1

a) thousandths, 3 thousandths

b) ten-thousandths, 2 ten-thousandths

Answer to Activity 3.3



Answer to Activity 3.4

a) A discount of 30%

b) A discount of $\frac{1}{10} = \frac{1 \times 10}{10 \times 10} = \frac{10}{100} = 10\%$

c) A discount of 25%

d) A discount of 20%

e) A discount of $\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{20}{100} = 20\%$

f) A discount of $\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$

Therefore (d) and (f) show the same discount

Answers to Group Work 3.2

$$\frac{2}{5} \times 200 = 80$$

Kemal's new clothes cost =Birr 80

200-80=120

$$\frac{120}{200} \times 100\% = 60\%$$

Kemal is left with 60% of his money.

Assessment

You can give problems on conversion of fractions to decimals and percentage in the form of class work, homework, assignment, quiz or test in order to assess students' performance. You may also give the following problems to fast learners or interested students as additional assessment.

Additional Assessment

Convert the following fractions to decimals

a) $\frac{9}{500}$ b) $\frac{31}{20}$ c) $\frac{37}{50}$

Answers to additional assessment

a) 0.018   b) 1.55   c) 0.74

Answer to Exercise 3.B

1. a) $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$ (terminating)

   b) $\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 0.40$ (terminating)

   c) $\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$ (terminating)

   d) $\frac{11}{32} = \frac{11}{32} \text{ x } \frac{100}{100} = \frac{1100}{32}\% = 34.375\% = 0.34375$ (terminating)

   e) $\frac{8}{125} = \frac{8}{125} \times \frac{100}{100} = \frac{800}{125}\% = 6.4\% = 0.064$ (terminating)

   f) $\frac{5}{6} = \frac{5}{6} \text{ x } \frac{100}{100} = \frac{500}{6}\% = 83.333\ldots\% = 83.\dot{3}\%$ (repeating)

   g) (repeating)

   h) $\frac{5}{9} = \frac{5 \times 100}{9 \times 100} = \frac{500}{9}\% = 55.555...\% = 55.\dot{5}\% = 0.56$ (repeating)

   i) $\frac{20}{99} = \frac{20 \times 100}{99 \times 100} = \frac{2000}{99}\% = 20.2020...\% = 0.20$ (repeating)

2. a) 75% b) 40% c) 28% d) 34.375%

e) 6.4% f) 83.3% g) $118.\overline{18}$ % h) $55.\dot{5}$%

i) 20.2%

3.

Decimal	One decimal place	Two decimal places	Three decimal places
0.121212…	0.1	0.12	0.121
2.3636…	2.4	2.36	2.364
4.257257…	4.3	4.26	4.257

4. a) $\frac{13}{25}$ b) $\frac{12}{25}$

c) $\frac{13}{25} = \frac{13\times 4}{25\times 4} = \frac{52}{100} = 52\%$

d) $\frac{12}{25} = \frac{12\times 4}{25\times 4} = \frac{48}{100} = 48\%$

5. a) Belaynesh's Score= $\frac{14}{25}$

Tewabech's score = $\frac{10}{40}$

Awol's Score = $\frac{23}{50}$

b) percentage of Belaynesh's score= $\frac{14}{25} = \frac{14\times 4}{25\times 4} = \frac{56}{100} = 56\%$

Percentage of Tewabech's score =

$\frac{10}{40} = \frac{10}{40}\times\frac{100}{100} = \frac{1000}{40}\% = 25\%$

Percentage of Awol's score = $\frac{23}{50} = \frac{23\times 2}{50\times 2} = \frac{46}{100} = 46\%$

c) Tewabech's score, Awol's score, Belayneh's score (from the lowest to the highest) because 25% < 46% < 56%

3.2.2 Conversion of Terminating Decimals to Fractions and percentages

Encourage students to practice on the method of multiplying and dividing decimals by powers of 10. You may give example as follows:

Convert 0.25 and 0.5 to fraction and percentage

$$0.25 = 0.25 \times \frac{100}{100} = \frac{25}{100} = 25\% = \frac{1}{4}$$

$$0.5 = 0.5 \times \frac{100}{100} = \frac{50}{100} = 50\% = \frac{1}{2}$$

Motivate students to practice on conversion of repeating decimals to fractions and percentages.

Answers to Activity 3.5

$$\frac{1}{50} = \frac{1 \times 2}{50 \times 2} = \frac{2}{100} = 0.02$$

Answers to Group Work 3.3

$$\frac{20}{800} \times 100\% = 2.5\% = 0.025$$

Assessment

You can give problems on converting terminating decimals to fractions and percentages in the form of class work, homework, assignment, quiz or test in order to assess students' level of understanding of the lesson.

Answers to Exercise 3.c

a) $0.36 = \frac{36}{100} = 36\%$

$0.36 = \frac{36}{100} = \frac{36 \div 4}{100 \div 4} = \frac{9}{25}$

b) $0.82 = \frac{82}{100} = 82\%$

$0.82 = \frac{82}{100} = \frac{82 \div 2}{100 \div 2} = \frac{41}{50}$

c) $0.23 = \frac{23}{100} = 23\%$

d) $0.465 = \frac{465}{1000} = \frac{46.5}{100} = 46.5\%$

e) $0.032 = \frac{32}{1000} = \frac{3.2}{100} = 3.2\%$

$0.032 = \frac{32}{1000} = \frac{32 \div 8}{1000 \div 8} = \frac{4}{125}$

f) $0.345 = \frac{345}{1000} = \frac{34.5}{100} = 34.5\%$

$0.345 = \frac{345}{1000} = \frac{345 \div 5}{1000 \div 5} = \frac{69}{200}$

g) $0.751 = \frac{751}{1000} = \frac{75.1}{100} = 75.1\%$

h) $0.259 = \frac{259}{1000} = \frac{25.9}{100} = 25.9\%$

i) $1.25 = \frac{125}{100}$

$= 125\%$

j) $24.3 = \frac{243}{10}$

$= 2430\%$

3.2.3 Conversion of percentage to Fractions and Decimals

Assist students to practice conversion of percentage to fraction. Make sure that students are able to convert percentage to fractions and decimals.

Answers to Activity 3.6

a) 50% b) 25% c $\frac{100}{3}\%$ d)60%

b) 40% f) $\frac{250}{3}\%$

Answers to Group Work 3.4

$$\frac{90{,}000}{250{,}000} \times 100\% = 36\%$$

Assessment

You can give problems on converting percentage to fractions and decimals in the form of class work, home work, assignment, quiz or test in order to assess students' performance.

You can also give the following problems to fast learners or interested students to answer as additional assessment.

Additional Assessment

Convert the following percentage to decimals

1. a) 3.5% b) 0.431% c) 27.15%

Answers to additional assessment

a) 0.035 b) 0.00431 c) 0.2715

Answers to Exercise 3.D

1. a) $15\% = \frac{15}{100} = \frac{3}{20}$

$$15\% = \frac{15}{100} = 0.15$$

b) $28\% = \frac{28}{100} = \frac{7}{25}$

$$28\% = \frac{28}{100} = 0.28$$

c) $72\% = \frac{72}{100} = \frac{18}{25}$

$$72\% = \frac{72}{100} = 0.72$$

d) $2\frac{1}{4}\% = \frac{9}{4}\% = \frac{9}{400}$

$$2\frac{1}{4}\% = \frac{9}{4} = \frac{2.25}{100} = \frac{225}{10000} = 0.0225$$

e) $8\frac{1}{2}\% = \frac{17}{2}\% = \frac{17}{200}$

$8\frac{1}{2}\% = \frac{17}{2}\% = \frac{8.5}{100} = \frac{85}{1000} = 0.085$

f) $12\frac{1}{6}\% = \frac{73}{6}\%$

$= 12.\dot{6}\% \quad = 0.121\dot{6}$

2.

Fraction	Decimal	Percent
$\frac{2}{5}$	0.40	40%
$\frac{2}{25}$	0.08	8%
$\frac{1}{25}$	0.04	4%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%
$3\frac{3}{5}$	3.6	360%

Selected problems to slow learners

1. Identify whether each of the fractions given below represent terminating or repeating decimal.

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{4}{9}$

2. Compare

$\frac{1}{2}, \frac{1}{5}$ and $\frac{1}{4}$

3. Write each percentage as fraction in simplest form.

   a) 36% b) 24% c) 70% d) 15%

4. Write each percentage as a decimal.

   a) 26% b) 32% c) 44% d) 77%

5. Write each fraction as a percentage.

   a) $\frac{8}{10}$ b) $\frac{9}{15}$ c) $\frac{5}{24}$ d) $\frac{3}{80}$

6. Compare using < , > or =

   a) 38% $\square$ 0.38 c) 86% $\square$ $\frac{43}{50}$

   b) 45% $\square$ 0.46 d) $\frac{9}{20}$ $\square$ 40%

7. Which inequality is a true statement?

   a) $13\% > \frac{1}{2}$ c) $4\% < 0.4$

   b) $0.83 < 83\%$ d) $\frac{1}{5} < 2\%$

8. Which of the following is equal to $50\frac{2}{3}\%$?

   a) $\frac{38}{75}$ b) $\frac{8}{3}$ c) $\frac{4}{3}$ d) $\frac{5}{3}$

9. Write " 5 out of 50" as percentage.

Selected problems to fast learners

1. Compare

   $\frac{5}{6}, \frac{3}{5}$ and $\frac{5}{8}$

2. Express $20\frac{1}{6}\%$ as

   a) a fraction   b) a decimal

3. Write each percentage as a fraction in simplest form.

   a) 27%   b) 53%   c) 58%   d) 76%

4. Write each percentage as a decimal.

   a) 26%   b) 9.6%   c) 40.7%   d) 7.03%

5. Write each fraction as percentage.

   a) $\frac{8}{50}$ b) $\frac{12}{36}$ c) $\frac{18}{48}$ d) $\frac{5}{90}$

6. Compare using < , > or =

   a) $\frac{18}{100}$ $\square$ 22%   c) $\frac{35}{52}$ $\square$ 72%

   b) $\frac{3}{20}$ $\square$ 15%   d) $\frac{11}{20}$ $\square$ 56%

7. Which inequality is a true statement?

   a) $24\% > \frac{1}{4}$ c) $8\% < 0.8$

   b) $0.76 < 76\%$ d) $\frac{1}{5} < 5\%$

8. Nineteen out of the 25 students participate in club A, and 68% of the students participate in club B. Which team had a greater percentage of students' participation in the club?

3.3 comparing and ordering Fractions

Periods allotted: 5 periods

Competency

At the end of this sub unit, students will be able to:

- compare fractions
- order fractions

Introduction

This sub unit begins by revising the concept of equivalent fractions and then deals with comparing and ordering fractions.

Activity and exercise are included in order to involve students in the discussion of this sub unit.

Teaching notes

You can start the lesson by revising the concept of equivalent fractions. After revising the concept of equivalent fractions, ask students if every number can be expressed using its equivalent fraction. You may use Activity 3.7 for this purpose.

Assist students to compare and order fractions and decimals. Make sure that students understand ordering fractions either in ascending order or in descending order.

Answers to Activity 3.7

$$\frac{1}{2} = \frac{2}{16} = \frac{5}{40} = \frac{6}{48}$$

Answers to Group work 3.5

Fast: Elevator D;

Slowest: Elevator B;

To compare the speeds, write them as decimals.

Assessment

You can give problems on comparing and ordering fractions as a form of class work, home work, assignment, test or quiz in order to asses students' progress. You can also give the following problems to fast learners or interested students as additional assessment.

Additional Assessment

Arrange the following in an ascending order.

$\frac{9}{5}, 1\frac{11}{20}, \frac{5}{3}, \frac{7}{25}, \frac{2}{5}$

Answers to additional assessment

$\frac{9}{5} = 1.8$, $1\frac{11}{20} = 1.55$, $\frac{5}{3} = 1.67$ (to the nearest),

$\frac{7}{25} = 0.28$ and $\frac{2}{5} = 0.4$

Thus $\frac{7}{25}$, $\frac{2}{5}$, $1\frac{11}{20}$, $\frac{5}{3}$ and $\frac{9}{5}$ are in ascending order.

Answers to Exercise 3.E

1. a) $\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24}$ (many possible answers)

   b) $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$

   c) $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{12}{28}$

   d) $\frac{9}{8} = \frac{18}{16} = \frac{27}{24} = \frac{36}{32}$

   e) $\frac{11}{3} = \frac{22}{6} = \frac{33}{9} = \frac{44}{12}$

2. $\frac{2}{5} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$

3. $\frac{9}{10} = \frac{18}{20} = \frac{27}{30} = \frac{36}{40} = \frac{63}{70}$

4. a) True $\frac{9}{10}=\frac{81}{90}$, $\frac{8}{9}=\frac{80}{90}$

$\frac{81}{90}>\frac{80}{90}$ Therefore $\frac{9}{10}>\frac{8}{9}$

b) True. $\frac{3}{15}=\frac{1}{5}=\frac{5}{25}$

c) True $\frac{9}{11}=\frac{72}{88}$, $\frac{7}{8}=\frac{77}{88}$

$\frac{72}{88}<\frac{77}{88}$

Therefore $\frac{9}{11}<\frac{7}{8}$

$\frac{72}{88}<\frac{77}{88}$, therefore $\frac{9}{11}<\frac{7}{8}$

d) False. $\frac{4}{5}=\frac{16}{20}$

e) False. $\frac{5}{7}=\frac{15}{21}$

f) True. $\frac{13}{30}=\frac{52}{120}$, $\frac{19}{40}=\frac{57}{120}$

$\frac{52}{120}<\frac{57}{120}$, Therefore, $\frac{13}{30}<\frac{19}{40}$

5 a) LCM of 5, 8, 25, 10, and 4 is 200.

$\frac{3}{5}=\frac{120}{200}$, $\frac{7}{8}=\frac{175}{200}$, $\frac{2}{25}=\frac{16}{200}$, $\frac{7}{10}=\frac{140}{200}$ and $\frac{3}{4}=\frac{150}{200}$

Now $\frac{16}{200}<\frac{120}{200}<\frac{140}{200}<\frac{150}{200}<\frac{175}{200}$ Therefore,

$\frac{2}{25}<\frac{3}{5}<\frac{7}{10}<\frac{3}{4}<\frac{7}{8}$

b) LCM of 8, 2, 20, 5 and 10 is 40.

$$\frac{7}{8}=\frac{35}{40}, \frac{1}{2}=\frac{20}{40}, \frac{11}{20}=\frac{22}{40}, \frac{4}{5}=\frac{32}{40} \text{ and } \frac{9}{10}=\frac{36}{40}$$

$$\frac{20}{40}<\frac{22}{40}<\frac{32}{40}<\frac{35}{40}<\frac{36}{40}$$

$$\text{Now } \frac{1}{2}<\frac{11}{20}<\frac{4}{5}<\frac{7}{8}<\frac{9}{10}$$

Therefore, $\frac{1}{2}<\frac{11}{20}<\frac{4}{5}<\frac{7}{8}<\frac{9}{10}$

c) LCM of 25, 2, 4, 50 and 10 is 100

$$\frac{14}{25}=\frac{56}{100}, 1\frac{1}{2}=\frac{3}{2}=\frac{150}{100}, \frac{1}{4}=\frac{25}{100}, \frac{47}{50}=\frac{94}{100} \text{ and } \frac{3}{10}=\frac{30}{100}$$

$$\frac{25}{100}<\frac{30}{100}<\frac{56}{100}<\frac{94}{100}<\frac{150}{100}$$

Therefore, $\frac{1}{4}<\frac{3}{10}<\frac{14}{25}<\frac{47}{50}<1\frac{1}{2}$

d) LCM of 2, 4, 5, 25 and 8 is 200

$$\frac{1}{2}=\frac{100}{200}, \frac{1}{4}=\frac{50}{200}, \frac{3}{4}=\frac{150}{200}, \frac{4}{5}=\frac{160}{200}, \frac{7}{25}=\frac{56}{200}, \text{ and } \frac{7}{8}=\frac{175}{200}$$

$$\frac{50}{200}<\frac{56}{200}<\frac{100}{200}<\frac{150}{200}<\frac{160}{200}<\frac{175}{200}$$

Therefore, $\frac{1}{4}<\frac{7}{25}<\frac{1}{2}<\frac{3}{4}<\frac{4}{5}<\frac{7}{8}$

6. a) LCM of 5, 2, 4, 3 and 10 is 60

$$\frac{4}{5}=\frac{48}{60}, \frac{1}{2}=\frac{30}{60}, \frac{3}{4}=\frac{45}{60}, \frac{1}{3}=\frac{20}{60} \text{ and } \frac{9}{10}=\frac{54}{60}$$

$$\frac{54}{60}>\frac{48}{60}>\frac{45}{60}>\frac{30}{60}>\frac{20}{60}$$

Therefore, $\frac{9}{10}>\frac{4}{5}>\frac{3}{4}>\frac{1}{2}>\frac{1}{3}$

b) $\frac{12}{25} = \frac{288}{600}, \frac{13}{20} = \frac{390}{600}, \frac{7}{40} = \frac{105}{600}, \frac{4}{15} = \frac{160}{600},$

and $\frac{49}{50} = \frac{588}{600}$

$$\frac{588}{600} > \frac{390}{600} > \frac{288}{600} > \frac{160}{600} > \frac{105}{600}$$

That is, $\frac{49}{50} > \frac{13}{20} > \frac{12}{25} > \frac{4}{15} > \frac{7}{40}$.

c) $\frac{3}{7} = \frac{675}{1575}, \frac{2}{3} = \frac{1050}{1575}, \frac{4}{5} = \frac{1260}{1575},$

$\frac{6}{25} = \frac{378}{1575}$, and $\frac{5}{9} = \frac{875}{1575}$.

$$\frac{1260}{1575} > \frac{1050}{1575} > \frac{875}{1575} > \frac{675}{1575} > \frac{378}{1575}.$$

That is $\frac{4}{5} > \frac{2}{3} > \frac{5}{9} > \frac{3}{7} > \frac{6}{25}$.

d) $2\frac{1}{4} = \frac{9}{4} = \frac{225}{100}, \quad 2\frac{1}{2} = \frac{5}{2} = \frac{250}{100},$

$2\frac{3}{5} = \frac{13}{5} = \frac{260}{100}, \quad 2\frac{2}{25} = \frac{52}{25} = \frac{208}{100}$ and

$1\frac{49}{50} = \frac{99}{50} = \frac{198}{100}$.

$$\frac{260}{100} > \frac{250}{100} > \frac{225}{100} > \frac{208}{100} > \frac{198}{100}$$

That is, $2\frac{3}{5} > 2\frac{1}{2} > 2\frac{1}{4} > 2\frac{2}{25} > 1\frac{49}{50}$

7. a) $\frac{5}{9}$? $\frac{8}{15}$

LCM of 9 and 15 is 45

$\frac{5}{9} = \frac{25}{45}$ and $\frac{8}{15} = \frac{24}{45}$

Therefore 15 books for Birr 8 is cheaper.

b) $\frac{50}{3}$? $\frac{100}{9}$

$\frac{50}{3} = \frac{150}{9} > \frac{100}{9}$

Therefore 9 lemons for 100 cents is cheaper.

8. i) $24.75 + \frac{6.25}{100} \times 24.75$

$= 24.75 + 1.55$

$= 26.30$

ii) $20.25 + \frac{7.5}{100} \times 20.25$

$= 20.25 + 1.52 = 21.77$

Therefore, mine is expensive.

Selected problems to slow learners

1. Write an equivalent fraction to each of the following fractions.

a) $\frac{1}{2}$ b) $\frac{3}{5}$ c) $\frac{5}{7}$ d) $\frac{8}{9}$

2. Arrange

a) $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{8}$ in ascending order.

b) $\frac{5}{6}, \frac{15}{16}$ and $\frac{9}{10}$ in descending order.

3. Compare the fractions $\frac{5}{6}$ and $\frac{7}{10}$.

4. Order $\frac{3}{5}, \frac{77}{99}, \frac{1}{10}$ and $1\frac{1}{5}$ from least to greatest.

5. Compare $\frac{2}{5}$ and $\frac{3}{7}$.

6. Order $\frac{4}{5}, \frac{93}{100}$ and $\frac{9}{10}$ from least to greatest.

7. Which one is greater, $\frac{9}{10}$ or $\frac{7}{8}$?

8. Which number is the greatest?

a) 0.71 b) $\frac{5}{8}$ c) 0.65 d) $\frac{5}{7}$

Selected problems to fast learners

1. Arrange

a) $\frac{4}{9}, \frac{5}{10}$ and $\frac{2}{5}$ in ascending order.

b) $\frac{5}{6}, \frac{15}{16}$ and $\frac{9}{10}$ in descending order.

2. Find a fraction that lies between

a) $\frac{17}{5}$ and $\frac{7}{2}$

3. Which number is the greatest?

b) $\frac{5}{6}$ b) 28% c) 0.8 d) $\frac{826}{100}$

4. Which number is the greater, $3\frac{4}{5}$ or $\frac{308}{100}$?

5. Order $\frac{5}{3}, \frac{525}{100}$, and $5\frac{2}{5}$ from least to greatest.

6. Find a fraction x such that $\frac{1}{2} < x < \frac{2}{3}$

3.4 Further on Addition and subtraction of fractions and decimals

Periods allotted:10 periods

Competencies

At the end of this sub unit, students will be able to:

- compute the sum of fractions and decimals.
- compute the difference of fractions and decimals.
- solve word problems on addition and subtraction.

Introduction

In this sub unit students will deal more with addition and subtraction of fractions and decimals. The presentation of the sub unit is sub divided in to two topics: addition of fractions and decimals, and subtraction of fractions and decimals.

Teaching notes

You are expected to involve and motivate students in the discussion of the sub unit.

3.4.1 Addition of fractions and decimals

You can start the lesson by revising addition of fractions with the same denominator (e.g.$\frac{3}{5} + \frac{1}{5}$).

Motivate students to add fractions and decimals by changing to the convinient form. You may use examples of the following type.

$$\frac{1}{5} + 0.3 = 0.2 + 0.3 = 0.5$$

$$\frac{1}{5} + 0.3 = \frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

Answers to Activity 3.8

a) 2 b. $\frac{3}{5}$ c) $\frac{5}{8}$ d) 0.6 e) 0.86 f) 1.18

Assessment

You can give problems on addition of fractions and decimals in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding of the lesson. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

Add. Give the answer in decimal.

a) $\frac{2165}{100}+14.2+10.03$ b) $\frac{41729}{1000}+14.92$

c) $4.018+\frac{7052}{1000}$ d) $7+\frac{5826}{1000}$

Answers to Additional assessment

a) 45.88 b) 56.649 c) 11.07 d) 12.826

Answers to exercise 3.F

a) $\frac{3}{5}+0.1=0.6+0.1=0.7=\frac{7}{10}$

b) $\frac{5}{8}+0.6=0.625+0.600=1.225=\frac{1225}{1000}=\frac{49}{40}=1\frac{9}{40}$

c) $2\frac{1}{2}+5.6=2.5+5.6=8.1=\frac{81}{10}$

d) $5\frac{1}{4}+1.375=5.250+1.375=6.625$

e) $4\frac{7}{8}+3.4=4.875+3.400=8.200$

f) $14\frac{1}{2}+7.2=14.5+7.2=21.7$

2. $2\frac{1}{2}+0.75=\frac{5}{2}+\frac{3}{4}=\frac{10}{4}+\frac{3}{4}=\frac{13}{4}$ kg of vegetable or 3.25 kg of vegetable

3. $3\frac{3}{4}+2.5+6.875=\frac{15}{4}+\frac{5}{2}+\frac{55}{8}=\frac{30+20+55}{8}=\frac{105}{8}$

$\frac{105}{8}$ hours or 13.125hours

4. She should remove the cakes from the oven after $2\frac{1}{2}$ hours.

That is, at 5:45 P.M.

3.4.2 Subtraction of fractions and decimals

Motivate students to subtract fractions and decimals (when subtracting avoid negative results). Assist students to solve word problems on addition and subtraction.

Answers to Activity 3.9

$$\frac{1}{2}-0.1=\frac{1}{2}-\frac{1}{10}=\frac{5-1}{10}=\frac{4}{10}=\frac{2}{5}$$

or $\frac{1}{2}-0.1=0.5-0.1=0.4$

Assessment

You can give problems on subtraction of fractions and decimals in order to assess students' level of understanding of the lesson. Check their work and give feedback. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

Add or subtract

a) $4.99+\frac{2289}{100}$

b) $\frac{1809}{100}-11.87$

c) $23-\frac{8905}{1000}$

d) $7.75-4\frac{1}{3}$

Answers to Additional Assessment

Add or subtract

a) $4.99+\frac{2289}{100}=4.99+22.89=27.88$

b) $\frac{1809}{100}-11.87=\frac{1809}{100}-\frac{1187}{100}=\frac{662}{100}=6.22$

c) $23-\frac{8905}{1000}=23.000-8.905=14.095$

d) $7.75-4\frac{1}{3}=\frac{775}{100}-\frac{13}{3}=\frac{2325-1300}{300}=\frac{1025}{300}$

$=\frac{41}{12}=3\frac{5}{12}$

Answers to Exercise 3.G

1 a) $\frac{4}{5}-0.32=0.8-0.32=0.48=\frac{48}{100}=\frac{12}{25}$

b) $\frac{1}{2}-0.125=0.500-0.125=0.375=\frac{375}{1000}=\frac{3}{8}$

c) $2\frac{3}{8}-1.75=2.375-1.750=0.625=\frac{625}{1000}=\frac{5}{8}$

d) $4\frac{1}{2}-1.375=4.5-1.375=3.125=\frac{3125}{1000}=\frac{25}{8}=3\frac{1}{8}$

e) $7\frac{15}{16}-2.375=7.9375-2.3750=5.5625=\frac{55625}{10000}=\frac{89}{16}=5\frac{9}{16}$

f) $13.125-1\frac{7}{10}=13.125-1.700=11.425$

$=\frac{11425}{1000}=\frac{457}{40}=11\frac{17}{40}$

2. $21\frac{1}{4}-2.125=21.25-2.125=19.125=\frac{19125}{1000}=\frac{153}{8}=19\frac{1}{8}$

The skirt is $19\frac{1}{8}$ cm long.

3. $5\frac{1}{2} - 1.25 = 5.5 - 1.25 = 4.25$ hours or $4\frac{1}{4}$ hours. Therefore, the number of hours available for actual teaching work = $4\frac{1}{4}$ hours.

4. $7\frac{1}{2} - 6.75 = 7.50 - 6.75 = 0.75 = \frac{3}{4}$.

   Therefore, Abdu walked for 0.75km or $\frac{3}{4}km$.

5. $\frac{3}{4} + 0.875 - \frac{5}{8} = 0.750 + 0.875 - 0.625$

   $= 1$

   Whereas, $\frac{7}{8} + \frac{5}{8} - 0.75 = 0.75$

   Therefore. $\frac{3}{4} + 0.875 - \frac{5}{8} \neq \frac{7}{8} + \frac{5}{8} - 0.75$

6. Let x represent original length of the string, then

   $\frac{x}{2} - \frac{x}{10} = 2$ That is $\frac{5x}{10} - \frac{x}{10} = 2$ or $\frac{4}{10}x = 2$

   Thus, x=5 metre.

Selected Problems to slow learners

1. Find the sum.

a) $\frac{2}{5} + 0.1$

b) $2.75 + \frac{1}{5}$

c) $\frac{1}{2} + 0.2$

d) $0.4 + \frac{1}{5}$

2. Find the difference.

a) $0.7 - \frac{1}{10}$

b) $4.875 - \frac{1}{8}$

c) $2.6 - \frac{1}{5}$

d) $3.4 - \frac{2}{5}$

3. Find the value of

$$4\frac{1}{4}+2\frac{1}{2}-1\frac{1}{4}$$

4. Marta $5\frac{3}{8}$km on saturday and $4\frac{9}{10}$km on sunday. Find the total number of kilometres she run.

5. Which one is greater $\frac{5}{8}+\frac{1}{4}$ or $\frac{3}{4}+\frac{4}{5}$

6. Find the sum .

$$3\frac{1}{8}+4\frac{1}{4}+2\frac{1}{2}+\frac{3}{4}+2\frac{1}{8}$$

7. Find the value of

$$4\frac{1}{3}-1\frac{5}{6}+1\frac{7}{12}-\frac{2}{3}$$

Selected problems to fast learners

1. Find the sum.

a) $3\frac{2}{5}+2.5$

b) $4\frac{1}{8}+2\frac{1}{4}+1.25$

c) $20\frac{1}{5}+4.125+1\frac{1}{2}$

d) $12\frac{1}{2}+4.6+25\frac{1}{4}$

2. Find the difference.

a) $4.6-3\frac{1}{8}$

b) $4.875-\frac{1}{8}$

c) $\left(6.125-2\frac{1}{4}\right)-0.25$

d) $\left(7\frac{1}{8}-0.24\right)-0.9$

3. Find the sum.

a) $3\frac{7}{8}+\frac{2}{15}$ b) $6\frac{1}{3}-\frac{5}{6}$

4. Find the difference.

a) $8\frac{3}{4}-6\frac{2}{5}$ b) $6\frac{1}{3}-\frac{5}{6}$

5. Find the value of $14\frac{2}{3}+1\frac{7}{9}-11\frac{14}{29}$

6. Abebe ran $12\frac{1}{2}$ km on Monday, $10\frac{1}{4}$ km on Tuesday and $15\frac{1}{5}$ km on Wednesday. Find the total number of kilometer he run.

7. Which one is greater, $\frac{5}{11}+\frac{4}{9}$ or $\frac{6}{13}+\frac{8}{9}$?

8. Find the sum.

$$12\frac{3}{8}+9\frac{11}{15}+3\frac{1}{4}+8\frac{1}{2}+4\frac{3}{4}+5\frac{2}{5}$$

9. Find the value of

$$47\frac{3}{4}+3\frac{2}{3}-2\frac{3}{5}$$

3.5 Further on multiplication and division of Fractions and Decimals

Periods allotted:11 periods

Competencies

At the end of this sub unit, students will be able to:

- find product of fraction and decimal.
- divide a decimal by decimals.
- express a given natural number in scientific (standard) notation.

Introduction

This sub unit deeply deals with multiplication and division of fractions and decimals. The steps to follow when multiplying (or dividing) fractions and decimals are stated in the sub unit.

Group Work, Activities and exercises are included in order to involve students in the discussion of the sub unit.

Teaching Notes

To address this sub unit, the presentation is divided in to two: multiplication of fractions and decimals, and division of fractions and decimals. Each is discussed in detail in this sub unit. Participation of students is required in the discussion of the sub unit.

3.5.1 Multiplication of Fractions and Decimals

You may start the discussion of the lesson by revising multiplication of fractions with fractions and decmals with decimals.

Assist students to multiply fraction by decimals after changing to the convenient form (you can take decimals with 2 decimal places)

Answers to Activity 3.10

a) $\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$ b) $\frac{5}{12}\times\frac{6}{25}=\frac{30}{300}=\frac{1}{10}$ c) $0.2\times 0.4 = 0.08$

Assessment

You can give problems on multiplication of fractions and decimals in the form of class work, homework, assignment, quiz or test in order to assess students' progress. You can also give the following questions to fast learners or interested students to answer as additional assessment.

Multiply

a) $3.6\times 1\frac{1}{12}$ b) $4.2\times 2\frac{1}{7}$ c) $6\frac{2}{3}\times 4.4$ d) $1.5\times\frac{3}{5}\times\frac{7}{9}$

Answers to additional assessment

a) $3.6\times1\frac{1}{12}=\frac{36}{10}\times\frac{13}{12}=\frac{39}{10}=3.9$

b) $4.2\times2\frac{1}{7}=\frac{42}{10}\times\frac{15}{7}=9$

c) $6\frac{2}{3}\times4.4=\frac{20}{3}\times\frac{44}{10}=\frac{88}{3}=29\frac{1}{3}$

d) $1.5\times\frac{3}{5}\times\frac{7}{9}=\frac{15}{10}\times\frac{3}{5}\times\frac{7}{9}=\frac{7}{10}=0.7$

Answers to exercise 3.H

1. a) 0.8 x 0.5 =0.4

   b) 0.12 x 0.3 = 0.036

   c) 0.042 x 0.4 =0.0168

   d) 0.153 x 0.5 = 0.0765

   e) 2.235 x 1.35 = 3.01725

   f) $\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}=\frac{3}{10}=0.3$

   g) $1\frac{3}{4}\times3\frac{1}{2}=\frac{7}{4}\times\frac{7}{2}=\frac{49}{8}=6.125$

   h) $1\frac{3}{7}\times\frac{7}{10}=\frac{10}{7}\times\frac{7}{10}=1$

2. a) $\frac{5}{8}\times0.4=\frac{5}{8}\times\frac{4}{10}=\frac{1}{4}$

   b) $2.6\times\frac{1}{4}=\frac{26}{10}\times\frac{1}{4}=\frac{13}{20}$

   c) $7.5\times2\frac{3}{4}=7.5\times2.75=20.625$

   d) $7\frac{3}{4}\times3.2=7.75\times3.2=24.8$

   e) $12\frac{1}{8}\times0.25=\frac{97}{8}\times\frac{1}{4}=\frac{97}{32}$

   f) $4\frac{1}{5}\times0.375=4.2\times0.375=1.575$

3. 0.6 x38 = 22.8 cm

4. 2.5 x5 = 12.5 ℓaps

5. 8.25 x 7 = 57.75 kms

3.5.2 Division of Fractions and decimals

You can start the discussion of the lesson by revising division of decimal by decimal after changing the dividend and divisor to natural numbers by multiplying with powers of 10 (i.e. 10, 100, 1000, …).

You may use examples of the following type:

$0.5 \div 0.1 = \frac{0.5}{0.1} = \frac{0.5\times10}{0.1\times10} = \frac{5}{1} = 5$

$19.6 \div 0.14 = \frac{19.6}{0.14} = \frac{19.6\times100}{0.14\times100} = \frac{1960}{14} = 140$

Encourage students to divide fraction and decimal after changing to the convenient form.

By introducing what is meant by scientific notation, assist students to write a given natural number in its scientific (standard) notation.

You may use examples of the following type:

$342 = 3.42\times10^2$

$1256 = 1.256 \times 10^3$

Answers to Activity 3.11

a) $0.2 \div 0.1 = \frac{0.2}{0.1} = \frac{0.2\times10}{0.1\times10} = \frac{2}{1} = 2$

b) $0.4 \div 0.01 = \frac{0.4}{0.01} = \frac{0.4\times100}{0.01\times100} = \frac{40}{1} = 40$

c) $0.5 \div 0.001 = \frac{0.5}{0.001} = \frac{0.5\times1000}{0.001\times1000} = \frac{500}{1} = 500$

d) $0.01 \div 0.004 = \frac{0.01}{0.004} = \frac{0.01\times1000}{0.004\times1000} = \frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$

Answers to Group Work 3.6

$9.17 \times 10^7 = 91{,}700{,}000$ and $6.287 \times 10^8 = 628{,}700{,}000$.

Observe that

$9.17 \times 10^7 < 6.287 \times 10^8$. Thus Mercury is closer to Earth.

Assessment

You can give problems on division of fractions and decimals in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding of the lesson. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

The life span of a golden dollar coin is 30 years, while paper currency lasts an average of 1.5 years. How many times longer will the golden dollar stay in circulation?

Answer to additional assessment

$30 \div 1.5 = \frac{30}{1.5} = \frac{30 \times 10}{1.5 \times 10} = \frac{300}{15} = 20$

Thus, the golden dollar will stay in circulation for about 20 times longer than paper currency.

Answers to Exercise 3.I

1. a) $0.24 \div 0.3 = \frac{24}{100} \div \frac{3}{10} = \frac{24}{100} \times \frac{10}{3} = \frac{8}{10} = 0.8$

   b) $0.725 \div 0.5 = \frac{725}{1000} \div \frac{5}{10} = \frac{725}{1000} \times \frac{10}{5} = \frac{145}{100} = 1.45$

   c) $0.12 \div 1.5 = \frac{12}{100} \div \frac{15}{10} = \frac{12}{100} \times \frac{10}{15} = \frac{2}{25} = 0.08$

   d) $2.9 \div 0.25 = \frac{29}{10} \div \frac{25}{100} = \frac{29}{10} \times \frac{100}{25} = \frac{58}{5} = 11.6$

   e) $\frac{7}{8} \div 1\frac{3}{4} = \frac{7}{8} \div \frac{7}{4} = \frac{7}{8} \times \frac{4}{7} = \frac{1}{2} = 0.5$

   f) $3 \div 12\frac{1}{2} = 3 \div \frac{25}{2} = 3 \times \frac{2}{25} = \frac{6}{25} = 0.24$

   g) $12\frac{1}{2} \div 0.25 = \frac{25}{2} \div \frac{25}{100} = \frac{25}{2} \times \frac{100}{25} = 50$

   h) $0.04 \div \frac{5}{2} = \frac{4}{100} \div \frac{5}{2} = \frac{4}{100} \times \frac{2}{5}$

   $= \frac{2}{125} = 0.016$

2. 1.32 x 38 = 50.16

   Since 53.12 > 50.16, there will be enough dosages for all her patients.

3. a) $9,600 = 9.6 \times 10^3$ b) $80700 = 8.07 \times 10^4$

   c) $500,000 = 5.0 \times 10^5$ d) $8,300,000 = 8.3 \times 10^6$

   e) $2,563,000 = 2.563 \times 10^6$

4. a) $2.38 \times 10^3 = 2380$ b) $4.917 \times 10^5 = 491700$
c) $8.11 \times 10^2 = 811$ d) $8.007 \times 10^1 = 80.07$
e) $4.321 \times 10^7 = 43,210,000$

Assessment

Different Assessment techniques such as group work, assignment, quiz or test should be given at the end of this unit and record should be kept so that you can further assess students' progress, see if there are areas they need to go over again and review your methodology.

Selected problems to slow learners

1. Multiply
   a) 0.2×0.3 c) 1.3×2.4
   b) 1.2×1.4 d) $\frac{1}{2}\times 3.6$
2. Divide
   a) $0.4\div 0.1$ c) $0.6\div 0.001$
   b) $0.3 \div 0.01$ d) $0.08\div 0.004$
3. A man goes $45\frac{5}{8}$ km in $3\frac{3}{4}$ hours. How far will he go in 1 hour?
4. Find the value of $\frac{1}{2}$ of $\left(\frac{3}{4}\times\frac{4}{9}\right)$.
5. Which expression is greater than $5\frac{5}{8}$?
   a) $8\left(\frac{9}{16}\right)$ b) $\frac{7}{9}\left(8\frac{2}{7}\right)$ c) $\left(3\frac{1}{2}\right)\left(\frac{5}{7}\right)$ d) $\left(\frac{3}{7}\right)\left(\frac{14}{27}\right)$
6. One cup of dry dog food weighs $1\frac{4}{5}$ grams. A dog eats $6\frac{1}{3}$ cups of food a day. How many grams of food does the dog eat each day?

7. Abiy has 72 videos in his collection. Mebratu has $\frac{5}{8}$ as many videos as Abiy. How many videos does Mebratu have?
8. How many books of width 2.5cm can be put on a shelf of length 0.5m?

Se1ected Problems to fast learners

1. Multiply

a) $\frac{1}{16}\times 3.2$ c) 4.3×0.226

b) $1.4\times \frac{7}{50}$ d) $\frac{123}{10}\times 0.5$

2. Divide

a) $\frac{1}{10}\div 3.2$ c) $\frac{1}{20}\div 0.001$

b) $6.875\div 20\frac{5}{8}$ d) $0.004\div \frac{1}{1000}$

3. A man purchased 5 $\frac{1}{2}$ kg sugar for Birr $50\frac{1}{2}$. How much does 1 kg sugar cost?
4. I purchased $25\frac{3}{4}$ meters of cloth costing Birr $17\frac{1}{4}$ per meter. How much have I to pay?
5. The sum of $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$ parts of a number is 28. What is the number?
6. Find the value of

$$\frac{2}{5} \text{ of } \left(1\frac{1}{2}\times\frac{2}{9}\right)$$

7. Find the value of

$$\left(3\frac{1}{3}\times 2\frac{5}{4}\right)\div\frac{4}{7}$$

8. A boy read $\frac{1}{8}$ of a book on the first day, $\frac{1}{4}$ of the remaining on the second day and $\frac{1}{5}$ of the rest on the third day. If 42 pages are left to be read, what is the total number of pages in the book?

Answers to Review Exercise

1. a) $\frac{800}{1000} = \frac{4}{5}$

   b) $\frac{450}{4050} = \frac{1}{9}$

   c) $\frac{2160}{2880} = \frac{3}{4}$

   d) $\frac{3150}{5040} = \frac{5}{8}$

2. a) $\frac{33}{12} = \frac{11}{4} = 2.75 = 2.75 \times 100\% = 275\%$

   b) $\frac{37}{5} = 7.4 = 7.4 \times 100\% = 740\%$

   c) $14\frac{3}{5} = 14.6 = 14.6 \times 100\% = 1460\%$

   d) $\frac{9}{10} = 0.9 = 0.9 \times 100\% = 90\%$

3. a) $0.45 = \frac{45}{100} = \frac{9}{20}$ or $0.45 = \frac{45}{100} = 45\%$

   b) $0.65 = \frac{65}{100} = \frac{13}{20}$ or $0.65 = \frac{65}{100} = 65\%$

   c) $3.2 = \frac{32}{10} = \frac{16}{5} = 3\frac{1}{5}$ or

   $3.2 = \frac{32}{10} \times \frac{10}{10} = \frac{320}{100} = 320\%$

   d) $10.25 = 10\frac{25}{100} = 10\frac{1}{4}$ or

4. a) $0.5 = \frac{5}{10} = \frac{1}{2}$

   LCM of 7, 2 and 3 is 42

$$\frac{2}{7} = \frac{2 \times 6}{7 \times 6} = \frac{12}{42}, \quad 0.5 = \frac{1}{2} \times \frac{21}{21} = \frac{21}{42} \text{ and } \frac{1}{3} = \frac{1\times14}{3\times14} = \frac{14}{42}$$

$$\frac{12}{42} < \frac{14}{42} < \frac{21}{42}$$

That is $\frac{2}{7} < \frac{1}{3} < 0.5$

b) $0.83 = \frac{83}{100}, \quad \frac{17}{10} = \frac{170}{100}$, and $\frac{5}{2} = \frac{250}{100}$

$$\frac{83}{100} < \frac{170}{100} < \frac{250}{100}$$

Therefore $0.83 < \frac{17}{10} < \frac{5}{2}$

c) LCM of 10, 4 and 5 is 20

$\frac{1}{10} = \frac{2}{20}, \frac{5}{4} = \frac{25}{20}$ and $\frac{3}{5} = \frac{12}{20}$

$$\frac{2}{20} < \frac{12}{20} < \frac{25}{20}$$

Therefore $\frac{1}{10} < \frac{3}{5} < \frac{5}{4}$

d) $0.4 = \frac{4}{10} = \frac{2}{5}$

LCM of 5, 6 and 9 is 90

$0.4 = \frac{2}{5} = \frac{2\times18}{5\times18} = \frac{36}{90}, \frac{1}{6} = \frac{1\times15}{6\times15} = \frac{15}{90}$ and $\frac{5}{9} = \frac{5\times10}{9\times10} = \frac{50}{90}$

$$\frac{15}{90} < \frac{36}{90} < \frac{50}{90}$$

Therefore $\frac{1}{6} < 0.4 < \frac{5}{9}$

5. a) $\frac{3}{2} + 0.8 = \frac{3}{2} + \frac{8}{10} = \frac{15+8}{10} = \frac{23}{10} = 2.3$

b) $\frac{3}{8} + 0.625 = \frac{3}{8} + \frac{625}{1000} = \frac{375}{1000} + \frac{625}{1000} = \frac{1000}{1000} = 1$

c) $45.5 - \frac{4}{15} = \frac{455}{10} - \frac{4}{15} = \frac{455\times3}{10\times3} - \frac{4\times2}{15\times2} = \frac{1365}{30} - \frac{8}{30} = \frac{1357}{30}$

d) 28.1-0.25 = 28.10 – 0.25 = 27.85

e) $\frac{21}{8} \times 0.4 = \frac{21}{8} \times \frac{4}{10} = \frac{21}{20} = 1\frac{1}{20}$

f) $\frac{10}{3} \times 3\frac{1}{2} = \frac{10}{3} \times \frac{7}{2} = \frac{35}{3} = 11\frac{2}{3}$

g) $1.5 \div 2\frac{1}{10} = \frac{15}{10} \div \frac{21}{10} = \frac{15}{10} \times \frac{10}{21} = \frac{5}{7}$

h) $12 \div 2.5 = 12 \div \frac{25}{10} = 12 \times \frac{10}{25} = \frac{24}{5} = 4\frac{4}{5}$

i) $0.224 \div 1.6 = \frac{224}{1000} \div \frac{16}{10} = \frac{224}{1000} \times \frac{10}{16} = \frac{14}{100} = 0.14$

j) $0.0032 \div 0.4 = \frac{32}{10000} \div \frac{4}{10} = \frac{32}{10000} \times \frac{10}{4} = \frac{8}{1000} = 0.008$

6. a) $8,900=8.9\text{x}10^3$ b) $400,000=4.0\text{x}10^5$

   c) $1,290,000=1.29\text{x}10^6$ d) $98,000,000=9.8\text{x}10^7$

7. a) $6.03 \text{ x } 10^5 = 603,000$ b) $3.89\text{x}10^6=3,890,000$

   c) $5.66\text{x}10^9 = 5,660,000,000$ d) $9.9923\text{x}10^{10} = 99,923,000,000$

8. 0.2 x 21.4

$= \frac{2000}{10,000} \times 2.14 \times 10$

$= \frac{4280}{10,000} \times 10 = \frac{428}{100} = 4.28$

9. $\frac{1}{8} = \frac{2}{16}$ Thus, both a t e the same amount of the cake.

$1-\left(\frac{1}{8}+\frac{2}{16}\right) = 1-\left(\frac{2+2}{16}\right) = 1-\frac{4}{16} = 1-\frac{1}{4} = \frac{3}{4}$. Therefore, $\frac{3}{4}$ of the cake left uneaten.

10. 1=100%

$\frac{1}{5}\% < 20\% < 100\% < 200\%$

That is , $\frac{1}{5}\% < 20\% < 1 < 200\%$

11. $0.9 = \frac{9}{10} = 90\%, 7 = 700\%, \frac{3}{8} = 0.375 = 37.5\%$

37.5 < 63 < 90 < 700

37.5% < 63% < 90% < 700%

Or $\frac{3}{8}$ % < 63% < 90% < 700%

$\frac{3}{8}$ is the least, and 7 is the greatest.

12. Number of months $= \frac{1772.64}{73.86} = 24$

13. a) $\frac{11}{5} - 2(0.8) = \frac{11}{5} - 2\left(\frac{8}{10}\right) = \frac{11}{5} - \frac{8}{5} = \frac{3}{5}$

b) $\left(\frac{1}{10}\right)^2 + (1.6)(2.1) = \left(\frac{1}{10}\right)\left(\frac{1}{10}\right) + 3.36 = \frac{1}{100} + 3.36$

= 0.01 + 3.36 = 3.37

c) $\frac{3}{8}\left(5.9 - \frac{47}{10}\right) = \frac{3}{8}(5.9 - 4.7) = 0.375(1.2)$

= 0.45

14. $\frac{6(4.8)}{2(3.2)} + \frac{3(4.8)^2(3.2)}{2(3.2)} + (4.8)(3.2)$

$= \frac{9}{2} + 34.56 + 15.36$

= 4.5+34.56+15.36

= 54.42

15.Store's cost = $\frac{486.50}{3.5}$ = Birr 139

16. School C

17. c

UNIT FOUR

INTEGERS

Introduction

The **integers** are the set of whole numbers and their opposites. After introducing integers this unit gives emphasis to comparing and ordering integers, addition and subtraction of integers, and the use of integers to describe different situations in the real life. By using integers, you can express elevations above, below, and at sea level.

Definition, descriptive examples, various activities and exercise are included in order to involve students in the discussion of integers.

Unit outcomes

At the end of this unit, students will be able to:

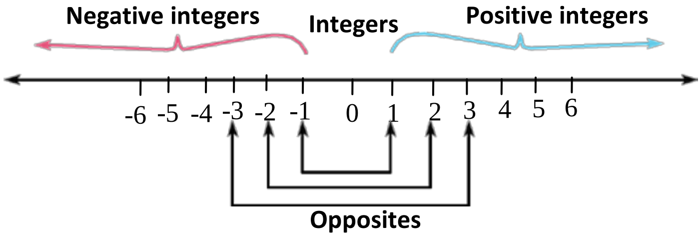
- understand the concept of integers.
- represent integers on a number line.
- perform the operations addition and subtraction on integers

Suggested Teaching Aids in Unit 4

Teaching aids are meant to reinforce instruction. A teacher as well as students are expected to make use of them.

Recommended teaching aids for the unit include thermometer, charts showing graphs of integers, locating integers on a number line, a number line for comparing integers and addition of integers. Encourage students to produce some of the teaching aids in this unit by themselves.

Charts containing the following are recommended.



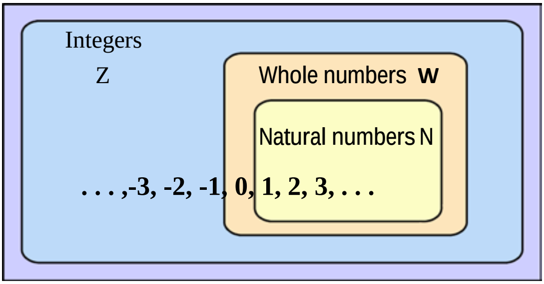


Figure 4.3

4.1. Introduction to Integers

Periods allotted: 5 periods

Competencies

At the end of this sub unit the students will be able to:

- define the set of integers.
- indicate integers on the number line.
- describe the relations, among natural numbers, whole numbers and integers.

The use of integers to describe real – world situations is a quite common experience in mathematics. This sub unit introduces integers and emphasizes on the use of integers in order to describe the real world situations.

You are expected to ensure active participation of students in the discussion of the sub unit.

Teaching Notes

To introduce students to integers, ask them to give examples of opposites. Focus on pairs that can be represented by numbers (e.g. up and down, hot and cold, winning and losing).

Give examples of real – world uses of integers (e.g. temperatures, foot ball yardage, and altitude).

Use students' input to write a definition of the opposite of a number. Point out that with opposites, the direction is different, but the distance between each point and zero is always the same.

Define the set of integers as

Z = {…, -3, -2, -1, 0, 1, 2, 3, ….}

Assist students to represent integers on the number line. Let students discuss the relation between the sets N, W and Z and show using venn diagram.

Answers to Activity 4.1

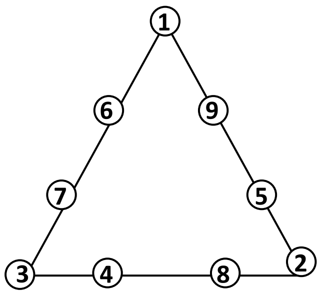


Figure 4.1

Answers to Activity 4.2

NCWCZ

Assessment

You can give problems on indicating integers on the number line, and describing situations using the idea of integers in the form of class work, home work, assignment, quiz or test in order to assess students' progress. You can also ask the following questions to fast learners or interested students as additional assessment.

Additional Assessment

The table shows the average temperatures in a city, for several months. In which month is the average temperature lowest?

a) January c) May

b) March d) July

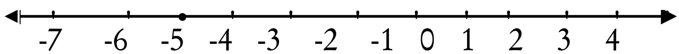
Monthly Temperatures	
January	$-20^{o}F$
March	$-13^{o}F$
May	$10^{o}F$
July	$30^{o}F$

Answer to additional assessment

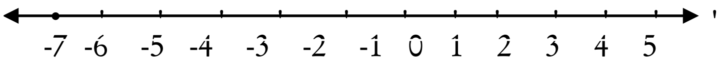
b) January

Answers to Exercise 4.A

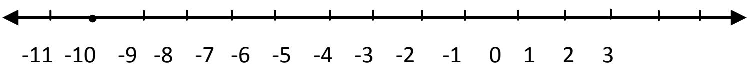
1. a)



b)



c)



2. a= -4, b= -3, c= 3, d= 5

3. a) -10 b) 8 c) -120 d) 1000 e) 0

4. a) 4 b) -5 c) -12 d) 9 e) -20 f) -10
   g) -16 h) 20 i) 30

5. -1,1

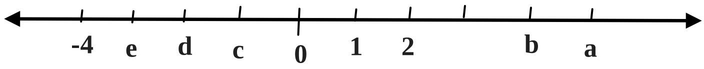
6. a) e.g. -3 (answers may vary) c) 0
   c) e.g. 6

Selected problems to slow learners

1. What is the opposite of

   a) -2 b) 3 c) -8 d) 10

2. What integers are represented by the variables on the give number line?



3. Indicate each integer and its opposite on a number line.

   a) -1 b) 2 c) -3 d) 4

4. Compare the integers by using > , < or =

   a) -3 $\square$ 3 c) 5 $\square$ -4

   b) -4 $\square$ -6 d) 0 $\square$ -1

5. Use a number line to order the integers -5 , -2 , 1 , 3, -3, 0

6. Which list shows the integers in order from least to greatest?

   a) – 4, -5, -6, 1, 2 c) -6, -5, -4, 1 , 2

   b) 1, 2, -4, -5, -6 d) 1, 2 , -6 , -5, -4,

7. Which one is greater, the distance from 0 to1 or the distance from 0 to -2?

Selected problems to fast learners

1. Write an integer for each situation.

   a) a gain of Birr 20.

   b) 10 steps back word.

   c) 15^0 C above zero.

   d) a loss of Birr 100.

2. Indicate each integer and its opposite on a number line.

   a) -5 b) 6 c) -7 d) 9

3. Use a number line to order the integers from least to greatest.

   -7 , - 10, -2, 0, -5

4. Which one is greater, the distance from 0 to 2 or the distance from 0 to -8?
5. Which list shows the integers in order from least to greatest?

    a) -5 , -6 , -7, 2, 3 b) 2 , 3 , -5 , -6, -7

    b) -7, -6, -5, 2, 3 d) 3, 2 , -7 , -6 , -5

4.2. Comparing and ordering Integers

Periods allotted: 7 periods

Competencies

At the end of this subunit, students will be able to:

- compare and order integers using a number line.
- determine the predecessor and successor of a given integer.

Introduction

This sub unit deals with comparing and ordering integers. The sub unit emphasizes comparing and ordering integers by graphing them on a number line. Activity and exercise are included in order to involve students in the discussion of the sub unit.

Teaching notes

In this sub unit, students learn to compare and order integers. As students begin to work with ordering integers, encourage them to place the integers on the number line before ordering them.

The students at this level should be able to generalize that when points corresponding to two numbers are plotted on the number line, the number corresponding to the point to the left is less than the number corresponding to the point which is to the right side.

Motivate students to determine the predecessor and successor of some given integers.

Answers to Activity 4.3

1. 3, 5
2. a) -5, -4, -3 b) -13, -12, 11 c) -26, -25, -24

Answers to Group Work 4.1

March

Assessment

You can give problems on comparing and ordering integers in the form of class work, home work, assignment, quiz or test in order to assess students' progress. You can also give the following problems to fast learners or interested students to answer as additional assessment.

Additional Assessment

For one day, Lelo recorded the low temperatures in five countries of the world. The temperatures were S^{o}c, -1^{o}c, -3^{o}c, 2^{o}c, and O^{o}c. Write the temperatures in order from least to greatest.

Answers to additional assessment

-3^{o}c, -1^{o}c, O^{o}c, 2^{o}c, 5^{o}c

Answers to Exercise 4.B

1. a) True b) True c) False d) True
2. a) 2 $\boxed{>}$ - 10 b) 3 $\boxed{>}$ - 87 c) -5 $\boxed{<}$ 5

   d) -101 $\boxed{>}$ - 1001 e) -20 $\boxed{>}$ - 90 f) -81 $\boxed{<}$ -51
3. a) -78, -11, -4, 0, 7, 29

   b) -91, -67, -13, -6, 2, 18, 35

   c) -44, -9, -1, 0, 5, 17, 44
4. a) 18, 10, 8, -3, -5, -70

   b) 15, 6, -3, -19, -55, -77

   c) 31, 26, 17, -28, -40, -52

5.

Predecessor	integer	Successor
28	29	30
-16	-15	-14
-75	-74	-73
-153	-152	-151

6. a) -8, -7, -6, -5, -4, -3, -2 and -1
   b) -19, -18, -17 and -16

Selected Problems to slow learners

1. order the integers from least to greatest.
   -2, -4, -6 , -8 , 0, 2
2. Order the integers from greatest to least
   -1 , -5 , -3 , -7, 1, -6
3. Draw the following integers on a number line
   -2, -3, -5, 4, and 0.

Selected problems to fast learners

1. Order the integers from least to greatest.
   a) – 200, -198, -174, - 250, -300
   b) -175 , -150 , 0, 4, -2, -6
2. Draw the following integers on a number line -9 , -7, -4, -3, 0, and 2
3. Which one is the greatest -100, -1000, -4, -28 or -70?
4. Which distance is greater, the distance from zero to -23 or the distance from zero to 12?

4.3 Addition and subtraction of Integers

Periods allotted: 8 periods

Competencies

At the end of this sub unit, students will be able to:

- find the sum of integers.
- find the difference between two integers.

Introduction

This sub unit deals with addition and subtraction of integers. Students can describe real world situations in which integer addition is used (e.g changes in temperature, altitude, or prices). This sub unit is devoted to explaining how students can find the sum and difference of integers.

Group work, activities and exercise are included to involve students in the discussion of the sub unit.

Teaching Notes

To introduce students to adding integers, discuss examples of saving and spending. Be sure to use only whole Birr amounts. Ask students to explain how they would know whether a particular combination of saving and spending results in a total saving or in a total spending. Encourage students to explain their methods for finding the total.

Relate negative integers to borrowing money to buy something you do not have enough money for in order to introduce subtracting integers.

Let students practice identifying 'plus' sign and 'positive' sign. Motivate students to practice adding integers (or subtracting integers). You may use examples of the following type.

$-3 + 4$ means '-3 add 4'. Start at -3 and go 4 units to the right. This shows $-3 + 4 = 1$. Encourage students to write rules for adding integers or subtracting integers in their own words. Have students share their rules with the class. Discuss the common points.

The same properties students learned for whole number addition are used for adding integers. The commutative and associative properties allow addition of three or more addends to be rewritten in the most convenient order. Generally, this involves first grouping the positives and then grouping the negatives. Be sure students keep the sign with its number when they change the order. By using integers you can talk about the opposite of a number. The sum of a number and its opposite is always zero.

Answers to Activity 4.4

1. That is -4 + 3 = -1
2. 0

Answers to Group Work 4.2

(i) Yes, 8 + (-3) = 5 and -3 + 8 = 5

(ii) Yes.

(iii) The sum is the integer itself.

Assessment

You can give problems on addition of integers and subtraction of integers in the form of class work, home work, assessment, quiz or test in order to assess student's performance. You can also ask the following problems to fast learners or interested students to answer as additional assessment.

Additional Assessment

Find the value of each word. Each vowel has a value as shown in the table. All consonants have a value of 2.

A	E	I	O	U
-5	-11	-8	-3	-6

a) ALGEBRA = ___________ f) ADDITION = ______

b) INTEGER = ____________ g) GREATER = _______

c) POSITIVE = ___________ h) LESS = ____________

d) NEGATIVE = __________ i) EQUAL = ______

e) SIGN = ____________

Answers to additional assessment

a) -13 b) -22 c) -22 d) -27

e) -2 f) -16 g) -19 h) -5 i) -18

e.g. To find the value of 'ALGEBRA' = A+ L + G + E + B + R + A

= -5 + 2 + 2 + (-11) + 2 + 2 + (-5)

= -21 + 8

= -13

Answers to Exercise 4.C

1. a. True b. True c. True d. False e. True
2. a. -13 b. 16 c. 96 d. -38 e. 23 f. 77
3. i. 1 ii. -1
4. a. -8 – (-12) $\boxed{>}$ 0 because -8 – (-12) = -8 + 12 = 4

   a) -27 + 30 = 3. Thus -27 + 30 $\boxed{>}$ 0

   b) -38 – 57 = -95 and 57 – 38 = 19

   Thus -38 – 57 $\boxed{<}$ 19

   c) -45 – (-45) = -45 + 45 = 0 and 54 – 54 = 0

   -45 – (-45) $\boxed{=}$ 54 – 54

   e) -38 – 57 = -95 and 57 – 38 = 19

   Thus -38 – 57 $\boxed{<}$ 57 – 38
5. x + y = 42 + (-71) = 42 -71 = -29
6. The club’s income = Birr 286

   Expenses = Birr 198

   Income is greater than expenses as 286 > 198, therefore club’s total

   profit = 286 – 198

   = Birr 88

Assessment

Different techniques of assessment such as group work, assignment should be used in order to assess the students' level of understanding of the unit under consideration and to enable you see if there are areas you need to go over again, and also review your methodology. You should assess student’s progress and keep record as this is the end of unit 4.

Selected problems to slow learners

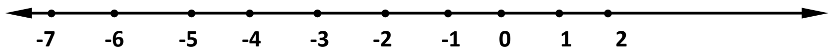
1. Subtract
   a) 5- 10 c) 7 - 15
   b) 3- 9 d) 8- 20
2. Evaluate x +y if
   a) x = -2 and y = 3
   b) x = -5 and y = 4
3. Evaluate a +b if a = 6 and b = -8
4. Find each sum
   a) -3 + (-6) c) -3 +4
   b) 4+ (-7) d) -12 +9
5. Find the difference
   a) -2 -3 c) 5 -6
   b) -4 –(-5) d) 7- (-3)
6. Find the value of
   -1 + [(-6) – (-7)]

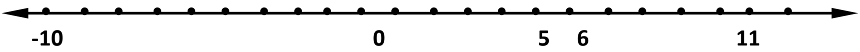
Selected problems to fast learners

1. Subtract
   a) – 14 – 15 c) -27 + 50
   b) 23 – (-23) d) 40 – 70
2. Evaluate x +y if
   a) X = -20 and y = 18
   b) X = - 50 and y = -65
3. Evaluate a+ b if a = 6 and b = -8
4. Find each sum
   a) – 3 + (-6) c) -3 +4
   b) 4 + (-7) d) 12 – 9
5. Find the difference
   a) -2 -3 c) 5 – 6
   b) -4 – (-5) d) 7 – (-3)
6. Find the value of
   -1 + [(-6) – (-7)]

Answers to review exercise

1. a. True b. True c. True d. False e. True f. False

2. 

3. 

4. a) -18, -14, -10, -6, -2, 2, 6

   b) -37, -32, -27, -22, -17, -12, -7

   c) -100, -82, -64, -46, -28, -10, 8

5. -541, -123, -120, -97, -58, 0, 2

6. a) 12 + 13 = 25, 25 – 30 = -5

   b) 354 + 147 = 501, 501 – (-420) = 501 + 420 = 921

7. a) 3 + (-24) + 13 – 8 = 3 + 13 + (-24) – 8 = 16 – 32 = -16

   b) 1 – 10 + 63 – (-21) + (-54) – 27

   = 1 – 10 + 63 + 21 - 54 – 27

   = 1 + 63 + 21 – 10 – 54 – 27

   = 85 – 91

   = - 6

   c) 10 – [10 – (10 – (10 – 1))] = 10 – [10 – (10 – 9)]

   = 10 – [10 – 1] = 10 – 9 = 1

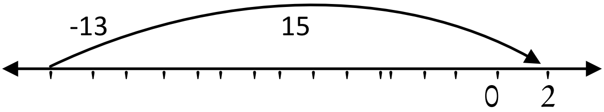
   d) -10 – [-10 – (-10 – (-10 – 10))]

   = -10 – [-10 – (-10 – (-20))]
   = -10 – [-10 – (-10 + 20)]
   = -10 – [-10 – 10]
   = -10 – (-20)
   -10+20
   =10

8. a) 

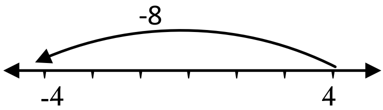
-7+(-5)= -12

b)



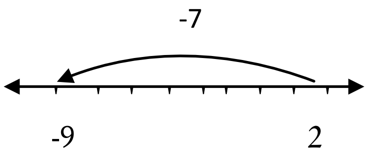
-13+15=2

c)



4-8 = -4

d)



-2-7=-9

9. a) -4
   b) 4
   c) -14

10. -12^0c

UNIT FIVE

LINEAR EQUATIONS, LINEAR INEQUALITIES AND PROPORTIONALITY

Introduction

The main task of this unit is to develop students skills in solving linear equations and linear inequalities, and introducing the concept of direct and inverse proportionalities.

Dealing with linear equations, linear inequalities, direct and indirect proportional relations enables students to determine the slope of a line and to graph a line. Students also learn to use rates of change in graphs. Students can gather information about rate of change both from a graph and from an equation. If the equation is linear, they will realize that the rate of change is constant. If the equation is not linear, then they will be able to conclude that the rate of change is variable.

In general, the basic ideas presented in this unit enables students to use graphs to show relationships between two variables (e.g. speed and time, time and distance or speed and distance, etc.)

Unit out comes

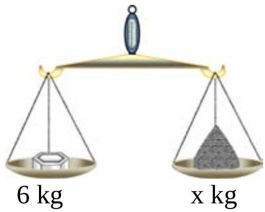
At the end of this unit, students will be able to:

- develop their skills in solving linear equations and inequalities (of the form $x + a = b$, $x + a > b$).
- understand the concept of direct and inverse proportionalities and represent them graphically.

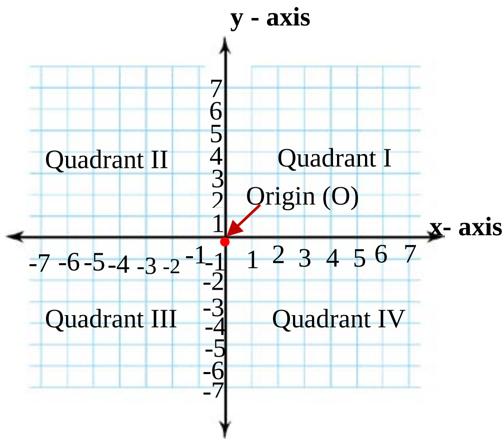
Suggested teaching Aids in Unit 5

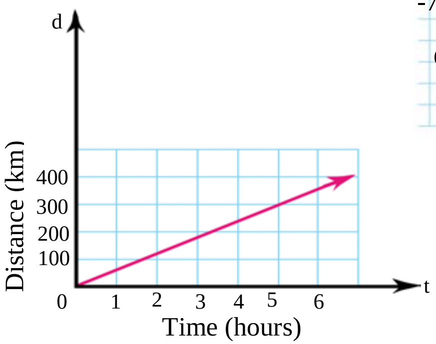
You can present different charts that show graphs of linear equations, direct proportional relations and inverse proportional relations. Encourage students to prepare different representative graphs illustrating direct and indirect proportional relations.

Charts containing the following are recommended

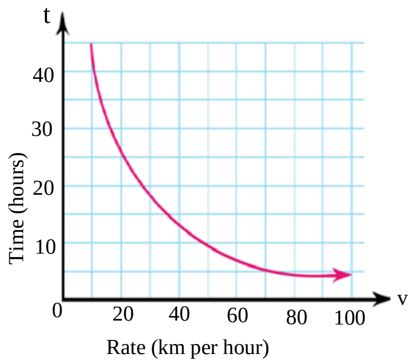


A balance

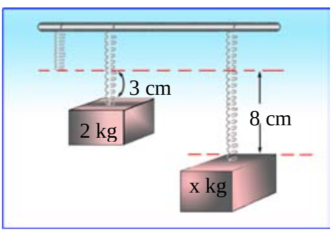




Direct proportion



Inverse proportion



Proportionality

Figure 5.1

5.1 Solution of simple linear equations and Inequalities

Periods allotted: 7 periods

Competencies

At the end of this sub-unit, students will be able to:

- solve one step linear equation of the form x +a = b.
- solve one step linear inequalities of the form x + a >b or x + a < b.

Teaching Notes

Dealing with linear equations and linear inequalities is a quite common experience in Mathematics.

Activities and exercises are included in order to involve in the discussion of the sub-unit.

5.1.1 Solution of one Step Linear Equations

You can start the discussion of the topic "solution of one step linear equations" by revising the method of solving linear equation by substituting values from a given list of numbers.

You may use a balance to introduce the concept of linear equation as follows.

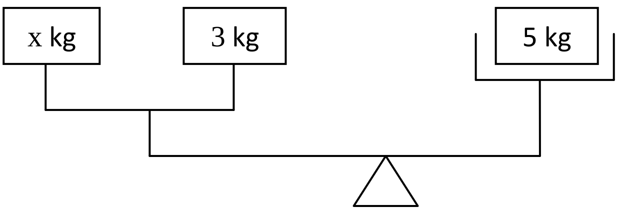


Figure 5.2

x + 3 = 5

Assist students to discuss the rules of transformation:

- Adding or subtracting the same number to and from both sides of an equation. a=b implies a + c = b + c where a, b, c, ∈ Q.
- Encourage students to solve one step equation by using the rule stated above

  x + 3 = 5

  x + 3 – 3 = 5 – 3

  x = 2 is the solution

Answers to Activity 5.1

1. i) $6+k$ ii) $18+x$ iii) $10-y$ iv) $54 \div d$ v) $16h$
2. i) $a + b + c = 6 + 3+ 2 = 11$ ii) $4ab - c = 4(6)(3) -2 = 70$

   iii) $2(a + b) - c = 2(6+3)-2 = 2(9) -2 = 18-2 =16$

   iv) $\frac{6(a+c)}{b} = \frac{6(6+2)}{3} = \frac{6(8)}{3} = 16$

   v) $c (b + a) - a = 2(3+6)-6 = 2 (9) - 6 = 18-6 = 12$

3. i (e), ii(c), iii (i), iv (f), v (g), vi (b), vii(a), viii (h), ix(j), x(d)
4. a) equation c) equation

   b) inequality d) inequality

Answers to Activity 5.2

i) $x=5$ ii) $y=-6$ iii) $z=5$ iv) $x=8$

v) $a=4$ vi) $n=3$

Assessment

You can give problems on solving one-step linear equations in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

In 3 years, Saba's sister will be twice as old as Saba. If Saba is now 3 years old, will her sister be 6, 9, or 12 years old in 3 years?

Answer to Additional assessment

12

Answers to Exercise 5. A.

1. a) $x-3 = 9$

   $x-3+3=9+3$

   $x=12$

Therefore, $x= 12$ is the solution of the equation $x-3 = 9$.

b) $x-7 =14$

   $x-7+7=14+7$

   $x=21$

therefore, $x= 21$ is the solution of the equation $x -7 = 14$.

c) $x+9 = 37$

$x+9-9 = 37 - 9$

$x=32$

Therefore, $x= 32$ is the solution of the equation $x+9 = 37$.

e) $34-x = 0$

$34 - x + x = 0 + x$

$x = 34$

Therefore $x = 34$ is the solution of the equation $34-x =0$.

g) $\frac{y}{17} = 4$

$(17)\left(\frac{y}{17}\right) = (4)(7)$

$y = 68$

Therefore, $y = 68$ is the solution of the equation $\frac{y}{17} = 4$.

i) $\frac{10}{3}x = 20$

$\left(\frac{3}{10}\right)\left(\frac{10}{3}x\right) = (20)\left(\frac{3}{10}\right)$

$x = 6$

Therefore, $x=6$ is the solution of the equation $\frac{10}{3}x = 20$.

k) $\frac{11}{6}m = 3$

$\left(\frac{6}{11}\right)\left(\frac{11}{6}m\right) = 3\left(\frac{6}{11}\right)$

$m = \frac{18}{11} = 1\frac{7}{11}$

d) $16 - y = 5$

$16 - y+y = 5+y$

$16= y+5 -5$

$y = 11$

Therefore, $y = 11$ is the solution of the equation $16-y = 5$.

f) $2 = y - 17$

$2+17 = y - 17 + 17$

$19 = y$

therefore, $y = 19$ is the solution of the equation $2= y-17$.

h) $13 - m = 59$

$13 - m + m = 59 + m$

$13 = m + 59$

$13-59 = m+ 59 - 59$

$m = - 46$

Therefore, $y = -46$ is the solution of the equation $13-m = 59$.

j) $\frac{2}{5}$ $y= 4$

$\left(\frac{5}{2}\right)\left(\frac{2}{5}y\right) = (4)\left(\frac{5}{2}\right)$, $y = 10$

Therefore, $y = 10$ is the solution of the equation $\frac{2}{5}y = 4$.

Therefore, $m = 1\frac{7}{11}$ is the solution of the equation $\frac{11}{6}m = 3$.

l) $\frac{4}{7}n = 8$

$\left(\frac{7}{4}\right)\left(\frac{4}{7}n\right) = 8\left(\frac{7}{4}\right)$, n = 14

Therefore, n= 14 is the solution of the equation $\frac{4}{7}n = 8$.

m) $\frac{n}{10} = 4$

$10\left(\frac{n}{10}\right) = 10(4)$

n = 40

Therefore, n= 40 is the solution of

the equation $\frac{n}{10} = 4$.

n) 200x = 0.1

$\frac{(200x)}{200} = \frac{0.1}{200}$

x = 0.1 ÷ 200

$x = \frac{1}{10} \div \frac{200}{1}$

x = $\frac{1}{2000}$

Therefore, x = $\frac{1}{2000}$ is the solution of the equation 200x = 0.1.

o) 0.01n = 10

$\frac{1}{100}n = 10$

100 $\left(\frac{1}{100}n\right) = 100(10)$

n = 100

Therefore, n = 1000 is the solution of the equation 0.01 n = 10

2. a) 7 + x = 34

7 – 7 + x = 34 -7

x = 27

Therefore, x = 27 is the solution of the equation 7+ x = 34.

b) x-3 =19

x-3 +3 = 19 +3

x = 22

therefore, x = 22 is the solution of the equation x-3 = 19.

c) 2x = 26

$\frac{2x}{2} = \frac{26}{2}$

x = 13

Therefore, x= 13 is the solution

of the equation 2x = 26.

d) 8x – 5 = 0

8x – 5 + 5 = 0 +5

8x = 5

$\frac{8x}{8} = \frac{5}{8}$

therefore, x = $\frac{5}{8}$ is the solution of the equation 8x-5=0.

e) $x + 27 = 316$

$x + 27 - 27 = 316 - 27$

$x = 289$

Therefore, the number of grade eight students enrolled is 289.

f) $12 = \frac{1}{6}x$

$(6)(12) = (6)\left(\frac{1}{6}x\right)$, $x = 72$

Therefore, the object weighs 72 kg on earth.

g) $3x = 225$

$\frac{3x}{3} = \frac{225}{3}$

$x = 75$

Therefore, the cost of the less expensive dress is Birr 75.

5.1.2 Solution of one step linear inequalities

Assist students to demonstrate that the existence of the solution of an inequality depends on the domain of the variable using examples of the following type:

Solve $x + 4 < 9$ if the domain is

a) The set of whole numbers
b) The set of natural numbers
c) The set of integers

Solution. a) $x \in \{0,1,2,3,4\}$

b) $x \in \{1,2,3,4\}$

c) $x \in \{\ldots -3,-2,-1,0,1,2,3,4\}$

Assist students how to represent solutions of an inequality using number line.

You may use examples of the following type:

Represent the solution of $x + 3 < 5$ on the number line if the domain of the variable is the set of natural numbers.

Solution.

$X + 3 < 5$

$X + 3 - 3 < 5 - 3$

$x < 2$

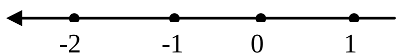


Figure 5.3

Point out that students can check the graph of an inequality by selecting a point on the shaded portion of the graph. Any point selected should satisfy the inequality (i.e, make the inequality a true inequality).

Answers to Activity 5.3

a) Let x represents number of people. Then you can represent the given inequality by $x \geq 20$

b) Let y represents room capacity. Then you can represent the given inequality by $y \leq 150$

Answers to Group Work 5.1

You are able to conclude the rules of transformation.

Assessment

You can give problems on solution of one step linear inequalities in the form of class work, home work, assignment, quiz or test in order to assess students' performance. You may use Exercise 5.B for this purpose. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

Write an inequality for each situation

1. No more than 300 people (x) are in the theater.
2. There are at least a dozen eggs(y) left.
3. Fewer than 14 people (z) attended the meeting.

Answers to additional assessment

1. $x \leq 300$ 2. $y \geq 12$ 3. $z < 14$

Answers to Exercise 5.B

1. a) $x+4 < 8$

$x + 4 - 4 < 8 - 4$

$x < 4$ and $x \in W$

solution set = {0,1,2,3}

b) $y - 2 < 7$

$y - 2 + 2 < 7 + 2$

$y < 9$ and $y \in Z$

solution set = {…-3,-2,-1,0,1,2,3,4,5,6,7,8}

c) $y - 3 > -5$

$y-3+3 > -5+3$

$y > -2$ and $y \in N$
solution set = N = The set of all natural numbers

d) $\frac{1}{5}y > 2$

$5\left(\frac{1}{5}y\right) > 5(2)$, $y > 10$ and y is a negative integer Thus, solution set = ∅

e) $\frac{2}{3}a < 4$

$\left(\frac{3}{2}\right)\left(\frac{2}{3}a\right) < \left(\frac{3}{2}\right)(4)$

$a < 6$ and $a \in W$

Solution set = { 0,1,2,3,4,5}

f) $\frac{3}{4}x > 2$

$\left(\frac{4}{3}\right)\left(\frac{3}{4}x\right) > \left(\frac{4}{3}\right)(2)$

$x > \frac{8}{3}$ and x is a positive integer

Solution set = {3,4,5,6, …}

2. a) $x < 6$ and $x \in W$. thus, $x \in \{0, 1, 2, 3, 4, 5,\}$

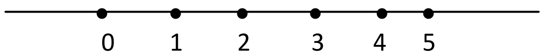


Figure 5.4

b) $2y > 5$, and $y \in Z$

$y > \frac{5}{2}$ and $y \in Z$. thus, $y \in \{ 3,4,5,6, \ldots\}$

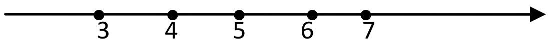


Figure 5.5

c) $\frac{1}{4}a < 2$

$4(\frac{1}{4}a) < 4\ (2)$

$a < 8$ and $a \in N$. Thus, $a \in \{ 1,2,3,4,5,6,7\}$

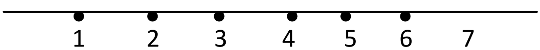


Figure 5.6

d) $y + 3 < -3$

$y + 3 - 3 < -3-3$

$y < -6$ and $y \in Z$. thus, $y \in \{ \ldots -9, -8, -7 \}$

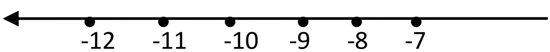


Figure 5.7

e) $\frac{1}{6}x < \frac{3}{4}$

$6(\frac{1}{6}x) < \left(\frac{3}{4}\right)6$

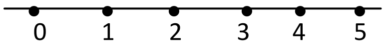


Figure 5.8

$x < \frac{9}{2}$ and $x \in W$. Thus, $x \in \{0,1,2,3,4\}$

3. a, b, d and e
4. Let $x \in W$

   $6x < 18$

   $\frac{6x}{6} < \frac{18}{6}$

$x < 3$ and $x \in W$. Thus, $x \in \{0,1,2\}$

Solution set = $\{0,1,2\}$

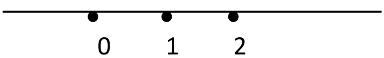


Figure 5.9

Selected Problems to slow learners

1. Which of the following value satisfy the inequality $3x<15$ when the domain is the set of whole numbers?

   0, 1, 2, 3, 4, or 5
2. Draw the graph of the solution set of the inequality $x < 3$ on a number line when the domain is the set of whole numbers.
3. Which of the following is the solution of the equation $2x -1 = 17$?

   a) 7 b) 8 c) 9 d) 10
4. Which of the following numbers satisfy the equation?

   $19 = 5n + 7 – 3n$?

   a) 8 b) 6 c) 4 d) 2
5. For which of the following equation is $x = 9$ a solution?

   a) $14x +2-7x= 37$ c) $12 –x -3 = 10$

   b) $10x-11-4x =43$ d) $3x = 9$

6. Solve

a) $n+9 < 20$, n is a whole number

b) $-2 < x+16$, x is an integer

Selected problems to fast learners

1. Solve
   a) $2x + 3 < 8$ on the set of whole numbers.
   b) $3x - 1 < 9$ on the set of integers.
2. Which of the following is the solution of the equation $\frac{x-4}{2} = 8$?

   a) 30 b) 20 c) 16 d) 12
3. Which of the following numbers satisfy the equation $\frac{0.5x+7}{8} = 5$

   a) 66 b) 60 c) 40 d) 30
4. For which of the following equation is $x = 3$ a solution?

   a) $2x + 5 + 3x = 10$ c) $\frac{4x}{6} = 3$

   b) $\frac{-x+7}{2} = 2$ d) $\frac{2x-2}{4} = 7$
5. Solve
   a) $72 \geq 40 + x$,
      x is a whole number.
   b) $-14 \leq y - 8$, y is an integer.

5.2 Coordinates

Periods allotted: 6 periods

Competencies

At the end of this sub unit, students will be able to:

- determine the coordinates of a point in the first quadrant.
- represent a point in the first quadrant given its coordinates.

Introduction

In mathematics, a coordinate system, or coordinate plane, is used to plot points in a plane. Archaeologists use a coordinate system at dig sites to record the original location of each artifact found. This sub-unit introduces the coordinate plane and discuss the way how you plot a point in a coordinate plane by using descriptive examples and graphs. Group Work, Activities and Exercises are included inorder to involve students in the discussion of the subunit.

Teaching notes

You can start the discussion of the lesson by revising the concept of data handling to read and indicate the relation in the form of ordered pairs.

To introduce students to a coordinate plane, have them discuss their experiences with locating a place by using the intersection of two lines. Motivate students to plot points on a coordinate plane and to name the coordinates of a point. Have students identify the components of a coordinate plane: the two number lines, one placed horizontally (the x-axis) and the other placed vertically (the y- axis), dividing the plane in to four quadrants. Point out that the quadrants are numbered counter clockwise, beginning at the upper right, with the Roman numerals I, II, III and IV.

Answer To Activity 5.4

1. B (4,2)
2. D (-3,-2)

Answers to Group Work 5.2

MATHS CAN BE FUN.

Assessment

You can give problems on the coordinate system in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

1. Use the coordinate plane shown below in which quadrant (s) would the figure drawn by connecting points J, Kand N be?
2. Use the coordinate plane shown below in which quadrant(s) would the figure drawn by connecting points C,F and M be?

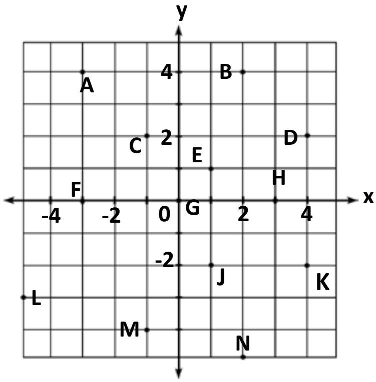


Figure 5.10

Answers to additional assessment

1. Quadrant IV
2. Quadrants II and III

Answers to Exercise 5. C

1. a) True b) False c) True d) False
2. a) (2, 2), b) (5,-2) c) (0,-4) d) (-1,0), e) (-3,1) f) (-4,-4)

3.

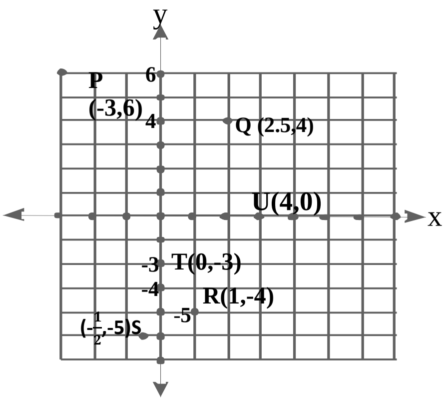


Figure 5.11

4. a) (-3, 3) b) (-5,-2) c) (3,3) d) (0,0)
   e) (3,-2) f) (2,-5.5)

Selected Problems to slow learners

1. Write the ordered pairs of the points P , Q and R

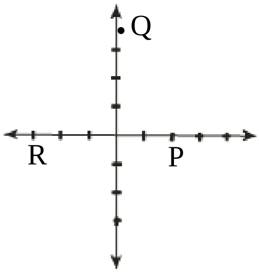


Figure 5.12

2. On graph paper, draw a coordinate plane. Then draw the graph and label each point.
   a) A (1, 3) b) B (-2, 3) d) D (-2, -3)
3. Write and plot the ordered pairs form the table
   i) How many ordered pairs are in Quadrant I?

x	y
-4	5
-1	3
2	1
5	-1

   ii) How many ordered pairs are in Quadrant II?
   iii) Which ordered pair is in Quadrant Iv?
4. Mamo plotted the ordered pairs (-3, -5), (-2, -1), (-1,) and (0,7). How many points did he plot in Quadrant III?

Selected Problems to fast learners

1. Which of the following points lie on the coordinate axes?
   a) (4, 0) b) (-5, 5) c) (0, 6) d) (4, 2)
2. Plot the points
   R (4, 3), S (-2, -5) and T (-1, 6) on the coordinate plane.

3. Write and plot the ordered pairs from the table.
   i) How many ordered pairs are in Quadrant I?
   ii) How many ordered pairs are in Quadrant III?
   iii) Which ordered pair is in Quadrant II?
4. A table of data has the ordered pairs (-2,5) , (1, 4) and (4, 3). Alemayehu plots the points and connects them with a straight line. At what point does the line cross the x- axis?

5.3 Proportionality

Periods allotted: 12 periods

Competencies

At the end of this sub-unit students will be able to:

- explain direct proportionality and factor of proportionality.
- determine the factor of direct proportionality.
- draw graphs to illustrate direct proportionality.
- apply the knowledge of direct proportionality to solve word problems.
- explain inverse proportionality and factor of proportionality.
- determine the factor of inverse proportionality.
- draw graphs to illustrate proportionality.
- apply the knowledge of inverse proportionality to solve word problems.

Introduction

In this sub-unit, the concept of direct proportionality, and inverse proportionality are thoroughly discussed by using definitions, descriptive examples and diagrammatic explanations.

Group Work, Activities and exercises are included in order to involve students in the discussion of this sub-unit.

Teaching notes

For the purpose of presentation, this sub – unit is sub divided in to two topics: direct proportion, and inverse proportion. You are expected to ensure maximum participation of students in the discussion of each topic.

5.3.1 Direct proportion

To introduce students to direct proportion, discuss situations in which they have heard the word direct proportion used. Ask students to explain what it means to say that two things are directly proportional. You may use examples of the following type in order to discuss about direct proportion.

A shop keeper is selling pencils for 50 cents each.

2 pencils cost 2 x 50 cents = 100 cents.

3 pencils cost 3 x 50 cents = 150 cents.

6 pencils cost 6 x 50 cents = 300 cents.

Then define direct proportionality and factor of proportionality. Encourage students to determine the constant of proportionality from a given table like the following.

x	1	2	3	4	5	6
y	10	20	30	40	50	60

y = 10x expresses the direct proportional relation between y and x.

Assist students to represent the idea of direct proportionality graphically.

Assist students to apply the definition of direct proportionality to solve problems like the following:

If 3 meters of cloth for your school uniform cost birr 60. How much will 6 meters cost you?

Assist students to describe the constant of direct proportionality expressed graphically as the slope of the line (graph).

Answers to Activity 5.5

a) Y also increases.
b) Y also decreases.

Assessment

You can give problems on direct proportion in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You may use Exercises 5. D for this purpose. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

Find the slope of the line whose graph is shown

Answer: 1

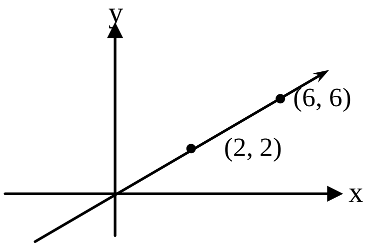


Figure 5.13

Answer to Group Work 5.4

a)

X	0	2	-2	-1
y	3	5	1	2

b) (0,3), (2,5), (-2,1), (-1,2)

c)

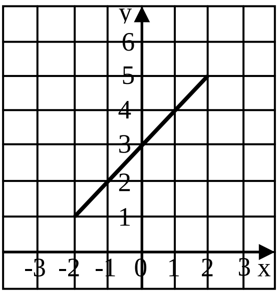


Figure 5.14

d) yes. Because 4=1+3

Answers to Exercise 5. D

1 **a)** $y = kx$

$12 = k(3)$

$\therefore\ k = 4$ and

$y = 4x$

b) $mw = k$

$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = k$

$\therefore\ k = \frac{1}{8}$ and $mw = \frac{1}{8}$

2. a)

y	X
$\frac{1}{3}$	$\frac{4}{9}$
8	$\frac{32}{3}$
$\frac{27}{4}$	9
2 0	$\frac{80}{3}$

b)

y	X
$\frac{1}{2}$	$\frac{3}{4}$
3	$\frac{9}{2}$
4	6
21	$\frac{63}{2}$

3. a) $y = kx$

$100 = k(20)$

$\therefore\ k = 5$ and

$Y = 5x$

When $x = 5$, $y = 25$

b) $nq = k$

$39(3) = k$

$\therefore\ k = 117$

$nq = 117$

when $q = 8$, $n = \frac{117}{8}$

4.

Time (hours)	1	2	3	4	5
Distance (kms)	65	130	195	260	325

$d = 65t$

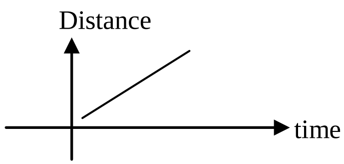


Figure 5.15

5. $p = k\ell$

   $20 = k(5)$

   $\therefore k = 4$ and

   $P = 4\ell$. Thus when $\ell = 6$, $P = 4(6) =$ Birr 24

6. $d = kt$

   $25 = k(2)$

   $k = \frac{25}{2}$ and $d = \frac{25}{2}t$

   When $t = 5$, $d = \frac{25}{2}(5) = \frac{125}{2}$ km

7. $y = kx$

   $3 = k(21)$

   $\therefore k = \frac{1}{7}$ and $y = \frac{1}{7}x$

   When $x = 84$, $y = \frac{84}{7} = 12$ kg

5.3.2 Inverse proportion

You may start the lesson by revising the concept of direct proportionality and introduce inverse proportionality by considering examples like the following:

Two children can clean a class room in 20 minutes. How long would four children take?

Define inverse proportionality, Encourage students to give examples of things (or situations) that are inversely proportional.
Motivate students to determine the constant of proportionality from a given table.

Answers to Activity 5.6

a) y decreases
b) y increases

Assessment

You can give problems on inverse proportion in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You may use exercise 5.E for this purpose. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional Assessment

What will be the value of x, in the relation

$yx = \frac{1}{5}$, when $y = \frac{1}{3}$

Answer: $\frac{3}{5}$

Answers to Exercises 5.E

1. a) $K = yx = (3)(2) = 6$

   b) $k = cd = (5)(2) = 10$

   c) $a = kb$, $k = \frac{a}{b} = \frac{3}{4}$

2. a) $y = kx$

   $100 = k(5)$

   $k = 20$

   $y = 20x$

   when $x = 5$, $y = 20(5) = 100$

   b) $k = ab$

   $k = (9)(2) = 18$

   $18 = ab$

   when $b = 6$, $18 = 6a$

   Thus, $a = 3$

3. i) b varies inversely with a

   $b = \frac{300}{a}$

a	b
$\frac{1}{2}$	600
1	300
30	10
900	$\frac{1}{3}$

ii) b varies inversely with a

   $b = \frac{500}{a}$

a	b
4	125
1000	$\frac{1}{2}$
250	2
$\frac{1}{8}$	4000

iii) b= $\frac{3}{2}$a

b) varies directly with a

a	b
12	18
16	24
$\frac{3}{2}$	$\frac{9}{4}$
15	$\frac{45}{2}$

4. a) y varies directly with x

   y= $\frac{1}{2}$x

   b) y varies inversely with x

   yx = 20

   c) y varies directly with x

   y = $\frac{7}{2}$x

5.

Speed (km/hr)	20	40	50	150
Time (hours)	15	7.5	6	2

6. $20 \times 8 = 32$ (a)

$\frac{160}{32} = a = 5$

If the sweets were divided among 32 children, then each child receives 5 sweets.

7. $80 \times 9 = 60 \times (t)$

$\frac{720}{60}$ = t = 12 hr.

A car traveling at 60 km/hr will take 12 hr to cover the same distance.

Assessment

As this is the end of unit 5, you need to use different assessment techniques such as group work, assignment, quiz or test in order to assess students' performance. You are expected to check their work, give feedback and keep record of students' progress.

Selected problems to slow learners

1. For each table, determine whether y varies directly or inversely with X.

a)

x	y
1	4
2	8
3	12
4	16
5	20

b)

x	y
3	10
2	15
10	3
30	1

2. Suppose, in the following table, x represents number of kilos of orange and y represents the price in Birr, what is the constant of proportionality?

x	1	2	3	4	5
y	10	20	30	40	50

3. Use the formula $y = \frac{4}{5}x$ to fill the missing entries in each table.

x	5		50	100
y		20		

4. If y varies inversely with x, and y = 6 when x = 4. The write a formula that expresses the indicated variation.

Selected problems to fast learners

1. Solve each variation Problem.
   a) y varies directly with x and y = 200 when x = 50. Find y when x = 10
   b) a varies inversely with b, and a = 3 when b = 10 . Find a when b = 5
2. The table shows the number of hours a student studies for a test and the score. Graph the data to find the score after studying for 6 hours. What appears to be the relationship between the number of hours of study and the score?

Hours	1	2	3	4
Score	15	30	45	60

3. The table shows the total cost of buying different number of bags of peanuts. Graph the data.

Number of Bags	1	2	3	4
Total Cost	0.50	1	1.50	2

What appears to be the relationship between the number of bags of peanuts and total cost?

4. Use the formula $t = \frac{800}{v}$ to fill the missing entries in each table.

v	10		40	80	
t		20			8

Selected problem to fast learners

1. Solve each variation problem.
   a) y varies directly with x, and
      y = 200 when x = 50. Find y when x = 10
   b) a varies inversely with b , and a = 3
      When b = 10. Find a when b = 5
2. Suppose, in the following table x represents number of kilos of orange and y represents the price in Birr, what is the constant of proportionality?

x	1	2	3	4	5
y	10	20	30	40	50

3. Use the formula $y = \frac{4}{5}x$ to fill the missing entries in each table.

x	5		50	100
y		20		

4. If y varies inversely with x, and y = 6 when x = 4. Then write a formula that expresses the indicated variation.

Answers to Review Exercise

1. a) $4xyz = 4(10)(8)(12) = 3840$

b) $2xz-3y=2(10)(12)-3(8)=240-24$
$= 216$

c) $x + 5y + 2z = 10+5(8) + 2(12)$
$= 10+40+24$
$= 74$

d) $6xyz + 8 = 6(10)(8)(12)+8$
$= 5760 + 8$
$= 5768$

Therefore, the answer is (c)

2. a) $10x^2 - x^2 - 3 = 9x^2 - 3$

b) $18 + 4x - 15 + 5x = 3 + 9x$

c) $3x + 7 - 4 + 3x = 6x + 3$

d) $7x^2 + 2x + 6 - 4 = 7x^2 + 2x + 2$

Therefore, the answer is (b)

3. $810 = x - 625$
$810 + 625 = x - 625 + 625$
$1435 = x$
Therefore, the answer is (d)

4. a) $x - \frac{1}{4} = \frac{3}{5}$

$x - \frac{1}{4} + \frac{1}{4} = \frac{3}{5} + \frac{1}{4}$

Therefore, $x = \frac{17}{20}$ is the solution to the equation $x - \frac{1}{4} = \frac{3}{5}$.

b) $x + \frac{1}{5} = 2$

$x = \frac{10-1}{5} = \frac{9}{5} = 1\frac{4}{5}$

$x = \frac{12+5}{20} = \frac{17}{20}$

Therefore, $x = 1\frac{4}{5}$ is the solution of the equation $x + \frac{1}{5} = 2$.

c) $2x = \frac{1}{3}$

$\frac{1}{2}(2x) = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$

$x = \frac{1}{6}$

Therefore, $x = \frac{1}{6}$ is the solution of the equation $2x = \frac{1}{3}$.

d) $\frac{3}{4}x = 81$

$\left(\frac{4}{3}\right)\left(\frac{3x}{4}\right) = (81)\left(\frac{4}{3}\right)$

$x = (27)(4) = 108$

Therefore, $x = 108$ is the solution of the equation $\frac{3}{4}x = 81$.

5. a) $x - \frac{1}{4} < \frac{1}{5}$

$x - \frac{1}{4} + \frac{1}{4} < \frac{1}{5} + \frac{1}{4}$

$x < \frac{4+5}{20}$

$x < \frac{9}{20}$ and $x \in W$ Therefore 0 is the solution of the inequality $x - \frac{1}{4} < \frac{1}{5}$.

b) $x + \frac{2}{3} > 4$

$x + \frac{2}{3} - \frac{2}{3} > 4 - \frac{2}{3}$

$x > \frac{12-2}{3}$

$x > \frac{10}{3}$ and $x \in \mathbb{N}$

Therefore, the numbers 4,5,6,7 … are the solution of the inequality $x + \frac{2}{3} > 4$

c) $3x < \frac{3}{7}$

$\left(\frac{1}{3}\right)(3x) < \left(\frac{3}{7}\right)\left(\frac{1}{3}\right)$

$x < \frac{1}{7}$ and x is a negative integer

Therefore, the numbers …-4,-3-2,-1 are the solution of the inequality

$3x < \frac{3}{7}$.

d) $\frac{1}{2}x > \frac{3}{5}$

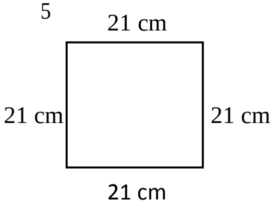
$2\left(\frac{1}{2}x\right) > 2\left(\frac{3}{5}\right)$

$x > \frac{6}{5}$ and x is a negative integer

Therefore, there are no elements which are the solution of the inequality

$\frac{1}{2}x > \frac{3}{5}$.

6.



perimeter = P = $4 \times 21 = 84$cm

Figure 5.16

7. a) A
b) B
c) C
d) D
e) E
f) F
g) G
h) H
i) I
j) J

8. a) Horizontal
b) Vertical
c) Vertical

9. a) $\frac{3}{2} = \frac{8}{x}$

$3x = 16$

$x = \frac{16}{3}$ kg

b) $\frac{3.6}{3} = \frac{6}{5} = \frac{a}{6}$

$a = \frac{36}{5} = 7.2$cm

10. (200) (12) = (150) (v)

$\frac{2400}{150} = V$

$V = 16cm^3$

11. a) inversely proportional.
b) inversely proportional.
c) inversely proportional.

12. 16 (12) + 6 (9) = 192 + 54 = Birr 246

UNIT SIX

GEOMETRY AND MEASURMENT

Introduction

Points, lines, and planes are the most basic figures of geometry. Other geometric figures, such as line segments and rays, are defined in terms of these building blocks.

This unit is devoted in the discussion of geometry and measurement. The unit gives much emphasis on identifying angles, proving congruency of triangles and constructing triangles. To understand special angle relationships, students must be able to distinguish different types of angles. These relationships are basic to the study of geometry. Here students are required to perform a few practice exercises to get accustomed to a ruler and compass. The discussion in each sub – units assumes students to have some background on fundamental geometric ideas and use the postulates in solving real life related problems.

Questions are included, in the form of Activity and Exercise, that help students see the need to learn classification of angles, congruency of triangles and constructing triangles in this unit.

In general, the concepts discussed in this unit enable students to be familiar with introductory geometric ideas.

Unit outcomes

At the end of this unit, students will be able to:

- identify angles.
- prove congruency of triangles.
- construct triangles.

Suggested teaching Aids in unit 6

Students are familiar with points, lines and planes, models of angles, line segments, parallel and perpendicular line segments around us. Edges of books, tables, doors, windows, boards, floor, and walls can be used to describe line segments, parallel and perpendicular line segments. Ruler, mathematical set, pieces of sticks, rope, clocks, charts representing classification of angles and triangles are helpful in the discussion of geometry and measurement. Encourage students to prepare models of a plane, and congruent triangles by themselves.

Charts containing the following are recommended

Measuring angles

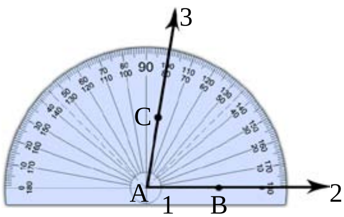


Figure 6.1

Angles formed by different figures

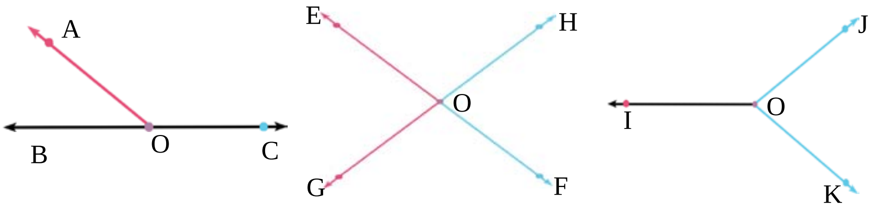
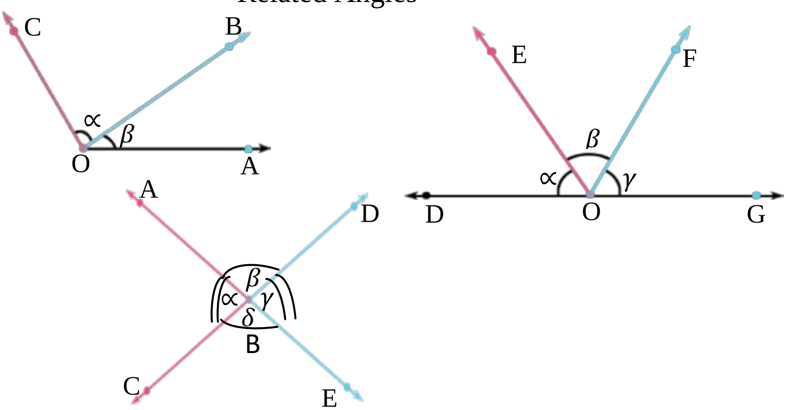


Figure 6.2

Related Angles



Parallel lines and a transversal

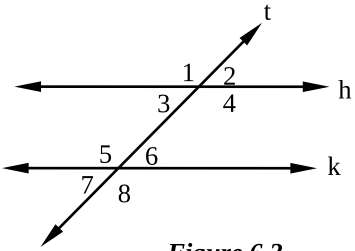
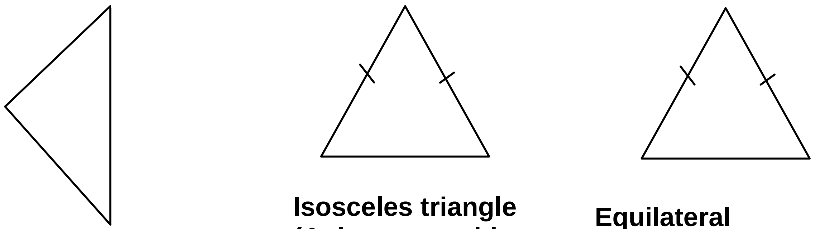


Figure 6.3

Triangles classified by their sides



Scalene triangle
(No sides congruent)

(a)

Isosceles triangle
(At least two sides congruent)

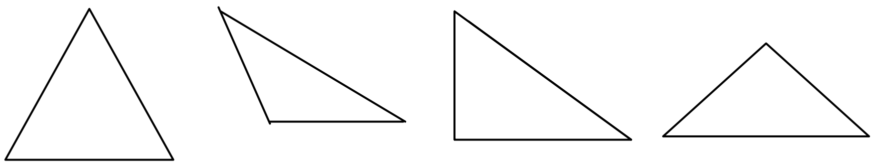
(b)

Equilateral
(All sides congruent)

(c)

Figure 6.4

Triangles classified by their angles:



Acute angled triangle
(Three acute Angles)

(a)

Obtuse angled triangle
(one obtuse angle)

(b)

Right angled triangle
(one right angle)

(c)

Equiangular triangle
(All angles are congruent)

(d)

Figure 6.5

Congruent triangles

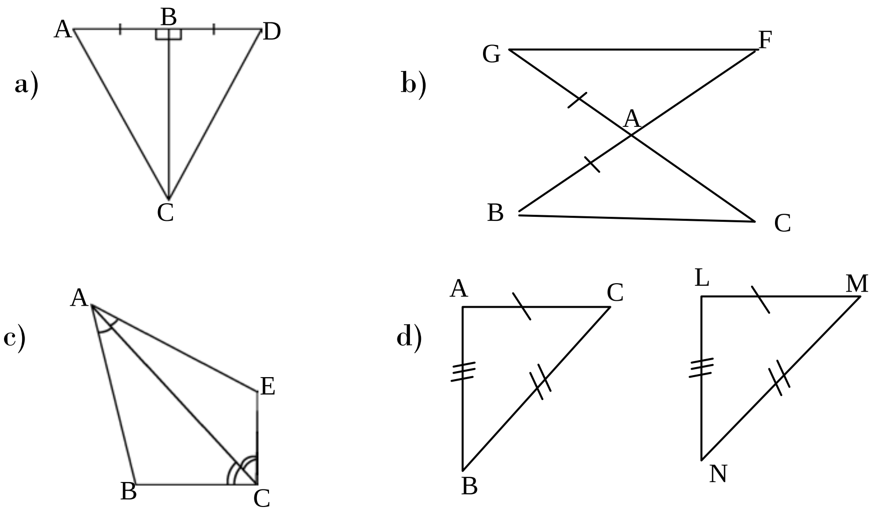


Figure 6.6

Volume of rectangular prism

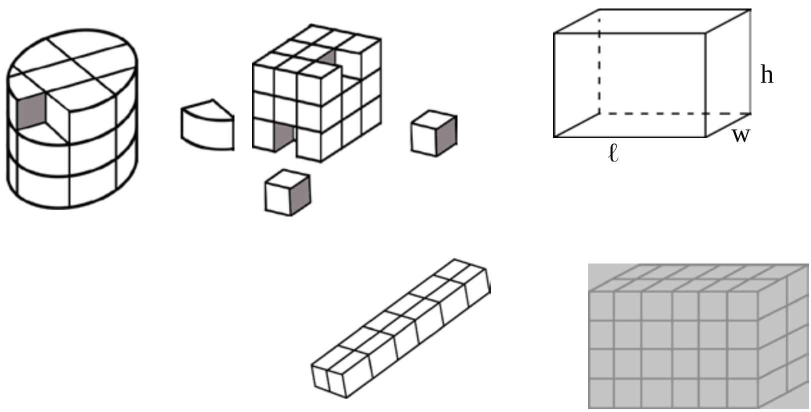


Figure 6.7

6.1 Angles

Periods allotted: 8 periods

Competencies

At the end of this sub unit, students will be able to:

- identify adjacent and vertically opposite angles.
- determine complementary angles.
- determine supplementary angles.
- identify a transversal.
- identify alternate interior angles.
- identify alternate exterior angles.
- identify corresponding angles.
- prove congruency of angles formed by two parallel lines and a transversal by measurement.
- solve problems related to angles formed by two parallel lines and a transversal.

Introduction

This sub-unit deals with further study on angles. It is sub divided in to two subtopics. The first sub topic, that is 6.1.1, deals with related angles (Adjacent angles, vertically opposite angles, complementary angles and supplementary angles). In subtopic 6.1.2 Postulates on angles and parallel lines are stated and discussed.

Teaching notes

The discussion in each of the sub-topics assumes students to have some background on introductory geometric ideas such as related angles and properties of angles on parallel lines. The ways the subtopic may be treated are explained as follows:

6.1.1 Related Angles

You can start the discussion of this sub-topic by revising what students had studied about angles. After introducing adjacent angles and vertically opposite angles encourage students to identify them.

Discuss the relations and explain the properties of complementary and supplementary angles.

Answers to Activity 6.1

1. a) True   b) True c) True   d) True   e) True

2. a) m (<ABC), B is its vertex $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are sides of angle ABC

   b) m (<PQR), Q is its vertex, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are sides of angle PQR

   c) m (<STU), T is its vertex, $\overrightarrow{TS}$ and $\overrightarrow{TU}$ are sides of angle STU

   d) m (<ZYX), Y is its vertex, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are sides of angle ZYX

3. a) A and B   b) C, D and E

4. a) < AOB, <COA, <COB

   b) <EOG, <HOE, <HOG, <GOF, <FOH, <GOH

   c) <JOI, < IOK, < KOJ

5. a) 33^0 b) 90^0 C) 135^0

6, a) m (<AOB) = 70^0

   M (<BOC) = 40^0

   M (<COA) = 110^0

   b) m (<QOP) = 70^0

   m (<ROQ) =60^0

   m (<SOR) =50^0

   m (<SOQ) = 110^0

   m (<ROP) = 130^0

   m (<POS) = 180^0

7. a) 55^0 ,   b) 90^0,   c) 135^0 ,   d) 295^0

8. a) acute   b) acute   c) right   d) obtuse

   e) straight   f) obtuse   g) reflex   h) reflex

Answers to Group Work 6.1

1. M(<A) + m(<B) + m(<C) = 180^0
2. $(x+15)^0 + (10x-20)^0\ (x + 5)° = 180^0$. Therefore $x=15^0$

Assessment

You can give problems on identifying angles in the form of class work, homework, assignment, quiz or test in order to assess students' level of understanding. You may use Activity 6.1 and Exercise 6.A for this purpose. You can also ask the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

Find the degree measures of angles marked x in the figure shown below, where $\overrightarrow{AB} \parallel \overrightarrow{CD}$

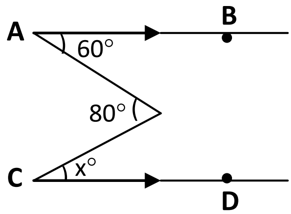
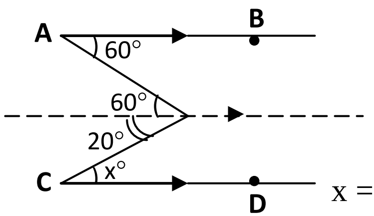


Figure 6.8

Answer to additional assessment



x = 20°

Figure 6.9

Answers to Exercise 6.A

1. a) True b) True c) False d) True
   e) True f) False

2.

Measure of Angle	Measure of complementary angle	Measure of supplementary angle
32^0	58^0	148^0
43^0	47^0	137^0
30^0	60^0	150^0
54^0	36^0	126^0
9^0	81^0	171^0

3. a) $x + 20^0 = 80^0$

$x + 20^0 - 20^0 = 80^0 - 20^0$

$x = 60^0$

b) $x - 10^0 = 130^0$

$x - 10^0 = 130^0 + 10^0$

$x = 140^0$

4. $\alpha = 60^0$ ….. Given

$\alpha = \gamma$ …… vertically opposite angles

Thus, $\gamma = 60^0$

$\beta + \gamma = 180^o$……. Angles on a straight line

$\beta + 60^0 = 180^o$……. implies that $\beta = 120^o$

$\delta = \beta$ ……. Vertically opposite angles

Thus, $\gamma = 120^0$

5. a

6. a) $\beta=110^o$ b) $\alpha = 120^o$

c) $\alpha + \beta = 180^0$ or $\beta - 20 + \beta = 180$ since $\alpha = \beta - 20$. That is, $2\beta = 200$.

It implies that $\beta = 100$. Therefore, $\alpha = 80^0$

d) $\alpha + \beta = 180^0$ or $½ \beta + \beta = 180^0$ since $\alpha = ½\beta$

that is $3/2\beta = 180^0$ or $\beta = 2/3\ (180^o) = 120^0$.

This implies that $\alpha = 60^0$

e) $\alpha = \beta = 90^0$

7. < 1 = <2 (vertically opposite angles), m (<2) $+88^0 = 180^0$ (adjacent supplementary angle). Therefore m (<1) = m (<2) = 92^0

6.1.2 Angles and Parallel lines

You can start the lesson by introducing by what we mean a transversal to two parallel lines Draw a transversal crossing two parallel lines and name the alternate interior angles, corresponding angles, same side interior angles, alternate interior angles, alternate exterior angles.

Let students identify alternate interior angles, corresponding angles, same side interior angles, and alternate exterior angles.

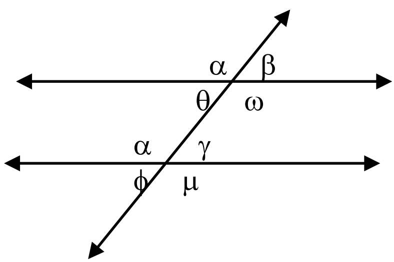


Figure 6.10

You may use Activity 6.2 for this purpose.

Assist students to demonstrate and to come to the conclusion of congruency of alternate interior angles, alternate exterior angles and corresponding angles, if and only if the two lines are parallel. That is when a transversal is drawn to two parallel lines by using measurement of angles.

Motivate students to work on problems related to angles formed by parallel lines and a transversal.

Answers to Activity 6.2

Alternate interior angles: θ and γ', α and β'

Alternate exterior angles: β and μ , δ and θ'

Answers to Activity 6.3

<1 and <2 are supplementary.

Hence m (<2) = 120^0

m(<3) = 120^0,

m(<4) = 60^0,

m(<5) = 60^0,

m(<6) = 120^0,

m(<7) = 120^0,

and m(<8) = 60^0

Answers to Group Work 6.2

$k // \ell$….. Given

m(<2) = m(<1) …. Corresponding angles

$t \perp \ell$ ….. Definition of a right angle and definition of perpendicular lines.

Answers to Activity 6.4

$\alpha + \beta = 180^0$

Assessment

You can give problems on identifying alternate interior angles, alternate exterior angles and corresponding angles when a transversal crosses two parallel lines in the form of class work, homework, assignment, quiz or test in order to assess students' level of understanding.

You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional assessment

In the figure shown ℓ//m. Find the measure of each angle.

a) <3 b) <4 c) <8

d) <6 e) <1 f) <5

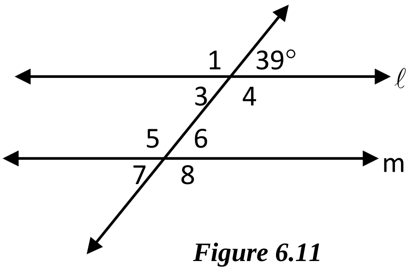


Figure 6.11

Answers to Additional assessment

a) 39^0 b) 141^0 c) 141^0 d) 39^0 e) 141^0 f) 141^0

Answers to Exercise 6.B

1. A

2) $\alpha = 80^0$, $\gamma = 100^0, \beta = 115^0$ and $\theta = 115^0$

3. a) $x+25^0 = 85^0$
   $x = 60^0$
   b) $x - 22^0 = 130^0$
   $x = 152^0$
   c) $200^0 - x = 150^0$
   $x = 50^0$
   d) $100^0 - x = 75^0$
   $x = 25^0$
   e) $x+20^0 = 140^0$
   $x = 120^0$
   f) $x + 16^0 = 90^0$
   $x = 74^0$

4.

θ	α	β	γ	δ	∊
35^0	145^0	35^0	145^0	145^0	35^0
76^0	104^0	76^0	104^0	104^0	76^0
138^0	42^0	138^0	42^0	42^0	138^0

5. a) m (<GBH)=m (<ADC) because m(<GBH) = m(<EAB) as $\overleftrightarrow{AD}//\overleftrightarrow{BC}$

This implies x + 14 = 80 (corresponding angles)

Or x= 66^0 again m (<EAB) = m(<ADC)

b) From (a), you have as $\overleftrightarrow{AB}//\overleftrightarrow{DC}$ (corresponding angles)

m(<ADC) = m(<GBH) = 80^0

and m(<EAB) = 80^o (Corresponding angle)

m (<EAB) + m(<BAD) = 180^0 (supplementary angles)

that is, 80^0 + m(<BAD) = 180^0

or m(<BAD) = 100^0

C) m(<JCI) = m (<GBA) … alternate exterior angles

And m(< GBA) + m (<GBH) = 180^0 … supplementary angles

That is, m(<GBA) + 80^0 = 180^0

Or m(<GBA) = 100^0

Therefore, m (<ICJ) = 100^0

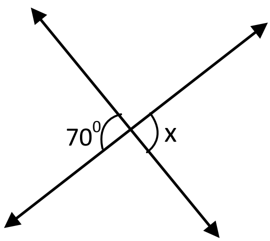
6. $\overleftrightarrow{BC} \parallel \overleftrightarrow{DF}$ because corresponding angles are congruent.

That is, $60^0 + 20^0 = 64^0 + 16^0$

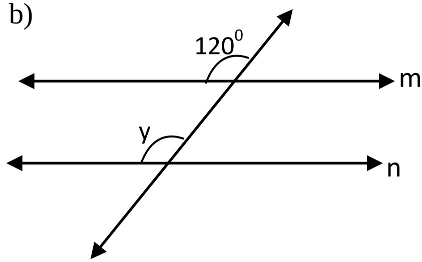
Selected problems to slow learners

1. Find the measures of angles x and y (where m//n) and t is a transversal.

a)



b)



c)

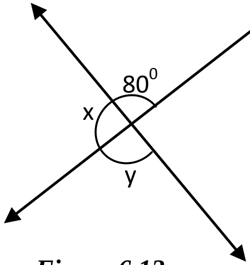


Figure 6.12

2. In the figure below
   $x = 20^0$, $y = 60^0$ and $z = 100^0$.
   Is ABC is a straight line? Why?

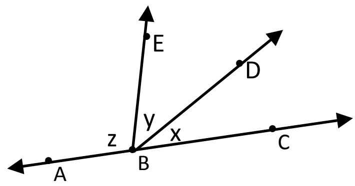


Figure 6.13

3. If $a = 110^0$, then find angles b, c and d.

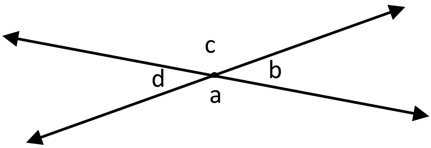


Figure 6.14

4. Angle is half of its supplement. Find the angle.
5. Find the measure of the angle x and y.

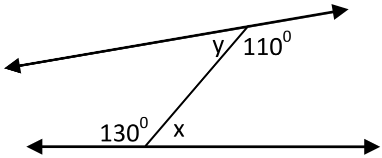


Figure 6.15

6. Find the measure of angle a.

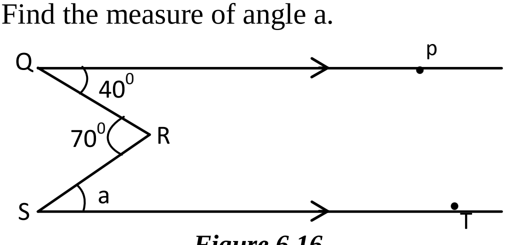
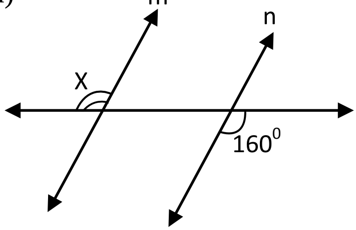


Figure 6.16

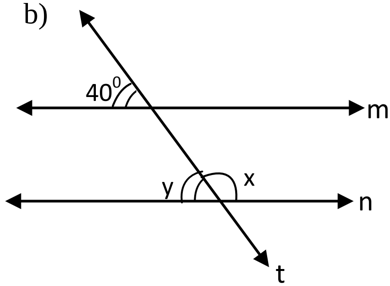
Selected problems to fast learners

1. Find the measures of angles x and y (where m // n) and t is transversal.

a)



b)



c)

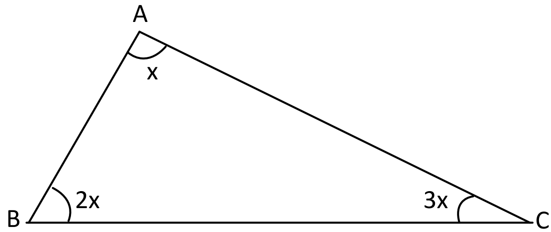


Figure 6.17

2. An angle is twice its complement. Find the angle.
3. Find the measure of angle x

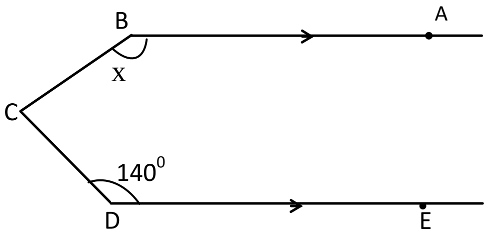


Figure 6.18

4. Find the measure of angle a

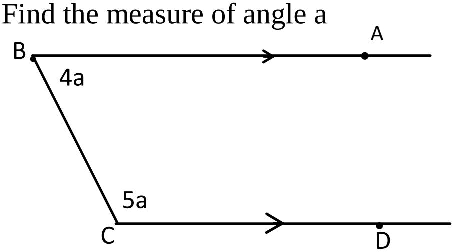


Figure 6.19

5. Find the measure of angle b

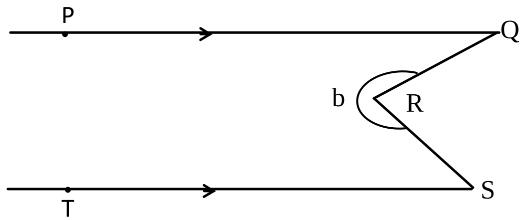


Figure 6.20

6.2 Construction of Triangles

Periods allotted: 12 periods

Competencies

- construct triangles given three sides.
- construct triangles given two sides and an included angle.
- construct triangles given two angles and an included side.
- explain the relation between angles and sides of a triangle.
- explain the relation between sides of a triangle.

Introduction

The students are already familiar with a triangle and its parts. In this sub unit they will be introduced to construction of triangles; given the length of three sides, two sides and the included angle, and two angles and the length of one side. Students are required to use a ruler, pair of compasses and protractor for the purpose of construction of triangles. Triangle inequality is stated to help students explain relation between sides of a triangle.

Activities and Exercises are included in order to involve students in the discussion of the sub-unit.

Teaching notes

As indicated in the text book, introduce construction of triangles (a) given the length of three sides, (b) given two sides and the included angle, (c) two angles and the length of one side. In order to help them understand the required concepts about construction of triangles, make them try Activity 6.7. you are required to ensure active participation of students in constructing triangles and explaining relation between angles and sides of triangles, also the relation between sides of a triangle.

Motivate students to construct using a ruler, pair of compasses and protractor if a triangle is given with

a) the length of the three sides.
b) the length of two sides and the included angle.
c) the measures of two angles and the length of the included side.

Answers to Group Work 6.3

a) Obtuse angled
b) Acute angled
c) obtuse angled
d) right angled

Answers to Group Work 6.4

i) AB + BC > AC
ii) AB + AC > BC
iii) BC + AC > AB

Conclusion: (1) If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

2. If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
3. **Triangle inequality**: the sum of the length of any two sides of a triangle is greater than the length of the third side.

Assessment

You can give problems on construction of triangles in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask the following question to fast learners or interested students to answer as additional assessment.

Additional assessment

The lengths of two sides of a triangle are 3cm and 7cm. Then find all possible whole numbers which can be the length of the third side of this triangle.

Answers to additional assessment

Let x be the length of the third side, where $x \in W$. Then $x + 3 > 7$, $3 + 7 > x$ and $x + 7 > 3$ according to triangle inequality. Thus, $x > 4$, $10 > x$ and $x > -4$. That means x is a whole number between 4 and 10. Therefore x = 5, 6, 7, 8, or 9.

Answers to Exercise 6.C

1. a) $\left.\begin{array}{l} 2 + 3 > 3 \\ 3 + 3 > 2 \\ 3 + 2 > 3 \end{array}\right\}$ yes.

   b) yes

   c) No, because $4 + 3 = 7$

   d) $\left.\begin{array}{l} 0.4 + 0.5 > 0.8 \\ 0.5 + 0.8 > 0.4 \\ 0.4 + 0.8 > 0.5 \end{array}\right\}$ yes.

2. The largest angle is opposite the longest side ($\overline{BC}$). Thus, <A is the largest angle in Δ ABC

   The smallest angle is opposite the shortest side ($\overline{AB}$). Thus, < C is the smallest angle in ΔABC.

3. Make sure that m (<D) +m (<E) + m(<F) = 180^o and so m (<F) = 60^o. Now, the longest side is opposite the largest angle (<D). Thus, $\overline{EF}$ is the longest side in ΔDEF.

The shortest side is opposite the smallest angle (<F). Thus, ED is the shortest side in   DEF.

4. m (<P) + m (<Q) + m (<R) = 180^0

   That is , $60^o + 75^o$ + m (<R) = 180^0

   135^o + m (<R) = 180^0

   Therefore, m (<R) = 45^0

   You can see that $\overline{PR}$ is the longest side and $\overline{QP}$ is the shortest side in ΔPQR.

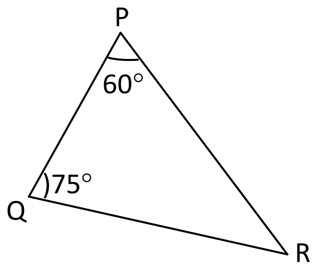


Figure 6.21

Selected problems to slow learners

1. Which of the following three numbers can represent measures of three sides of a triangle?

   a) 1, 2, 3   b) 3, 4, 5   c) 2, 3, 5   d) 2, 3, 6

2. Name the largest angle and the smallest angle of ΔABC.

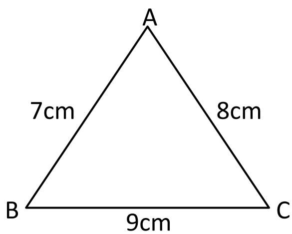


Figure 6.22

3. Try to construct a triangle whose sides are 3cm, 4cm and 7cm. what do you see and can you say why?
4. If two angles of a triangle are 90^0 and 40^0, then find the measure of the third angle.
5. What is the sum of the measures of the three angles of a triangle?
6. Construct ΔABC where AB = 6cm, BC = 4cm and CA = 5cm.

Selected problems to fast learners

1. What should be the possible values of x if x, 3 and 6 represent measures of three sides of a triangle, where x is a counting number?
2. Name the largest side and the shortest side of ΔPQR.

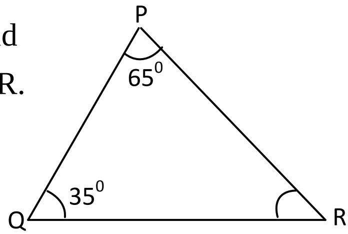


Figure 6.23

3. Try to construct a triangle whose sides are 5cm, 3cm and 8cm. what do you see and can you say why?
4. Two angles of a triangle are complement to each other. What is the measure of the third angle?
5. If 2x, 3x and 5x are the degree measures of the three angles of a triangle, then name this triangle.
6. Construct ΔABC, where AB = 6cm, m(<A)=55^0 and AC = 4cm.

6.3 Congruent Triangles

Periods allotted : 12 periods

Competencies

At the end of this sub-unit, students will be able to:

- explain the concept of congruency of triangles.
- check the congruence of given triangles by tracing, cutting and over lapping.
- identify the congruence of two given triangles by using the tests for congruence (SAS, SSS and ASA).

Introduction

This sub – unit deals with introductory ideas on the concept of congruency of triangles and the tests for congruence (SAS, SSS and ASA). Students can also use tracing, cutting and over lapping to check congruency of triangles. Activities and exercises are included in order to involve students in the discussion of the sub – unit.

Teaching notes

Students are expected to prepare a triangle congruent to a given triangle by tracing, cutting and overlapping.

6.3.1 Congruence of Triangles

You can start the discussion of this sub topic by showing pictures having equal size and same shape after introducing the concept of congruent figures and the symbol "≡" for congruence, assist students to explain congruent triangles.

Let them check congruent triangles by tracing, cutting and overlapping one over the other.

Encourage students to come to conclusion of congruence of triangles by classifying congruency of corresponding sides and corresponding angle (for example, you may use models of two congruent scalene triangles).

Answers to Activity 6.5

You may check congruency of the given figures by tracing, cutting and overlapping one over the other.

Assessment

You can give problems on congruency of triangles in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also give the following questions to fast learners or interested students to answer as additional assessment.

Additional assessment

Identify congruent triangles in the figure shown

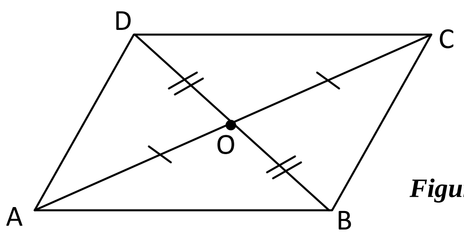


Figure 6.24

Answers to additional assessment

ΔAOD ≅ ΔCOB … by SAS (<AOD ≅ <COB…vertically opposite angles)

ΔDOC ≅ΔBOA…. by SAS (<DOC≅<BOA … vertically opposite angles)

Answers to Exercise 6.D

1. a) True b) False c) True

2. a) $\overline{NL} \cong \overline{QR}$, <L ≅ <R b) $\overline{FG} \cong \overline{YW}$, <J ≅ <X

   c) <A ≅ <D, <B ≅ <E, <C ≅ <F

   $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, $\overline{AB} \cong \overline{DE}$

3. a) No. Corresponding parts of the two triangles are not congruent.

   b) Yes, ΔLKJ ≅ ΔONM.

4. a) <ABC ≅ <FED b) $\overline{AB} \cong \overline{FE}$

   c) <F ≅ <A d) ΔABC ≅ ΔFED

   e) ΔBAC ≅ ΔEFD f) ΔCAB ≅ ΔDFE

6.3.2 Tests for congruency of Triangles (SAS, SSS and ASA)

In order to introduce SAS, you may start the discussion by taking two sides and an included angle where given the lengths of two sides and the degree measures of included angles are correspondingly equal.

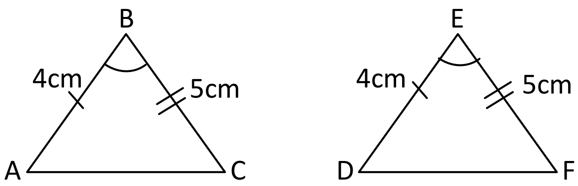


Figure 6.25

Assist students to measure the remaining side and two angles of each triangle and let them write their findings.

Encourage students to conclude the congruence of these two triangles by showing that all conditions in the definition are fulfilled.

Guide students to reach at the following statement:
SAS (side, Angle, side) postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle then the two triangles are congruent. You may use Activity 6.6, Activity 6.7 and Activity 6.8 to each SSS and ASA.

Answers to Activity 6.6

$\Delta ABC \cong \Delta\ EFG$

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

That is, if in the figure,
$\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong SU$ and
$QR \cong TU$, then
$\Delta\ PQR \cong \Delta STU$

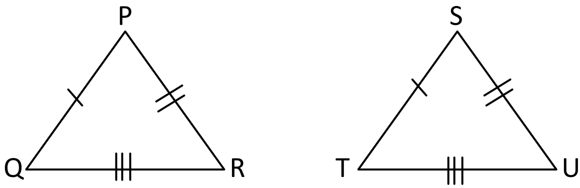


Figure 6.26

Answers to Activity 6.7

$\overline{BC} \cong \overline{EF}$ and $\Delta ABC \cong \Delta DEF$

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. That is if in the figure, $EF \cong HI$, $<E \cong <H$ and

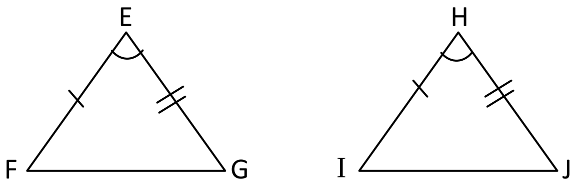


Figure 6.27

$\overline{EG} \cong \overline{HJ}$, then $\Delta\ EFG \cong \Delta\ HIJ$

Answers to Activity 6.8

$\overline{AB} \cong \overline{DE}$, <A ≅<D and also <B ≅<E

Δ ABC ≅ Δ DEF… ASA

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Assessment

You can give problems on congruency of triangles in the form of class work, home work, assignment, quiz or test in order to assess students' performance. You can also ask the following questions to fast learners or interested students to answer as additional assessment.

Additional assessment

Write the congruency relation if there is any congruency between the triangles shown below.

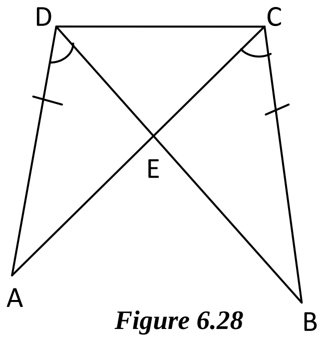


Figure 6.28

Answers to additional assessment

Δ ADE ≅ ΔBCE by ASA

Answers to Exercise 6E

a) ΔABC≅ ΔDBC by SAS

b) No congruence can be deduced

c) ΔGAF≅ ΔBAC by ASA

d) ΔABC≅ ΔAEC by ASA

e) ΔABC≅ ΔLNM by SSS

Selected problem to slow learners

1. If Δ ABC ≅ Δ DEF then state all corresponding congruent sides and congruent angles in the two triangles.
2. If Δ PQR≅ Δ STU then Δ QRT ≅ Δ ______
3. In the figure
   Δ ABC ≅ Δ DEF by

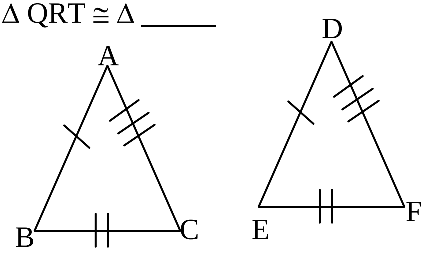


Figure 6.29

   a) AAS   b) SAS   c) SSS

4. Δ PQR ≅ Δ STU by
   a) SSS
   b) ASA
   c) SAS

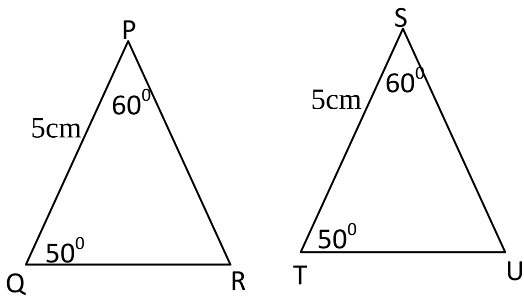


Figure 6.30

5. Two equilateral triangles are necessarily congruent (True or false)?
6. Identify congruent triangles in the figure shown

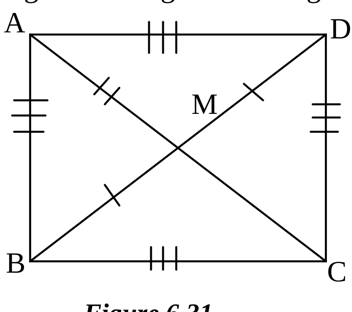


Figure 6.31

Selected problems to fast learners

1. If Δ ABC $\cong$ Δ PQR and Δ PQR $\cong$ Δ EFG then what can you say about congruency of Δ ABC and ΔEFG?
2. Are the figures below congruent? Why?

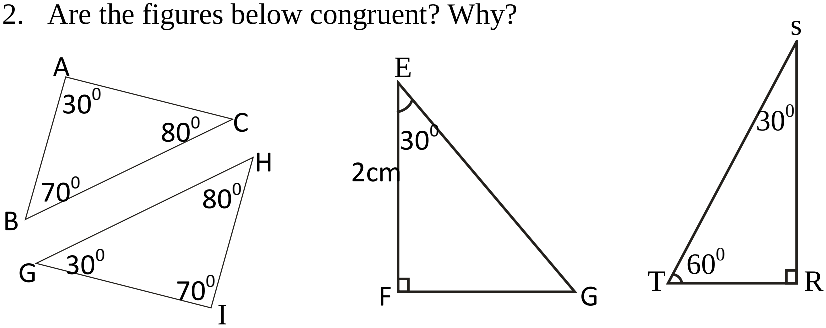


Figure 6.32

3. Can we always say that Δ ABC $\cong$ Δ DEF knowing only $<$ A$\equiv$ D, and B $\equiv$ $<$ E?

4. If in the figure

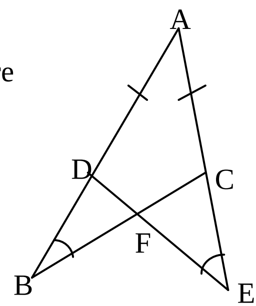


Figure 6.33

5. If in the figure $\overline{PQ} \cong \overline{RS}$ and $<$ Q $\cong$ the identify congruent triangles.

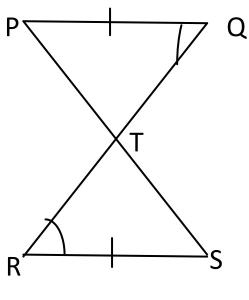


Figure 6.34

6.4 Measurement

Periods allotted: 12 periods

Competencies

At the end of this sub – unit, students will be able to:

- derive the formula of area of right angled triangle from the area of rectangle.
- calculate the area of right angled triangle.
- convert square meters to square centimeters and vice versa.
- calculate the perimeter of triangles.
- discover the formula for the volume of a rectangular prism.
- calculate the volume of a rectangular prism.
- convert cubic centimeters to liters and cubic meters and vice versa.
- convert milliliters to liters and vice versa.

Introduction

In this sub-unit, measurement of geometric figures is studied. Students will be able to compare perimeter, and area of geometric figures. They will also be able to calculate the volume of a rectangular prism and convert cm^3 to liters and vice versa.

Activities and Exercises are included in order to involve students in the discussion of the sub- unit.

Teaching notes

As far as possible, ensure active participation of students in the discussion of the sub-unit. The presentation of this sub – unit is divided in to two subtopics as follows.

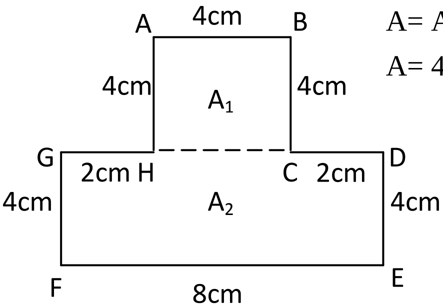
6.4.1 Areas of right angled triangles and perimeter of triangles.

Motivate students to write a formula for finding the area of a right angled triangle and perimeter of triangles.

Assist students to exercise computing areas of right angled triangles and perimeters of triangles. Discuss with students the method of converting units of area (cm^2 to m^2 or vice versa). Encourage students to practice conversion of units of areas.

Answers to Activity 6.9

a)



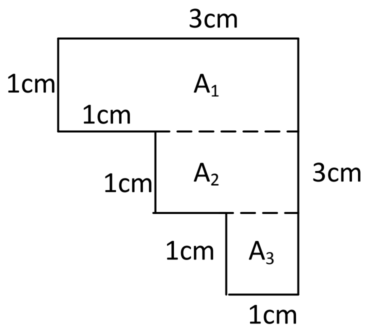
P = 4+4+2+4+8+4+2+4 = 32 cm

A= A_1 +A_2 = (4×4) + (8×4)

A= 48 cm^2

Figure 6.35

b)



P= 3+3+1+1+1+1+1+1= 12cm

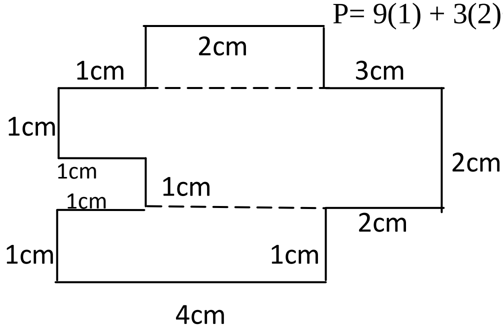
A= A_1 +A_2 + A_3

= (1×3) + (1×2) + (1×1)

A= 6cm²

Figure 6.36

c)



P= 9(1) + 3(2)+3+4

P= 22cm

A= A_1 +A_2 + A_3 +A_4

= (1×1) + (1×2)

+ (2×5) + (1×4)

A= 17cm²

Figure 6.37

d)

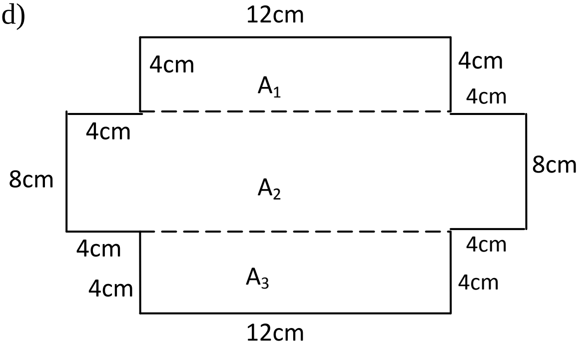


Figure 6.38

$P = 2(12) + 8(4)+2(8)$

$P = 72cm$

$A= A_1 + A_2 + A_3$

$= (4\times12) + (20\times8) + (4\times12)$

$\mathbf{A= 256cm^2}$

Answers to Group Work 6.5

a) Two right angled triangles.

b) They are congruent.

c) Area of rectangle = bh.

d) Area of each triangle = $\frac{1}{2}$bh.

Answers to Activity 6.11

1 hectare = 100m x 100m = 10,000m^2 or

1 hectare = 10,000cm x 10,000cm = 100,000,000cm^2

Assessment

You can give problems on computing areas of right angled triangles, perimeter of triangles, and also on conversion of units of area in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask fast learners or interested students to answer the following question as additional assessment.

Additional assessment

Convert 0.01m^2 to hectare.

Answer to additional assessment

1m^2 = 0.0001 hectare

0.01m^2 = ?

$\dfrac{0.01 \text{ x } 0.0001}{1} = 0.000001$

Therefore, $0.01m^2 = 0.000001$ hectare.

Answers to Exercise 6.F

1. a) $1m^2 = 10{,}000\ cm^2$

$50m^2 = 50 \text{ x} 10{,}000cm^2 = 500{,}000\ cm^2$

b) $1cm^2 = 0.0001m^2$

$100cm^2 = 0.01m^2$

c) 1 hectare $= 10{,}000m^2$

0.4 hectare $= 0.4 \times 10{,}000m^2 = 4000m^2$

d) $10{,}000m^2 = 1$ hectare

$1000m^2 = \dfrac{1000}{10000}$ hectare = 0.1 hectare

e) $1cm^2 = 100mm^2$

$7.5cm^2 = 7.5 \times 100mm^2 = 750mm^2$

f) $1mm^2 = 0.01cm^2$

$800mm^2 = 800 \text{x} 0.01 = 8cm^2$

g) 1 hectare $= 100{,}000{,}000cm^2$

0.09 hectare $= 0.09 \times 100{,}000{,}000 = 9{,}000{,}000cm^2$

2. a) $A = A_1 + A_2$

$= (4 \times 6) + (\frac{1}{2} \times 6 \times 3)$

$= 24 + 9 = 33$ square units

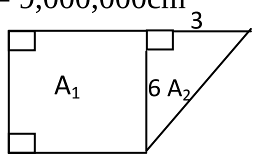


Figure 6.39

b) $A = A_1 + A_2 + A_3 = (\frac{1}{2} \times 4 \times 5) + (7 \times 4) + (\frac{1}{2} \times 6 \times 4)$

A= 10+28+12

A= 50 square units

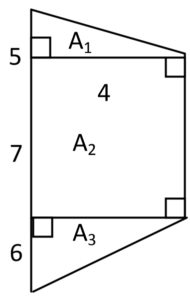


Figure 6.40

c) A= $(\frac{1}{2}\times4\times8)+\frac{1}{2}(6\times8)$

$= 16+24=40$ square units

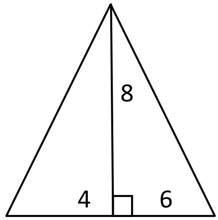


Figure 6.41

d) A= $(\frac{1}{2}\times44\times57)+(36\times37)$

$= 1144 + 1332$

$= 2476cm^2$

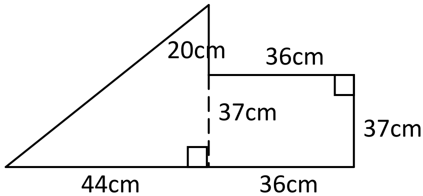


Figure 6.42

3. A= $48cm^2$, h= 12cm

   A= ½ bh, ½ b (12) = 48

   Thus, b = 8cm

4. Perimeter of ΔABC = 2 + 3 + 4 = 9cm

   Perimeter of ΔEDF = 4 + 3 + 5 = 12 cm

   Perimeter of Δ PQR = $1+1+\frac{3}{2}$ = 3.5cm

   Therefore, perimeter of ΔEDF is the greatest.

5. A= $\frac{1}{2}$ bh

   $160 = \frac{1}{2}(40)(h)$

   160 = 20h

   Therefore, h = 8m

6. Area of shaded = Total Area–(Area of the two right angled triangles + area of rectangle)

Area of shaded region = $\frac{1}{2}(18\times13)-(\frac{1}{2}\times6\times8+\frac{1}{2}\times5\times12+5\times6)$

$= (9\ 13) - (24 + 30 + 30)$

$= 117 - 84$

$= 33$ square units

6.4.2 Volume of Rectangular prism

Encourage students to discover the formula of rectangular prism by counting cubic centimeters. Motivate students to calculate the volume of rectangular prism and also practice converting units of volume. That is, liter to cm^3 and vice versa.

Answers to Activity 6.12

a) V= 12 cubic units b) V= 16 cubic units

Assessment

You can give problems on finding volumes of rectangular prisms in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask the following question to fast learners or interested students to answer as additional assessment.

Additional assessment

Convert $10cm^3$ to litre.

Answer to Additional Assessment

$1cm^3 = 0.001\ell$

$10cm^3 = 10 \times 0.001\ \ell$

Thus, $10cm^3 = 0.01\ \ell$

Answers to Exercise 6.G

1. a) $V = 6 \times 5\frac{1}{4} \times 7\frac{1}{2} = 6 \times \frac{21}{4} \times \frac{15}{2} = \frac{945}{4} m^3$

   b) $V = 3.5 \times 9 \times 7.2 = 226.8cm^3$

   c) $V = 3 \times 4.4 \times 1.5 = 19.8cm^3$

   d) $V = 7 \times 12 \times 14 = 1176cm^3$

   e) $V = 7 \times 2 \times 2 = 28\ m^3$

   f) $V = 8.8 \times 1.5 \times 0.5 = 6.6mm^3$

   g) $h = 10cm = 100mm$

   $\therefore V = 5 \times 7 \times 100 = 3500\ mm^3$

   h) $h = 7cm = 0.07m$

   $\therefore V = 12 \times 9 \times 0.07 = 4.41m^3$

   i) $h = 10.6mm = 1.06cm$

   $\therefore V = 12.1 \times 8.2 \times 1.06$

   $V = 105.1732cm^3$

2. a) $V = 7^3 = 343cm^3$
   b) $V = \ell^3$, where ℓ is the length of its side
3. a) $h = \dfrac{V}{\ell\omega} = \dfrac{122{,}500}{50 \times 35} = 70cm$

   b) $h = \dfrac{V}{\ell\omega} = \dfrac{22.05}{3.5 \times 4.2} = 1.5cm$

   c) $h = \dfrac{V}{\ell\omega} = \dfrac{3{,}375}{15 \times 15} = 15mm$
4. a) $20m^3 = 20{,}000{,}000cm^3$

   b) $100cm^3 = \dfrac{100}{1{,}000{,}000}\ m^3 = 0.0001m^3$

   c) $0.5m^3 = 500{,}000cm^3 = 500$ liters

   d) $5000m^3 = 5{,}000{,}000{,}000cm^3$
   e) 3 liters = $3000\ cm^3$
   f) $2000cm^3$ = 2 liters

   g) $100cm^3 = 100{,}000mm^3$

Assessment

As this is the end of this unit, you can give quiz or test or group work in order to assess students learning and understanding. Check students' responses and always keep record of students' progress, give immediate feedback or corrections to their work. Give individual assistance to those who are logging behind.

Selected Problems to slow learners

1. Find the perimeter.
   a)

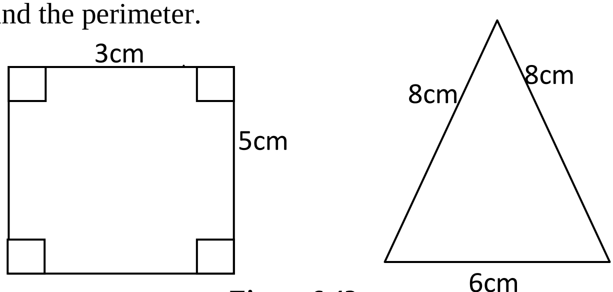


Figure 6.43

2. Find the area of a right angled triangle if its legs are 6 cm and 8cm.
3. Find the volume of a cube where the length of its sides is 6cm.

4. Find the volume of a rectangular prism whose length is 5 cm, width is 4cm and height is 10cm.
5. Convert 300 cm^3 in to liter.
6. Name the vertices and faces of the cube.

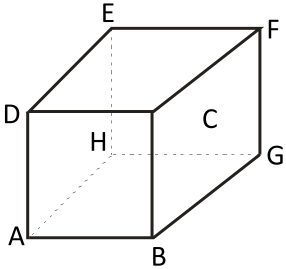


Figure 6.44

7. Cover $2m^3$ into cm^3.
8. Convert 4 liter in to cm^3.
9. A carpet is in the shape of a right angled triangle and has area $100m^2$. If the base of the carpet is 20m, then find its height.

Selected problems to fast learners

1. Find the length of legs of a right angled triangle whose area is 32 cm^2.
2. If the volume of a cube is $\frac{64}{27}$ cm^3, then find the length of one of its edge.
3. Find the volume of a cube where the length of its sides is 0.1m.
4. Find the length of a rectangular prism whose width is 7 centimeters, height is 12 centimeters and the volume is $840cm^3$.
5. Convert $200cm^3$ in to liter.
6. Convert $0.2m^3$ in to cm^3.
7. Write the edges, vertices and faces of the cube.

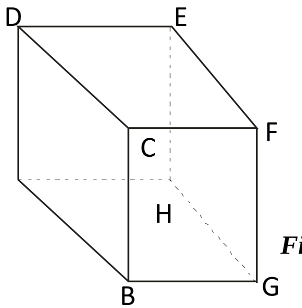


Figure 6.45

8. Convert $\frac{1}{2}$ liter in to cm^3.

Answers to Review Exercise

1. a) False b) True c) True
   d) True e) True f) False

2. a) $x = 45^0$ because $x + 3x = 180^0$
   b) $x = 18^0$ because $x + 4x = 90^0$
   c) $x + 10 = 60$. Hence $x = 50^0$
      $m(<B) = 55^0$

3. $x + y = 90^0$, $y + z = 90^0$, $x = z$ and $y = 2x$
   Therefore $x + 2x = 90^0$ or $x = 30^0$, $y = 60^0$, $z = 30^0$

4. Let m (<COB) = x, then
   $70^o + m(<BOA) = 110^0$
   $m(<BOA) = 40^0$
   $x + 40 = 80^0$
   Therefore $x = m(<BOC) = 40^0$

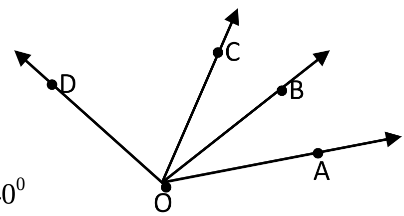


Figure 6.46

5. b) $60^0 + 56^0 = 156^0$
   Therefore, the lines marked are parallel.

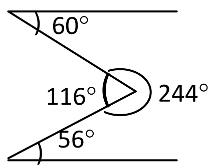


Figure 6.47

6. a) $60 + 55 + x = 180$
   $\therefore$ $x = 65^0$ and $y = 60^0$
   b) $x = 120^0$, $y = 40^0$

7. a) Δ ABC $\cong$ Δ ADE by SAS
   b) Δ ABC $\cong$ Δ ABD by SAS
   c) Δ ABC $\cong$ Δ FED by SSS
   d) No congruence can be concluded
   e) Δ CAB $\cong$ Δ CED by ASA

8. $\dfrac{(x+5)+(4x-6)+(2x+1)}{3} = 14$

$$\frac{7x}{3} = 14$$

$\therefore$ x = 6, that is, AB= 11, BC = 13 and AC = 18 and therefore, AC is the longest side

9. (x+3) + (3x +2) + (2x +3) = 20

That is, 6x + 8 = 20 or 6x =12 or x =2 Thus, AB = 5, AC =8 and BC= 7

It implies that Δ ABC is scalene

10. Eight : (1,1,16), (2,2,14), (3,3,12), (4,4,10), (5,5,8), (6,6,6), (7,7,4), (8,8,2) are triplets of side measures.

11. The area of the shaded region

$= \frac{1}{2}(19\times4) - \frac{1}{2}(10\times4) = 38\text{-}20 = 18$ square units

12. Volume = $V_1 + V_2$

$= (5\times2\times10) + (7\times10\times4)$

$= 100 + 280$

$= 380$ Cubic units

13. Base area = $\ell^2 = 16cm^2$

Thus, = ℓ =4cm

and Volume = $\ell^3 = 4^3 = 64\ cm^3$

14. Volume = 9x5x12=540 m^3

15. $\ell^3 = 125m^3$

Thus, ℓ=5m

and base area = $\ell^2 = 25m^2$

16. x^3=27. Thus, x=3cm

and Volume of the larger box $=(2x)^3=8x^3$

$= 8(27)=216cm^3$

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