MATHEMATICS

Grade 5

Teacher Guide

Author and Editor:

　　　　Abera Temesgen (B.SC.)

　　　　B.Janaki Mohan (M.A)

Evaluators:

　　　　Abdella Mohe

　　　　Meneberu Kebede

Acknowledgements

© Federal Democratic Republic of Ethiopia, Ministry of Education
First edition, 2003(E.C.)
ISBN : 978-999944-2-214-2
Developed, printed and distributed for the Federal Democratic Republic of Ethiopia, Ministry of Education by:

Al Ghurair Printing and Publishing House CO. (LLC)
PO Box 5613
Dubai
U.A.E.

In collaboration with
Kuraz International Publisher P.L.C
P.O. Box 100767
Addis Ababa
Ethiopia

Disclaimer
Every effort has been made to trace the copyright owners of material used in this document. We apologise in advance for any unintentional omissions. We would be pleased to insert the appropriate acknowledgement in any future edition.

Foreword

Education and development are closely related endeavours. This is the main reason why it is said that education is the key instrument in Ethiopia's development. The fast and globalised world we now live in requires new knowledge, skills, attitudes and values on the part of each individual. It is with this objective that the curriculum, which is a reflection of a country's education system, must be responsive to changing conditions.

It is more than fifteen years since Ethiopia launched and implemented the *Education and Training Policy*. Since then our country has made remarkable progress in terms of access, equity and relevance. Vigorous efforts also have been made, and continue to be made, to improve the quality of education.

To continue this progress, the Ministry of Education has developed a Framework for Curriculum Development. The Framework covers all pre-primary, primary, general secondary and preparatory subjects and grades. It aims to reinforce the basic tenets and principles outlined in the *Education and Training Policy*, and provides guidance on the preparation of all subsequent curriculum materials – including this teacher guide and the student textbooks that come with it – to be based on active-learning methods and a competency-based approach.

Publication of a new Framework and revised textbooks and teacher guides are not the sole solution to improving the quality of education in any country. Continued improvement calls for the efforts of all stakeholders. The teacher's role must become more flexible ranging from lecturer to motivator, guide and facilitator. To assist this, teachers have been given, and will continue to receive, training on the strategies suggested in the Framework and in this teacher guide.

Teachers are urged read this guide carefully and to support their students by putting into action the strategies and activities suggested in it. The guide includes possible answers for the review questions at the end of each unit in the student textbook, but these answers should not bar the students from looking for alternative answers. What is required is that the students are able to come up with, and explain knowledgeably, their own possible answers to the questions in the textbook.

Introduction

Mathematics is one of the school disciplines that focus on the enhancement of student's mathematical power and proficiency that lead to purposeful and worth while mathematical work. As a science of patterns and relationships, mathematics relies on **logic**, **reasoning, problem solving** and **creativity**. It is characterized by a cycle of learning that includes representation, manipulation and validation.

Nowadays learning mathematics is becoming helpful in almost every kind of human endeavor. It serves as a basic precise language for the other field of studies such as **science** and **technology.** All sciences use the language of mathematics to describe objects and events, to characterize relationships between variables, and to argue logically. It can be said that learning mathematics is essential in every day life.

Mathematics involves certain interrelated learning elements such as:-

- Comprehension of mathematical terms, concepts, operations and relationships.
- Skill in carrying our procedures flexibly, accurately, efficiently and appropriately.
- Ability to formulate, represent and solve mathematical problems.
- Logical thought, reflection, explanation and justification.

The need to develop continuous assessment implementation teacher guide arise from the following basic assumptions:-

- Effective mathematics instruction requires periodic and constant flow of information about students learning progress or learning deficiencies.
- Repeated and regular assessment of students provides better picture of the instructional process for mathematics teacher.
- A system of continuous assessment in mathematics teacher helps to measure a wide range of mathematical skills (such as problem solving and critical thinking) that cannot easily be assesses by time-limit terminal examinations.

- Implementation of continuous assessment improves the motivation of students to work hard and helps to get involved in learning mathematics.
- The other support systems such as teacher's resource materials (Syllabus, text books, teachers guides) and refreshment courses should be in place to effectively implement.
- Finally it is possible to implement a system of continuous assessment in mathematics in spite of the increased effort time and energy it demands form both teachers and students.

Organization of this teachers Guide

This teacher guide is organized unit by unit. It contains the following major themes:

i. **Introduction**: - includes the role and rational and special Characteristics of learning the subject matter, guidelines on how to use the teacher guide and the nature of continuous assessment.

ii. **Competencies of each unit**: - drawn from mathematics syllabus of grade 5.

iii. Suggested teaching aids.

iv. Sub-unit competencies of each unit.

v. Sub-unit introduction of each unit.

vi. Teaching notes of each unit.

vii. Answers to Activities and Exercises.

viii. Continuous assessment.

ix. Answers to Miscellaneous Exercises for unit by unit.

x. Topics, period allotment and location chart.

xi. Syllabus

Table of Contents

The Concept of Active learning and Continuous Assessment
What is Active Learning?

Active learning: as the name suggests, is a process where by students are actively engaged in the learning process, rather than "Passively" absorbing lectures. Students are rather encouraged to think, Solve problems, do activities carefully selected by the teacher, answer questions, formulate questions of their own, discuss, explain, debate or brain storm question , explore and discover, work cooperatively in group to solve problems and work out project.

Teachers' are strongly advised to discuss and work out difficult questions. As far as possible the class should not be teacher centered. Attention should be given to the following points in motivating students to participate in the lesson through activities, class work, home work, Group work and reading the text book independently.

- Give students a chance to express theorems, definitions, properties and rules in their own for each unit.
- Make students work the activities in class either individually, in pair or in small groups:
- Make the lesson lively be relating it with real examples from the students' environment.
- Use order to methods in teaching i.e from simple to complex methods in teaching.
- In order to evaluate students and find out individual weakness and help them, regular tests should be prepared carefully by referring to the unit out comes in the syllabus.
- Use different types of teaching aids based on each unit.

What is Assessment?

Assessment: is a process by which information is obtained relative to some known objective or goal. The teachers assess at the end of a lesson or unit or the end of a school year through

testing. Generally assessment is defined as collecting information on the progress of students learning using varieties of procedures (Example checklist, formal tests, self-assessment, creative writing,   portfolios).

Purposes of Assessment

Teachers have many purposes for assessment of students. Some of the main reasons are:

1. **Improving instructional materials**: - Teachers need information regarding how effective teaching procedures, activities, the text book and other materials are in teaching.
2. **Improving students learning**: Both teachers and students need to know how students are doing.
3. **Determining content mastery**: Teachers evaluate students to determine if and when they have mastered the subject matter.
4. **Teaching: Evaluation activities**, if appropriately planned and used, can be powerful learning activities.
5. **Grading Students**:- Parents, administrators, and sometimes employees need evidence of pupil progress.

Forms of Assessment

There are two forms of assessment. These are continuous assessment and summative assessment.

Continuous Assessment: of learners' progress could be defined as a mechanism whereby the final grading of learners in the **cognitive**, **affective and psychomotor** domains of learning systematically take account of all their performances during a given period of schooling. Continuous assessment is an assessment approach that involves the use of a variety of assessment so as the assess various components of learning:

- The thinking processes (cognitive skills),
- Behaviors, personality traits (affective characteristics) and
- Manual dexterity (psychomotor domain)

Summative Assessment

This is a summary assessment of the extent to which learners have mastered the intended objectives. It normally occurs at the end or the completion of a semester teaching.

The Need for Continuous Assessment

Continuous assessment as a method of evaluating the progress and achievement of students on a day – to – day basis is relevant to get a clear picture of every students' performance.

Most importantly, planning a continuous assessment system at school level is useful to gather adequate and reliable information about:-

- The present status of every students
- The students motivation to participate actively in the teaching – learning process;
- Students progress in his/her learning;
- Students learning difficulties for diagnosing problems and to take remedial measures;
- Students preferences, interests and attitudes; and
- The effectiveness of teaching methods, techniques, and learning material used by teachers.

Steps in the Continuous Assessment

The following are the major steps to follow in Continuous Assessment:-

Step i: Overview the unit out comes, contents, methods and tools of the unit.

Step ii: Produce a schedule of assessment for the unit.

Step iii. Determine the items for the suggested assessments of the unit.

Step iv: Construct questions for the types of assessments suggested for the unit based on the determined items.

Step v: Administer the suggested assessment tools constructed specifically on the bases of the schedule.

Step vi: Grade or mark what was done by students.

Step vii: Record the assessed results.

Teachers should have format (s) for recording the assessment results of students. The format (s) may be centrally or regionally designed or individually formulated by the teachers themselves. In any case, the recording format has to include at least, the names of students, grade level, subject type, and the marks allotted for each assessment task.

Step viii:- Report the recorded results.

Methods /Strategies of continuous Assessment

The methods of continuous assessment enables you to assess a wide range of learning competencies and behaviors using a variety of instruments some of which are:-

- Tests (quizzes)
- Classroom discussions, exercises, assignment or group works
- Project
- Observations
- Interview
- Group discussions
- Questionnaires

Different competencies may require different assessment techniques and instruments. For example, oral questions and interviews may serve to assess listening and speaking abilities.

Below are a descriptions of these methods of continuous assessment used in this assessment used in this assessment guide and their possible uses.

Tests

These usually consist of a range of questions covering almost all of the objectives of a unit. Students are required to respond to questions within a specified time, not more than half an hour. Tests could be phrased in different ways:

Close – ended (selection type such as true – false, multiple – choice, matching type) and open – ended (short – answer, essays, completion type).

Group Projects

A Project:-   is an exercise on a single objective or topic that requires investigation in with the time constraints more investigation in with the time constraints more relaxed than assignments. More over, projects require much more information than assignments and require the involvement of a group of learners working together.

Marking

Marking or grading:-   is the process of offering different types of symbols to academic progress or achievement of students. The marks given to students academic achievement are usually reported to the school administration in general and parents in particular. Designing a good marking scheme can help to be uniformly fair to all students.

The following are some suggestions on how to mark a semester's achievement.

1. One final semester examination 30%
2. Mid examination 20%
3. Tests 15%
4. Quizzed 10%
5. Home work 5%
6. Class activities, class work and presentation 10%
7. Project work, in groups or individually 10%

Recording and Reporting Students' progress and Achievement

Recording Students' achievement is an important aspect of continuous assessment. The reports on students' progress and performance may be miss- leading and incomprehensible unless records are properly kept.

The major records to be kept are teacher's records, student's cumulative report card and transcript.

a) **The teacher's record book: -** is a permanent record book which every teacher must keep in his/her class. The teacher's record book is expected to contain a detailed scheme of work, an accurate diary or daily record of work and progress report.

b) **The student's cumulative record card:-** this contains the most available information of students development through out the primary school course. The following main information should be including in the students cumulative record card.

   − Personal information about the students
   − Weekly or periodic report of academic performance.
   − Report on his/her character.
   − Report on the terminal tests
   − Report on the summary of progress in all areas of the school curriculum.

c) **The transcript:-** This includes the results of continuous and Summative assessments add up to 100%. Below is a record format of transcript.

Students	Weight	Continuous Assessment								Total
		Class work, Class activities	Home Work	Quizzes	Project; Group Work	Test 1	Test 2	Mid Exam	Final Exam	
1		10%	5%	10%	10	5%	10%	20%	30%	100%
2										
3										
4										
.										
.										
N										

Reporting Makes educators more accountable to learners, parents, the education system and the border community.

N.B: This plan is more preferably during the beginning of the semester (year).

Topics and period allotment

Unit	Sub-unit	Number of Periods	
		Sub units	Total
Unit 1 Whole numbers and the four operations	1.1 Whole numbers greater than 1,000,000	20	
	1.2. Operation on whole numbers	28	48
Unit 2 Working with Variables	2.1 Algebraic terms and values of terms	17	
	2.2 Equations and inequalities	7	24
Unit 3 Fractions, decimals and the four operations	3.1 Types of fractions	4	
	3.2 Percentage as fractions	5	
	3.3 Comparison and ordering of fractions	8	42
	3.4 Operations of fractions	13	
	3.5 Operations on decimals	12	
Unit 4 Data Handling	4.1 Further on construction and interpretation of bar graphs	9	
	4.2 The average of numbers	5	14
Unit 5 Geometric figures and measurement	5.1 Lines	8	
	5.2 Angles and the measurement of angles	8	42
	5.3 Classification of triangles	5	
	5.4 Lines of symmetry	8	
	5.5 Measurement	13	

.

UNIT ONE

WHOLE NUMBERS AND THE FOUR OPERATIONS

Introduction

The main task of this unit is to make the students familiar with the idea of whole numbers and operating with whole numbers. You know that addition, subtraction, multiplication and division are four fundamental operations of mathematics. In this unit, students will learn about the properties of these operations on whole numbers.

In Grade 4, students have already learned about whole numbers up to one million and dealt with solving real life problems using whole numbers upto one million. In this unit, students will learn about whole numbers greater than one million and perform the four fundamental operations on whole numbers.

The activities and exercises given in each sub unit are designed to motivate students to think critically about the lessons presented and to explore the key concepts.

Unit Outcomes

At the end of this unit, students will be able to:

- understand and have deep knowledge about whole numbers.
- perform the four fundamental operations on whole numbers.
- apply their knowledge of whole numbers to solve problems in their environment.

Suggested Teaching Aids in Unit 1

Apart from use of the student text book, bottle tops or similar objects for counting, structured base ten materials, cards, Birr notes, match sticks, different charts showing place value, charts showing predecessor and successor, charts showing multiples and divisors of whole numbers are useful teaching aids in this unit. The students should be encouraged to produce and select teaching materials.

Place value chart like the one given below is recommended
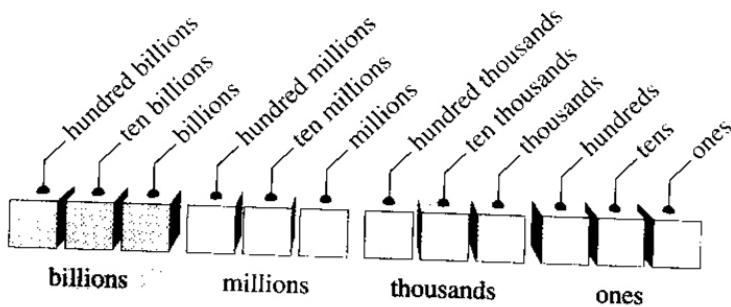
For example, the number 16,640,000,000 can be place in the diagram like this
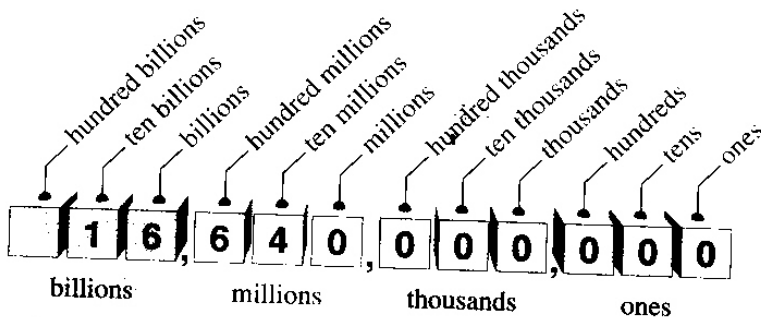
1.1. Whole Numbers Greater Than 1,000,000

Periods allotted: 20 periods

Competencies

At the end of this sub-unit, students will be able to:

- read whole numbers upto 1,000,000.
- write whole numbers upto 1,000,000.
- compare whole numbers upto 1,000,000.
- order whole number up to 1,000,000.
- read whole numbers greater than 1,000,000.
- write the whole numbers greater than 1,000,000.
- compare whole numbers greater than 1,000,000.
- order whole numbers greater than 1,000,000 either in increasing or decreasing order.
- determine the "successor" and the "predecessor" (except zero) of a given whole number.
- determine the place value of each digit in a given whole number.
- write a given whole number as a sum of integral multiples of 10.
- determine the order of given whole numbers by using either place value or number ray.
- identify even and odd whole numbers.

Introduction

This sub-unit is devoted to discussing whole numbers greater than one million. Here students will learn how to read, write, compare and order the whole numbers greater than one million.

Activities, Exercises and descriptive examples are included so as to involve students in the discussion of the sub-unit.

Teaching Notes

Most students will have heard of numbers up to one million and will have already used the idea- especially with reference to money and measurements. Here students will deal with formally developing appropriate abilities and skills in using the operating on whole numbers greater than 1,000,000 based on the knowledge and skills already acquired at grade 4 level.

In order to discuss the sub-unit follow the sub-topics discussed below.

1.1.1 Revision of Whole numbers up to 1,000,000

You may start the discussion of this topic by revising whole numbers up to 1,000,000 by reading a given number, by giving example from daily life which are expressed by these numbers, telling the place value, matching a number with its word expression, writing, comparing and ordering whole numbers. You may use Exercise 1.A for the purpose of revising. The revision work helps you assess what students have learned in previous lessons and determine the overall readiness of the class for new lesson.

Answers to Activity 1.1

1. Whole numbers   2. 0, No

3.

| a) | 10,000 | 20,000 | 30,000 | 40,000 | 50,000 | 60,000 | 70,000 |
| b) | 100,000 | 200,000 | 300,000 | 400,00 | 500,000 | 600,000 | 700,000 |

Assessment

You may ask students to read, write compare and order whole numbers up to 1,000,000 in the form of class work, home work, assignment, test or quiz in order to assess students level of understanding. For fast learners or interested students, you may give the following questions as additional assessment.

Additional Assessment

1. Write these numbers in words
   a) 23,007,000   b) 57,063,100   c) 100,000,400

2. Compare the numbers by using >, < or =
   a) 4,573,291 ☐ 4, 753, 291
   b) 13,082,288 ☐ 13,280,000
   c) 400,100,100 ☐ 400,001,000

Answers to additional assessment

1   a) Twenty three million, seven thousand
    b) Fifty seven million, sixty three thousand one hundred
    c) Hundred million, for hundred

2. a) 4,573,291 | < | 4,753,291

    b) 13,082,288 | < | 13,280,000

    c) 400,100,100 | > | 400,001,000

Answers to Exercise 1. A

1. a) 'One hundred thirty six thousand forty two'
   b) 'Two hundred eighteen thousand six hundred six'
   c) 'Three hundred ninety thousand seventy one'
   d) 'Four hundred sixty seven thousand three hundred nineteen'
   e) 'Five hundred twenty two thousand two hundred two'
   f) 'Six hundred fifty thousand five hundred five'
   g) 'Eight hundred thousand three hundred four'
   h) 'Four hundred thirty thousand seven hundred thirteen'
2. i) b   ii) c   iii) e   iv) a   v) d   vi) h   vii) g   viii) f
3.   a) One hundred thousand three hundred fifty
      b) Two hundred six thousand five hundred seventy

Mathematics Grade 5 Teacher's Guide   **5**

c) One hundred sixty thousand eighty

d) Three hundred twenty thousand ten

e) Four hundred eighty five thousand six hundred seventy five

f) Eight hundred sixty thousand three

g) Nine hundred seventy three thousand four hundred sixty eight

h) Ninety eight thousand seven hundred sixty four

4 a) Ten   b) Thousand   c) One   d) Ten thousand   e) Hundred

5 a) 140,000   b) 170,630   c) 205,380   d) 516,409

   e) 603,027   f) 90,074   g) 785,212

6 a) 705,000   b) 388,000   c) 689,400

1.1.2 Whole numbers greater than 1,000,000

You can start the topic by asking your students questions like: what is the population of Ethiopia? What is the population your region? What is the distance between the earth and the sun?

After introducing the number 1,000,000 by adding 1 to 999,999 encourage your students to generate numbers greater than 1,000,000 and let them read, write, compare and order these numbers.

You may also use Activity 1.2 in order to involve students in the discussion of whole numbers greater than 1,000,000.

Motivate students to read, write and order whole numbers greater than 1,000,000. Assist the students to generalize that any whole number, n, different from 0 has a predecessor "n-1" and a successor "n+1". Guide the students to conclude the following points:

- There is no largest whole number.
- Zero is the smallest whole number.

Make sure that students are able to determine the predecessor and successor of a given whole number.

Answers to Activity 1.2

  b) Seven million four hundred sixteen thousand thirty five

  c) Eight million forty two thousand one hundred seven

  d) Nine million one hundred four thousand sixty

  e) Twelve million

Answers to Group Work 1.1

1. 8000 km = 8000x1000m =8,000,000m

2. 900 km=900x100,000cm = 90,000,000 cm

Assessment

You may give problems to students on reading, writing and finding predecessor and successor of whole numbers greater than 1,000,000 in the form of class work, home work, assignment, quiz or test in order to assess students' progress.

Answers to Exercise 1.B

1. a) 5,804,020   b) 8,916,132   c) 9,030,403

2. b) five Million six hundred twenty five thousand

   c) one million forty eight thousand five hundred seventy six

   d) thirty million two hundred seventy one thousand.

3)

	Predecessor	**Successor**
a) 3,406,705	3,406,704	3,406,706
b) 5,167,428	5,167,427	5,167,429
c) 9,582,396	9,582,395	9,582,397
d) 8,005,104	8,005,103	8,005,105
e) 6,767,221	6,767,220	6,767,222

4.

Figure 1.1

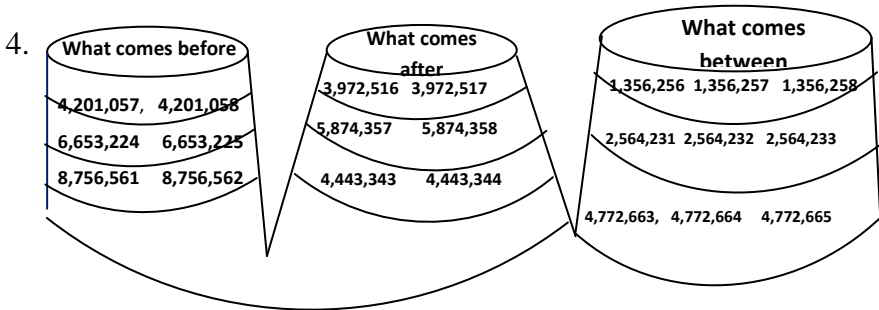5. b) 3,820,013>3,820,012   d) 7,630,009<7,630,010

   c) 6,540,000=6,540,000   e) 8,999,026>8,999,025

1.1.3 Place Value and Ordering of Whole numbers

Let some students carry number cards and stand in a row, so that the other students read the numbers formed and repeat this by swaping the students with the cards among.

You may discuss the place value of each digit in a given numbers by giving place value table.

Assist students to write the expansion of a given whole number by using examples like:

$$8432 = 8000+400+30+2$$
$$= (8\times1000)+(4\times100)+(3\times10)+2$$

Encourage the students to represent whole numbers on a number ray to compare as well as to order whole numbers.

Give exercise on place value, ordering and expansion of whole numbers.

Answers to Activity 1.3

  a) 124,578   b) 875,421

Answers to Group Work 1.2

1. Millions

2. 8x1,000,000+6x100,000+9x10,000+7x1000+3x100+5x10+1

Mathematics Grade 5 Teacher's Guide   **8**

Assessment

You may give problems on place value, ordering and expansion of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You need to check their work and the overall discussion during presentation. You can also give the following problems to fast learners or interested students to answer as additional assessment.

Additional Assessment

Write each digit of the number 3,428,756,901 according to its place-value position in the chart given below. Read the number.

Trillions			Billions			Millions			Thousands			Ones		
Hundred	Ten trillions	trillions	Hundred	Ten Billions	Billions	Hundred	Ten millions	Millions	Hundred	Ten thousands	Thousands	Hundreds	Tens	Ones

Answers to additional assessment

Trillions			Billions			Millions			Thousands			Ones		
Hundred	Ten trillions	trillions	Hundred	Ten Billions	Billions	Hundred	Ten millions	Millions	Hundred	Ten thousands	Thousands	Hundreds	Tens	Ones
					3	4	2	8	7	5	6	9	0	1

The given number is read as 'Three billion, four hundred twenty eight million, seven hundred fifty six thousand, nine hundred one'.

Answers to Exercise 1.C

1. a) Million   c) hundred thousand   e) Ten thousand
   b) Hundred   d) thousand

2. a) 2,536,879=
   $(2×1,000,000)+(5×100,000)+(3×10,000)+$
   $(6×1,000)+(8×100)+(7×10)+9$
   b) 1,546,308=
   $(1×100,000)+(5×100,000)+$
   $(4×10,000)+(6×1,000)+(3×100)+8$
   c) 7,089,461=$(7×1,000,000)+(8×10,000)+(9×1,000)$
   $+(4×100)+(6 ×10)+1$
   d) 8,571,026=
   $(8×1,000,000)+(5×100,000)+(7×10,000)+(1×1,000)+$
   $(2×10)+6$
   e)988,472=$(9×1,000,000)+(9×100,000)+(8×10,000)+(8×1,000)$
   $+ (4 ×100)+(7×10)+2$

3. a) 3,060,843   c) 4,005,607
   b) 6,807,039   d) 8,000,308

4.
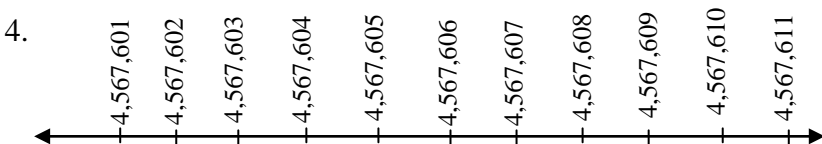
5. a) e.g. 781,053 (There are many other possible answers)
   b) e.g. 6,714,982
   c) e.g. 3,452,618

6. a) 4,325,270>4,246,370   b) 3,507,469>3,206,986
   c) 5,651,845>5,461,835   d) 2,453,578<2,453,587
   e) 9,678,450<9,768,675

7. b) 230,000   330,000 430,000   530,000   630,000
      730,000   830,00   930,000
   c) 376,000   476,000 576,000   676,000   776,000
8. a) 1,250,000   2,250,000 3,250,000 4,250,000
      5,250,000   6,250,000
   b) 4,600,000   5,600,000 6,600,000 7,600,000
      8,600,000   9,600,000
9. 408,893   423,635   469,743   480,982   837,209
   873,276   947,534   3,604,376   5,628,370   6,086,304

1.1.4 Even and Odd Numbers

You can let the students differentiate even and odd numbers by using examples from their experience like:

- dividing a given amount of money among themselves.
- grouping themselves according to their roll number where one group is formed by students having even roll numbers and the other by students having odd roll numbers.

Encourage students to list the first few even numbers and odd numbers and let them see the pattern of these numbers.

Guide students to conclude that:

even + even= even

odd + odd = odd

odd + even = odd = even + odd

Motivate students to list even and odd numbers.

Answers to Activity 1.4

1. 1,253,402   1,253,404   1,253,406   1,253,408 1,253,410
2. 2,430,679   2,430,681   2,430,683   2,430,685 2,430,687
3. a) 3,570,602   3,570,604   3,570,606   3,570,608 3,570,610
      3,570,612   3,570,614   3,570,616   3,570,618
   b) 6,620,403   6,620,405   6,620,407   6,620.409 6,620,411
      6,620,413   6,620,415   6,620,417   6,620,419

4. a) 2,438,672   b) 3,156,259
5. There are many possible answers
   a) e.g. 352   274   416   734
   b) e.g. 1243   1425   6753   7631

Assessment

You may give problems on even and odd numbers in the form of class work, home work, assignment, quiz or test in order to assess students' performance.

Answers to Group Work 1.3

   Even + Even = Even
   Odd + Odd = Even
   Even + Odd = Odd

Answers to Exercise 1.D

1. True   2. False   3. False   4. True   5. True   6. True
7. False   8. True

Selected problems to slow learners

1. Compare and order
    2,406,107   2,460,107   2,046,107
    2,604,107   2,640,107
2. What is the sum of three even numbers? Even or odd?
3. Write "three million, seven hundred seventy thousand, one hundred seven" in figures.
4. What is the place value of 3 in 5,384,600?
5. What whole number comes before 7,065,432?
6. What whole number comes after 8,109,067?
7. Write the number "five million, sixty thousand, seven hundred eight" in figure.

8. What even number comes in between 1,260,704 and 1,260,708?
9. Write 10,764,892 in expanded form.
10. Find the number which is represented by the expanded notation (8x1,000,000) + (6x100,000).

Selected problems to fast learners

1. If x is even and y is odd, then what can you say about $(x+y)^2$? Even or odd?
2. Write "six billion, six hundred six million" in figures.
3. Which one is greater? One billion or ten times one million?
4. What is the largest 8 digit number?
5. What is the least 7 digit number?
6. Which one is greater? 10,070,107 or 10,071,017?
7. Give an example of 7 digit even number whose thousand's place is 7.
8. Count in millions and list the numbers starting from 1,723,456 to 16,723,456.
9. Is the sum of 100 even numbers even?
10. Is the sum of 100 odd numbers odd?
11. If a seven digit number ends in 7, will it be divisible by 2?

1.2 Operation on whole numbers

Periods allotted: 28 periods

Competencies

At the end of this sub-unit, students will be able to:

- solve problems of addition.
- solve problems of subtraction.
- identify the basic properties of addition on whole numbers.
- solve problems on multiplication of whole numbers.

- identify the distributive property of multiplication over addition.
- divide whole numbers whose quotient are expressed in decimals (2 decimal places).
- solve problems containing several operations and brackets.
- find multiples of whole numbers.
- find divisors of whole numbers.
- express product of equal factors as powers.
- Compute using powers.

Introduction

This sub-unit discusses operation on whole numbers. For this purpose the sub-unit is subdivided in to six subtopics: Addition and subtraction of whole numbers, multiplication of whole numbers, Division of whole numbers, problems containing several operations, multiples and divisors of whole numbers, and powers of whole numbers.

Teaching Notes

Revise commutative property of addition/multiplication, associative property of addition/ multiplication of whole numbers. Point out that (i) subtraction and division are not commutative operations (ii) the distributive property involves both the operations of addition and multiplication in the same expression.

You are expected to involve or ensure active participation of the students in the discussion of each sub-topic.

1.2.1 Addition and Subtraction of whole numbers

You can let students work out problem involving addition and subtraction and check the result of addition by subtraction and vise-versa.

You can revise the commutative and associative property of addition to discuss Properties of zero on whole numbers, that is $0+x=x+0$ for all whole numbers x.

Motivate students to solve word problems related to students real life of the following type e.g. A factory produced 356,400 bottles on Monday, 527,200 on Tuesday, and 487,600 on Wednesday. How many bottles are produced in three days?
You can give students exercises on addition and subtraction.
Let them identify the properties of addition.

Answers to Activity 1.5

a=169,   b=141,   c=127

Answers to Group Work 1.4

a) 1,172,373   b) 1,611,812

Answers to Activity 1.6

1. a) 0   b) 0   c) 0   d) 8   e) 600   f) 2456

2. b)   432   701   701
        +<u>269</u>   -<u>269</u>   -<u>432</u>
           701   432   269

    c) 5445   7832   7832
        +<u>2387</u>   -<u>2387</u>   -<u>5445</u>
        <u>7832</u>   <u>5445</u>   <u>2387</u>

3. True

4. (a) 232,567+687,758=920,325
        And 354,743+467, 869=822,612
    therefore 232,567+687,758<354,743+467,869
    (b) 358,676+576,589=935,265
        And 2,121,342+3,436,536=5,557,878
        Therefore, 358,676 + 576,589<2,121,342+3,436,536

(c) 6,234,238-4,867,786=1,366,452

And 7,158,349-3,283,898=3,874,451

Therefore, 6,234,238-4,867,786<7,158,349-3,283,898

Assessment

You may give problems on addition and subtraction of whole numbers in the   form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also ask fast learners or interested students to answer the following question as additional assessment.

Additional Assessment

Find the difference: 1,000,000

- 899,989

Answer to additional assessment

100,011

Answers to Exercise 1.E

1.   a) 43257   b) 56674   c) 727585

   +15894   + 48486   +575869

      59151   105160   1303454

   d) 94328   e) 79024   f)   810731

   -56779   - 68968   -799843

      37549   10056   10888

2. 8,754+1,296=10,050 bags altogether.

3. 745,700+176,507=922,207= total number of cattle

4. Total population of the three towns=
   12,542+11,460+13,627=37,629

5. Number of women= 8,432-4,971=3,461

6. a) male   b) 37,296,657+36,621,848=73,918,505
   c) 37,296,657-36,621,848 = 674,809

7. 1,052,747-(905,002+87,445)

$$=1,052,747-992,447$$

$$= 60,300$$

8. Boxes of lemons $= 33,000,000 – 1,200,900 = 31,799,100$

1.2.2 Multiplication of whole numbers

You may start the discussion of this subtopic by asking students to give the connection between multiplication and addition both in words and by examples. (Answers may vary). Then point out that multiplication by a whole number can be thought of as repeated addition and demonstrate this by several examples.

Assists students to work out problems involving multiplication up to 3 digit number form real life.

You can revise the commutative and associative property of multiplication. Allow students to discuss the distributive property of multiplication over addition.

To consolidate you can give examples like:

a) $3\times6\times4 = (3\times6) \times4$

$$=12\times4$$

$$= 72$$

b) $21\times5 \times22=21\times (5\times22)$

$$=21\times110$$

$$=2310$$

c) $52\times3=(50+2) \times3= (50\times3)+(2\times3)$

$$= 150+6$$

$$=156$$

d) $12\times8+5\times8= (12+5) \times8$

$$=17\times8$$

$$=136$$

Assist the students to come to the multiplication properties of 1 and 0 by any whole number.

Encourage your students to work with approximate values of determining rough estimation for in multiplying large numbers and comparing it with the real result. You may take examples like:

$3124 \times 298 \approx 900,000$

$3124 \times 298 = 930,952$

Let students use the properties of multiplication to solve multiplication problems.

Answers to Activity 1.7

$3 \times 6 = 18$

There are 18 pencils in 3 packets $3+3+3+3+3+3 = 18$ and $6+6+6= 18$ therefore $3 \times 6 = 6 \times 3$.

Group Work 1.5

$78 \times 3 + 80 \times 3$

$= 234 + 240$

$= 474$ times

Answers to Activity 1.8

a) $4,326 \times 15 = 4,326 \times (10+5)$

$= (4,326 \times 10) + (4326 \times 5)$

$= 43,260 + 21,630$

$= 64,890$

b) $3,674 \times 28 = 3,674 \times (20+8)$

$= (3,674 \times 2 \times 10) + (3,674 \times 8)$

$= (7,348 \times 10) + (29,392)$

$= 73,480 + 29,392$

$= 102,872$

c) $4,318 \times 34 = 4,318 \times (30+4) = (4,318 \times 3 \times 10) + (4,318 \times 4)$

$= (12,954 \times 10) + (17,272)$

$= 129,540 + 17,272$

$= 146,812$

$$7,508 \times 63 = 7,508 \times (60+3)$$
$$= (7,508 \times 6 \times 10) + (7,508 \times 3)$$
$$= (45,048 \times 10) + (22,524)$$
$$= 450,480 + 22,524$$
$$= 473,004$$

Assessment

You may give problems on multiplication of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess the students' performance. You are expected to check their work and the overall discussion during presentation.

Answers to Exercise 1. F

1.   a) 28   b) 69   c) 252   d) 840
      e) 882   f) 15,988   g) 74,772   h) 6,528
      i) 18,575   j) 18, 000   k) 75, 392

2. a) 400,000   b) 1,200,000
3. $3 \times 25 = 75$ mangoes
4. $5 \times 42 = 210$ school days
5. $1,236 \times 24 = 29,664$ pages
6. $60 \times 3,762 = 225,720$ copies
7. Total profit $= 2,815,890$

1.2.3 Division of Whole numbers

You may start by revising commutative property of addition/ multiplication, associative property of addition/ multiplication of whole numbers.

  i) Subtraction and division are not commutative operations on whole numbers.

  ii) The distributive property involves both the operations of addition and multiplication in the same expression on whole numbers.

Assist students to exercise division of large whole numbers by two, three or four digits number.

Assist students to divide whole numbers and express the quotients in decimal terminating after 2 decimal places.

Let students discover that division is not commutative as well as associative.

Encourage students to do exercises on dividing of whole numbers by two, three or four digit number.

Answers to Activity 1.9

a) $12 \div 3 = 4$, quotient $= 4$, remainder $= 0$

b) $13 = 3 \times 4 + 1$, quotient $= 3$, remainder $= 1$

c) $18 \div 2 = 9$, quotient $= 9$, remainder $= 0$

d) $16 = 3 \times 5 + 1$, quotient $= 3$, remainder $= 1$

Assessment

You may give students problems on division of whole numbers in the form of class work, home work, assignment quiz or test,. Ask them in what way multiplication and division are related operations. You are expected to check their work and the overall presentation in the discussion of this sub-topic. You can also ask the following question to fast learners or interested students to answer as additional assessment.

Additional Assessment

$x \div y = 9$	x		2727		47,511
	Y	120		4351	

Answers to additional assessment

$x \div y = 9$	x	1080	2727	39159	47,511
	y	120	303	4351	5279

Answers to Exercise 1.G

1. a) quotient = 32   b) quotient =43   c) quotient =91
      remainder =5   remainder =1   remainder =2

   d) quotient = 97   e) quotient =60   f) quotient =29
      remainder =3   remainder =8   remainder =17

   g) quotient = 117   h) quotient =547   i) quotient =45
      remainder =43   remainder =62   remainder =4816

2.

a÷b=3	a	18	27	36	60	102	300	9000
	b	6	9	12	20	34	100	3000

3. 5887÷7=841 weeks
4. 360÷24=15 days
5. 2400÷96=25
6. Each child get 21 oranges and 140 oranges are left over.
7. 352×36 +27=12,699
   Therefore, the required number is 12,699

1.2.4 Problems containing several operations

Initiate a discussion about the importance of order in various activities. Ask students to describe activities or situations in which the order of the steps is important (e.g. cooking, getting dressed or building a house). Then have students evaluate the expression 3+7×4. Students may give either 31 or 40 as the answer. Have students share their methods and discuss why a standard order of operations is important.

Tell students that this lesson is a review of the rules they have learned to this point. Take time to have students restate the rules for the four operations on whole numbers and the order of operations. See if other students can improve on this response. When the class has reached some kind of consensus, write the concise rules on the

chalk board so that everyone can use them. Let students practice solving problems containing several operations.

Answers to Activity 1.10
Answers may vary. Here is one such example $(7+8) \times 2-5=25$.

Answers to Group Work 1.6
$8 \times (9+13)=8 \times 9+8 \times 13=72+104$
$$=176 \neq 85$$

Assessment
You may give students problems containing several operations in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You can also give the following problems to students as additional assessment.

Additional Assessment
Evaluate (a) $27-18 \div 6$ b) $30-18 \div 2 \times 3+8$

Answers to additional assessment
   a) $27-18 \div 6=27-3=24$
   b) $30-18 \div 2 \times 3+8=30-9 \times 3+8=30-27+8=3+8=11$

Answers to Exercise 1. H
1. a) True. Because $4 \times (20-10)=4 \times 10=40$
                  $(7 \times 5)+5=35+5=40$
  That is, $4 \times (20-10) = (7 \times 5)+5$
  b) True. Because $(27 \div 9)+9=3+9=12$
           $27 \div (3+6)=27 \div 9=3$
          and $12>3$
  c) True. Because $(20 \div 2) \div 2=10 \div 2=5$
          $20-(18 \div 3)=20-6=14$
          and $5<14$

d) False. Because $(3\times4)+(3\times5)= (4\times3)+(5\times3)$

e) True. Because $(32\div4)+(36\div4)=8+9=17$

$\qquad\qquad\qquad (4\times2)+ (3\times3)=8+9=17$

f) False. Because $(4\times7)-(20-10)$

$\qquad\qquad\qquad = 28-10=18$

$\qquad\qquad\qquad (7\times4)-(10-5)=28-5=23$

$\qquad\qquad\qquad$ and $18< 23$

g) False. Because $(25\div5) +10=5+10=15$

$\qquad\qquad\qquad (30\div6)+15=5+15=20$

$\qquad\qquad\qquad$ and $15<20$

h) True. Because $(36\div2)-15=18-15=3$

$\qquad\qquad\qquad (30\div2)-12 =15-12=3$

i) False. Because $(5\times4)+(6\times4)=20+24=44$

$\qquad\qquad\qquad (6\times3)+(7\times3)= 18+21=39$

$\qquad\qquad\qquad$ and $44>39$

j) False. Because $(18\div3) \times (30\div5)= 6\times6=36$

$\qquad\qquad\qquad (6\times6) \div(3\times2)=36\div6=6$

$\qquad\qquad\qquad$ and $36\neq6$

2. b) 309 c) 83 d) 106 e) 3 f) 2 g) 95

1.2.5 Multiples and Divisors of Whole numbers

Give students a number such as 225, and asks them to find all divisors of 225.

Encourage students to practice finding multiples and divisors of a given whole number.

Answers to Activity 1.11

1. a) $1\times6, 2\times3$ b) $1\times16, 2\times8, 4\times4$

    c) 1×17 d) $1\times36, 2\times18, 3\times12, 4\times9, 6\times6$

2. Meskerem 11

Assessment

You may give students problems on multiples and divisors of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess students' progress.

Answers to Exercise 1.I

1. 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60
2. 21, 28, 35, 42, 49, 56
3. 0, 8, 16, 24, 32, . . .
4. 1, 2, 4, 8, 16, 32
5. 1 and 2 are common divisors of 18 and 32

1.2.6 Powers of whole Numbers

Let the students understand the concept of powers by using examples. Assist the students to practice representing product of equal factors as powers.

Encourage students to identify the concepts "power", "base" and "exponent" and let them discover the rules for the product and quotient of powers having the same base. You may use examples like:

$$2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 2^{3+4}$$

$$\frac{3^5}{3^3} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3} \times 3 \times 3 = 3^2 = 3^{5-3}$$

Note: Exponents should be positive integers so as to avoid examples like $\dfrac{3^2}{3^4} = 3^{2-4}$

Motivate students to represent products of equal factors as powers and identify power, base and exponent.

Let students do exercise on the application of rules for the product and quotient of powers having the same base.

Answers to Activity 1.12

b) $3 \times 3 \times 3 \times 3 = 3^4$

c) $4 \times 4 \times 4 \times 4 \times 4 = 4^5$

Answers to Group Work 1.7

    c) $3\times5^2+2=3\times25+2$

              $=75+2$

              $=77$

Assessment

You may give problems on powers of whole numbers in the form of class work, home work, assignment, quiz or test in order to assess students' level of understanding. You are expected to check their work and the overall presentation in the discussion of this subtopic.

Answers to Exercise 1.J

1. b) $27=3\times3\times3=3^3$ d) $125=5\times5\times5=5^3$

   c) $32=2\times2\times2\times2\times2=2^5$ e) $1000=10\times10\times10=10^3$

2. a) $x=2$ b) $x=3$ c) $x=5$ d) $x=10$

3.

number	8	9	16	25	32	64	81
a^n	2^3	3^2	2^4	5^2	2^5	4^3	3^4
exponent	3	2	4	2	5	3	4
base	2	3	2	5	2	4	3

4. (a) $2^3<3^2$ (b) $4^3<3^4$ (c) $2^5>5^2$ (d) $2^{10}>10^2$

5.

Number	Product of sevens	Number of sevens	Number using exponent
7	7	1	7^1
49	7×7	2	7^2
343	$7\times7\times7$	3	7^3
2,401	$7\times7\times7\times7$	4	7^4
16,807	$7\times7\times7\times7\times7$	5	7^5
11,7649	$7\times7\times7\times7\times7\times7$	6	7^6
823,543	$7\times7\times7\times7\times7\times7\times7$	7	7^7
5,764,801	$7\times7\times7\times7\times7\times7\times7\times7$	8	7^8

6. a) $\dfrac{36}{3^2-3} = \dfrac{36}{9-3} = \dfrac{36}{6} = 6$

b) $(5^2+3) \div 7 = (25+3) \div 7 = 28 \div 7 = 4$

c) $(20+30) \div 5 \times 2 + (2^4-1)$

$\qquad = 50 \div 5 \times 2 + (16-1)$

$\qquad = 10 \times 2 + 15$

$\qquad = 20 + 15$

$\qquad = 35$

Assessment

Apart from the details mentioned above, you may use a different approach to assess students' performance. You can use different assessment techniques such as class work, group work, assignment, quiz or test. Check their work and keep record.

Selected Problems to slow learners

1. Perform the indicated operations.

   a. 4,378,599
      + 3,769,789

   b. 25,989,689
      - 17,886,472

   c. 4532
      × 43

   d. $45864 \div 63$

2. Write in power form.

   a. 32   b. 81   c. 10,000

3. There are 38,140 students in school A, and 15,245 students in school B. How many students are there altogether in the two schools?

4. Subtract 231,456 from 456,789.

5. From 971,849 take away 310,400.

6. What is the product of 5362 and 43?

7. Divide 5824 by 112.

8. How many groups of 650 can we make from 31,200?

9. Complete

$a \div b = 2$	a	20		80	100
	b		20		

10. Is 2^5 and 5^2 equal?
11. Simplify $2^2 \times 3^2$.

Selected problems to fast learners

1. Perform the indicated operations

   a. 92,469,989
   + 18,787,797

   b. 32,021,130
   − 19,986,764

   c. 58954
   × 386

   d. 46,351,680 ÷ 636

2. Write in power form.
   a. 2,048   b. 59,049   c. 15,625
3. There are 3,846,171 people in city A, and 4,712,909 people in city B. How many people are there altogether in the two cities?
4. Subtract 798,987 from 1,000,000.
5. From 2,001,001 take away 888,999.
6. What is the product of 874,562 and 785?
7. Divide 967,960 by 3457
8. How many groups of 6127 can we make from 1,801,340?
9. Complete

$a \div b = 5$	a	500			30,000
	b		500	1000	

10. If $3^4 = 9^x$, then what is the value of x?

11. Simplify $\dfrac{4^2 \times 2^4}{8^2}$

12. What is the next number in the pattern: $3, 3^2, 27, 3^4, 3^5, \ldots$

Answers to Review Exercise

1. a) Four million three hundred fifty thousand six hundred seventy two.

   b) Seven million five hundred eighty two thousand ninety one.

   c) Ten million ninety three thousand three hundred eighty five.

   d) Sixteen million seven hundred twenty four thousand one hundred nine.

   e) Twenty million.

   f) Eighty three million four hundred

2. a) 7,010,086   c) 14,000,016
   b) 12,700,103   d) 37,625,049

3. a) 5,907,182   b) 7,068,440
   c) 8,907,055   d) 12,000,401

4. a) 3,586,275<3,658,752
   b) 10,706,009<10,099,991
   c) 13,218,780<13,900,000
   d) 21,007,700<21,008,000
   e) 38,704,100>38,407,100

5. Hundred thousand

6. a) $(2\times10,000,000) + (1\times1,000,000)+(7\times100,000) + (6\times1000) + (4\times100) + (8\times10) + 9$

   b) $(3\times10,000,000) + (4\times1,000,000) + (6\times10,000) + (9\times1000) + (7\times100)+5$

   c) $(9\times10,000,000) + (1\times1,000,000) + (3\times100,000) + (6\times10,000) + (7\times10)+2$

7. a) 4,075,091   c) 9,000,863
   b) 7,900,680   d) 67,007,009

8. a) 1,300,200   2,300,200   3,300,200   4,300,200
      5,300,200   6,300,200   7,300,200   8,300,200

b) 13,407,500   14,407,500   15,407,500   16,407,500

17,407,500   18,407,500   19,407,500   20,407,500

c) 30,566,409   31,566,409   32,566,409   33,566,409

34,566,409   35,566,409   36,566,409   37,566,409

38,566,409   39,566,409

9. a) 30,708,970   30,708,972   30,708,974   30,708,976

30,708,978   30,708,980   30,708,982

b) 42,561,843   42,561,845   42,561,847   42,561,849

42,561,851

10. a) Even   b) Even   c) odd

11.

a) 8,346,271   b) 13,097,805 c) 24,681,967

+ 4,077,956   + 7,903,769   + 18,098,123

12,424,227   21,001,574   42,780,090

12.

a) 18,076,045   b) 21,606,909 c) 32,168,432

− 6,953,852   − 8,079,098   − 9,969,909

11,122,193   13,527,811   22,198,523

13.   a) 325,992   b) 205,494   c) 5,403,963

14.   a) 7894   b) 4669   c) 498   d) 6853

15.   a) 4264   b) 12,650   c) 631   d) 58   e) 125

16.   a) 3^5 b) 2^7 c) 7^4 d) 5^4

17.   If his dream was true, Zeberga would have

$\underbrace{85,234}$ + $\underbrace{750,000+480,000}$

amount in   amount from a

the bank   winning ticket

= Birr 1,315,234

18. Ato Wondimu has to pay the teachers an amount

= 854,550 – 45,680

= Birr 808,870

19. Since each bench can hold 16 children, total number of children on the 1432 benches

= 16 × 1432

= 22,912

20. Each student carries an amount

= 32,448 ÷ 52

= 624

UNIT TWO

WORKING WITH VARIABLES

Introduction

This unit introduces the use of variables in mathematics. The unit gives much emphasis to representing quantities with variables, finding the value of simple algebraic expressions, writing equations and inequalities. Different approaches, to translate simple mathematical statements in to mathematical expressions, compute values of terms and expressions by substitution, simplification of expressions, writing equations and inequalities, are used with descriptive examples. In addition, rules of addition and subtraction of algebraic expressions are discussed in good detail in the unit. Involvement of students in various aspects of this unit is sought to help for better understanding of the use of variables as well.

Unit outcomes

At the end of this unit, students will be able to:
- realize the use of variables in mathematics.
- understand mathematical terms, expressions and simplification of expressions.
- identify equations and inequalities and determine their value by substitution.

Suggested teaching aids in Unit 2

You may use a balanced scale, some coins, charts containing algebraic expressions, equations and inequalities that will enable you to represent mathematical statements.

Apart from the use of the student text book, you need to elaborate more real world problems from your surrounding so that students can best appreciate and see how useful algebraic terms, expressions, equations and inequalities are.

You may group students, give them hints on a problem and let them assess problems from their daily life to work with variables, where the problems are represented as algebraic expression, equation or inequality.

Charts containing the following are recommended

4 (5)	mean →	4 × 5
10x	mean →	10 × x
ab	mean →	a × b
4.9a	mean →	4×9×a
7ab²	mean →	7×a×b×b
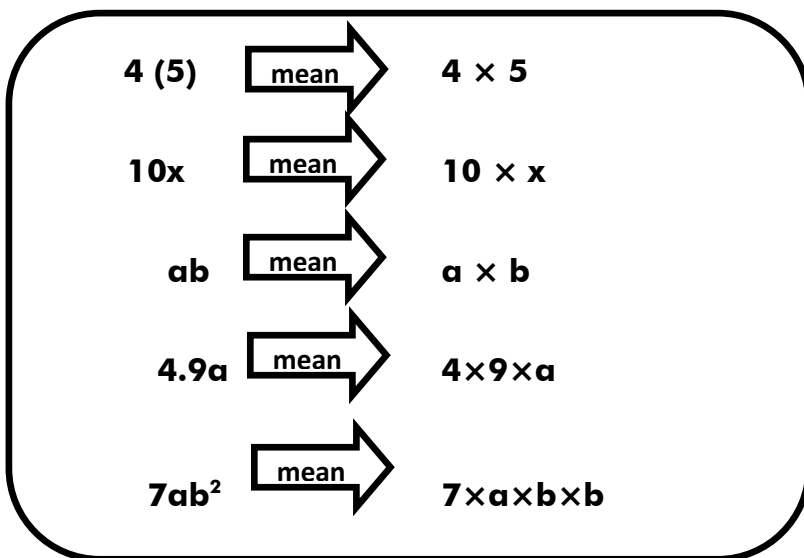
$$a[b(cd)] \quad \text{means} \quad a \times [b \times (c \times d)]$$

$$\frac{e}{8f} \quad \text{means} \quad e \div (8 \times f)$$

$$a\left(\frac{b}{4}\right) \quad \text{means} \quad a \times (b \div 4)$$

22   =   X + 9

left expression   right expression
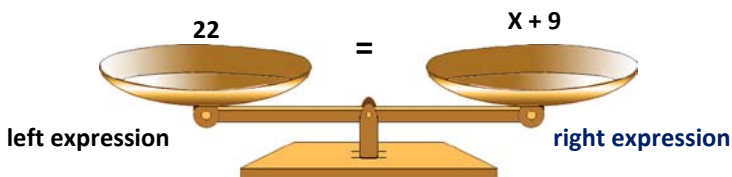
Mathematical statements	Algebraic Expressions
Three less than a number	$x - 3$
A number increased by 10	$y + 10$
One third of a number	$\dfrac{a}{3}$
Twice a number	$2c$
The sum of two numbers	$e + f$
Twice a number decreased by five	$2d - 5$
The quotient of a number and 8	$\dfrac{n}{8}$

2.1 Algebraic Terms and Expressions

Periods allotted: 17 periods

Competencies

At the end of this sub unit, students will be able to:

- change simple mathematical statements in to mathematical expressions.
- identify terms and expressions.
- simplify given expressions by collecting like terms.
- compute the values of terms and expressions by substituting whole numbers in the given variables.
- identify algebraic expressions based on their number of terms.

Introduction

Translating verbal descriptions in to algebraic expressions is a problem – solving strategy that is used in everyday life. For example, the amount of profit that you can make by selling a number of items for more than they cost you, and the number kilometers you have left to travel before reaching your destination, can be expressed using variables. In this sub- unit, an attempt will be made to discuss the use of variables. Algebraic terms, expressions, simplifying expressions, and computing the values of terms and expressions by substituting will be discussed in this sub-unit.

Teaching Notes

You may start this sub-unit by encouraging and assisting students to describe mathematical ideas they develop from their every day examples by using variables. In order to involve students with the use of variables, follow the sub- topics discussed below.

2.1.1 Algebraic terms and values of terms

Assist students to realize the use of variables. You may give examples like:

- I am twice of a number $= 2x$
- One third of a number $= \dfrac{1}{3}a$
- The sum of two numbers $= x + y$
- The difference of two numbers $= a - b$

Encourage students to explain the meaning of terms and expressions and that the use of letters for numbers is a short hand method for writing mathematical statements through everyday examples. You may use examples like:

If a book cost Birr x birr and soft drink cost Birr y, how much will 20 such books and 30 soft drinks cost?

Make sure that students can identify algebraic expressions based on their number of terms and change simple mathematical statements in to mathematical expressions.

Answers to Activity 2.1

- a) Difference (subtraction)
- b) Product (multiplication)
- c) Quotient (division)
- d) Sum (addition)
- e) Product (because $x^2 = x. x$)

Answers to Activity 2.2

1.
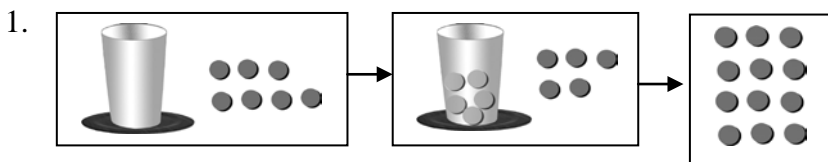
There are 12 counters (5+7)

2.
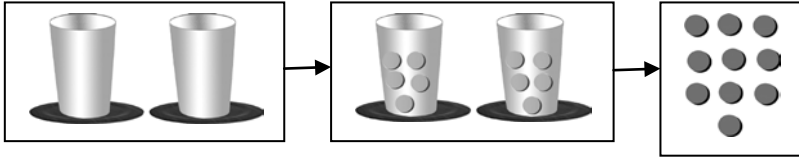
There are 10 counters (2×5)

3.
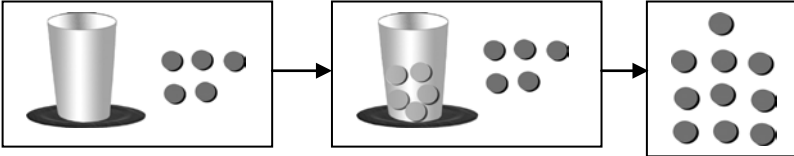
There are 10 counters (5+5)

4.
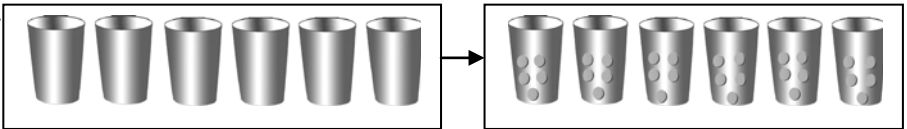
There are 30 counters (6×5)

Answers to Group Work 2.1

1. $\dfrac{x}{20}$ 2. y-10   3. 20(x-2)   4. 7+8a

Assessment

You may give students a set of problems to identify algebraic expressions based on their number of terms, and change simple mathematical statements in to mathematical expressions in the form of class work, home work, assignment, quiz or test so as to check their progress. You may also ask fast learners or interested students to answers the following question as additional assessment.

Additional Assessment

Complete the chart given below which shows algebraic expressions with the variable x

Number	Algebraic expressions	Verbal expressions	Value for x=10
1	x+ 4	x plus 4	?
2	?	20 minus x	10
3	5x	5 times x	?
4	?	100 divided by x	10
5	?	x subtracted from 35	?

Answers to additional assessment

Number	Algebraic expressions	Verbal expressions	Value for x=10
1	X+ 4	X plus 4	14
2	20-×	20 minus x	10
3	5x	5 times x	50
4	$\frac{100}{x}$	100 divided by x	10
5	35-x	X subtracted from 35	25

Answers to Exercise 2.A

1. Added to, sum of, plus, more than, increased by, etc

2. a) binomial   b) monomial   c) monomial

   d) binomial   e) binomial

3.   i. d   ii. a   iii. g   iv. i   v. j

   vi. f   vii. h   viii. e   ix. c   x. b

4. a) k+5   b) 40a   c) 36÷ b   d) c-14

   e) 60+d   f) 24w   g) p-q   h) r ÷t

   i) $\frac{a}{b}$ j) m-n   k) 2m +8   l) $\frac{1}{4}n$ m) $\frac{1}{3}(x+y)$

5. a) A number decreased by 12

    b) one fourth of a number

    c) A number increased by 28

    d) The sum of two numbers

    e) 100 times a number

    f) The quotient of 100 and a number

    g) t subtracted from 6

    h) Twice the difference of two numbers

    j) The quotient of the sum of two numbers and 10

6. $10x + 12y$

7. $300 \div p$

8. $200 + 0.50 K$

2.1.2 The value of Simple Algebraic Expressions

Assist students to collect like terms. You may use examples such as the following:

$3a + 5b + 6a - 2b = (3a + 6a) + (5b - 2b)$
$$= (3+6) a + (5-2)b$$
$$= 9a + 3b$$

Ask students these questions: How does grouping the terms of an expression allow you to simplify it? How do you know when an expression can be simplified no further?

Emphasize that a numerical expression names a unique (just one) number. Therefore, when a number is substituted for each variable in an algebraic expression, the expression names a unique number.

Review the meaning of the words sum, difference, product and quotient. Then allow students to practice writing expressions and

then substituting numbers for letters using every day examples. Emphasize that the rules for order of operations represent an agreement that is made to ensure that a given numerical expression will name a unique number.

Let students identify monomial and binomial expressions.

Answers to Activity 2.3

1. $(4^2+4) \div (2^2-2) = (16+4) \div (4-2) = 20 \div 2 = 10$
2. $24 + 8 \times 12 \div 4 - 2 = 24 + 8 \times 3 - 2 = 24 + 24 - 2 = 46$
3. $16 \div 8 + 9 \times 7 = 2 + 63 = 65$
4. $8 - [14 \div (2+5)] = 8 - [14 \div 7] = 8-2 = 6$

Answers to Group Work 2.2

1. 21   2. 12   3. 13   4. 31

Assessment

You may give a set of problems on finding the value of simple algebraic expressions as a form of class work, home work, assignment, quiz or test so as to check students' progress. You may also ask students, who are fast learners, or interested students to answer the following questions as additional assessment.

Additional Assessment

1. Evaluate (a) $\dfrac{y - 2(x+3)}{y - 3x}$ for x=3 and y = 12

   (b) $\dfrac{21}{x}$ for x= $\dfrac{1}{3}$

   (c) $\dfrac{x}{y}$ for x= $\dfrac{1}{2}$ and y = $\dfrac{1}{8}$

2. Simplify, if possible.

    a) $14x - x$

    b) $11d + 4 + d + 7$

    c) $\dfrac{3}{4}(12x + 8) + 5x + 2$

Answers to additional assessment

1. a) $\dfrac{y - 2(x+3)}{y - 3x} = \dfrac{12 - 2(3+3)}{12 - 3(3)} = \dfrac{12 - 2(6)}{12 - 6} = \dfrac{12 - 12}{3} = \dfrac{0}{3} = 0$

   b) $\dfrac{21}{x} = \dfrac{21}{\frac{1}{3}} = 21 \div \dfrac{1}{3} = 21 \times 3 = 63$

   c) $\dfrac{x}{y} = \dfrac{\frac{1}{2}}{\frac{1}{8}} = \dfrac{1}{2} \div \dfrac{1}{8} = \dfrac{1}{2} \times 8 = 4$

2. a) $14x - x = 13x$

   b) $11d + 4 + d + 7 = (11d + d) + (4 + 7) = 12d + 11$

   c) $\dfrac{3}{4}(12x + 8) + 5x + 2 = \dfrac{3}{4}(12x) + \dfrac{3}{4}(8) + 5x + 2 = 9x + 6 +$

    $5x + 2 = 14x + 8$

Answers to Exercise 2.B

1. a) $4x = 4(3) = 12$

   b) $\dfrac{x + y}{9} = \dfrac{12 + 6}{9} = \dfrac{18}{9} = 2$

   c) $8x - 1 = 8(2) - 1 = 16 - 1 = 15$

   d) $\dfrac{r - t}{8} = \dfrac{14 - 6}{8} = \dfrac{8}{8} = 1$

   e) $\dfrac{2u + 3v}{6} = \dfrac{2(3) + 3(2)}{6} = \dfrac{6 + 6}{6} = \dfrac{12}{6} = 2$

f) $\dfrac{r}{t} = \dfrac{16}{2} = 8$

g) $\dfrac{x + y}{7} = \dfrac{15 + 20}{7} = \dfrac{35}{7} = 5$

h) $\dfrac{9m}{q} = \dfrac{9(6)}{18} = \dfrac{54}{18} = 3$

i) $\dfrac{m^2 - n^2}{3} = \dfrac{6^2 - 3^2}{3} = \dfrac{36 - 9}{3} = \dfrac{27}{3} = 9$

2. a) 3x and x are like terms

   b) 7u and 5u are like terms; $3u^2$ and $4u^2$ are another like terms

   c) 3y and 5y are like terms

   2z, 8z and z are another like terms

3. a) $x \div y + xy = 6 \div 3 + 6(3) = 2 + 18 = 20$

   b) $x^2 + y^2 + z^2 = 6^2 + 3^2 + 2^2 = 36 + 9 + 4 = 49$

   c) $xy \div z - yz = 6(3) \div 2 - 3(2) = 18 \div 2 - 6 = 9 - 6 = 3$

   d) $x^2 - xy + z = 6^2 - 6(3) + 2 = 36 - 18 + 2 = 20$

   e) $\dfrac{x + y + z}{11} = \dfrac{6 + 3 + 2}{11} = \dfrac{11}{11} = 1$

4. a) $2x + 3x + 6x + x = (2 + 3 + 6 + 1)x = 12x$

   b) $2y^2 + 7y^2 + 9y^2 = (2 + 7 + 9)y^2 = 18y^2$

   c) $3xy + 7xy + 5xy = (3 + 7 + 5)xy = 15xy$

   d) $5b + 5b + 3b + 8b = (5 + 5 + 3 + 8)b = 21b$

5. a) $2x + 3y + 4z + 5x + 8y - 2z = 2x + 5x + 3y + 8y + 4z - 2z$

   $= (2 + 5)x + (3 + 8)y + (4 - 2)z$

   $= 7x + 11y + 2z$

   b) $4e + f + 3h + e - 2h + 2f = 4e + e + f + 2f + 3h - 2h$

   $= (4 + 1)e + (1 + 2)f + (3 - 2)h$

   $= 5e + 3f + h$

c) $d^2 + e^2 + 4f^2 + 3d^2 + 2e^2 - 3f^2 = d^2 + 3d^2 + e^2 + 2e^2 + 4f^2 - 3f^2$

$\qquad = (1+3)\, d^2 + (1+2)e^2 + (4-3)\, f^2$

$\qquad = 4d^2 + 3e^2 + f^2$

6. a) $10x - 2x = (10-2)x = 8x$

   b) $15y - 3y = (15-3)\, y = 12y$

   c) $31z - 20z = (31 - 20)\, z = 11z$

7. a) $4x + y + 6z - x + 2y - 3z = 4x - x + y + 2y + 6z - 3z$

   $\qquad = (4-1)\, x + (1+2)y + (6-3)\, z$

   $\qquad = 3x + 3y + 3z$

   b) $8r + 2q + 3t - 7r - q - 2t = 8r - 7r + 2q - q + 3t - 2t$

   $\qquad = (8-7)\, r + (2-1)\, q + (3-2)\, t$

   $\qquad = r + q + t$

   c) $10t - 4t + 8q + 2r - 3q + 5r$

   $\qquad = 10t - 4t + 8q - 3q + 2r + 5r$

   $\qquad = (10-4)\, t + (8-3)\, q + (2+5)\, r$

   $\qquad = 6t + 5q + 7r$

Selected problems to slow learners

1. Write algebraic expressions for each of the following.

   a. x plus 2.

   b. 4 minus x.

   c. 10 times x.

   d. 1000 divided by x.

2. Simplify

   a. $2x + 3x + 4x$

   b. $20x - 3x + 5x$

   c. $15x + 20x + 4y + 5y$

3. What is the variable in the term "8x"?

4. Is $3a + b$ a monomial or a binomial?

5. If a = 5 and b=3, then find the value of 2ab.
6. Evaluate $a^2 + b^2 + c^2$ if a = 1, b = 2 and c = 3
7. Write an algebraic expression for "2 is subtracted from x".
8. Write an algebraic expression for "sum of 3x and 4".
9. Are 6a and 6b like terms?
10. Is it true that x+1+x+1+x = 3x+2?

Selected problems to fast learners

1. Write algebraic expressions for each of the following.
   a. Twice the sum of x and y.
   b. One third of the difference of 6 and x.
   c. The square of the product of x and y.
   d. The cube of the quotient of x and y.

2. Evaluate $\left[\frac{a+2(b+3)}{a-3b}\right]^2$ when a = 12 and b = 3

3. Write the algebraic expression "3x - 4" in words.

4. Show that the expression
   $x^3 - 13x^2 + 44x = 32$, when x = 1, 4 or 8

5. Evaluate $\frac{a^3 + b^3}{c^3}$ when a = 2, b = 3 and c = 4.

6. Write the difference between 3a and 4b.

7. Evaluate $\frac{12x-10y}{6z-4x}$ if x = 3, y =1 and z =4

8. Are $8a^2b^2$ and $10a^2b^2$ like terms?

9. Simplify 6abc + 5abc + 4ab − 8abc + 3ab.

10. Which expression has a value of 74 when x = 10, y = 8 and
    z = 12?
    a) 4xyz   c) x + 5y + 2z
    b) 2xz − 3y   d) 6xyz + 8

2.2 Equations and Inequalities

Periods allotted: 7 periods

Competencies

At the end of this sub – unit, students will be able to:

- differentiate equations and inequalities.
- determine the possible solution of the given equations from the given numbers.
- determine the possible solution of the given inequalities.

Introduction

In the previous sub-unit students were discussing algebraic terms and expressions. In this sub-unit they are expected to discuss equations and inequalities. For this purpose, different activities are included in the sub unit. Students are expected to identify the difference between algebraic expressions, equations and inequalities, thereby give examples of equations and inequalities from everyday example. Basic examples which deal with important issues like shopping, saving, etc are discussed in the sub-unit.

Teaching notes

You may start the sub-unit by asking students to write an algebraic statement for these sentences: The sum of 6 and 4 equals 10, and 8 subtracted from x equals 20. Ask them whether the first statement is true or false. Ask them whether they can tell the second statement is true or false and why. Ask them whether the second statement is true for x = 30. Ask whether it is true for x = 28.

Discuss with the students the meaning of equation and inequality by using everyday examples. Assist students to practice making simple equations and inequalities related to important issues (like HIV, buying and selling). You may use examples such as the following:

If I think of a number and add 10 to it, my answer is 30. What was the number I thought of?

Facilitate students to work in pairs and ask each other simple problems like:

If the cost of 4kg of sugar is Birr 40, then what is the cost of 1kg of sugar?

Answers to Activity 2.4
a) True. Because $15 = 8+7$
b) False. Because $14+4 = 18 \neq 19$
c) True. Because $18-17 = 1$
d) False. Because $3(6) = 18 \neq 21$

Answers to Activity 2.5
i) An expression   ii) an equation   iii) an expression

iv) an equation   v) an expression   vi) an equation

Answers to Group Work 2.3
a) 75   b) 70   c) 9   d) 400

Answers to Group Work 2.4
$60 \times 9 = 540$ seconds

Assessment
You may give students problems on solving equations and inequalities in the form of class work, homework, assignment, quiz or test. You may also ask fast learners or interested students to answer the following questions as additional assessment.

Additional Assessment
1. Solve a) $x + \dfrac{1}{2} = 4$ b) $y - \dfrac{3}{4} = 1$ c) $\dfrac{1}{5}a = 10$

2. Which of the numbers (2, 4 or 6) can be in the solution of the inequality $2x > x + 5$?

Answers to additional assessment

1. a) $x + \dfrac{1}{2} = 4$. Thus, $x = 4 - \dfrac{1}{2} = \dfrac{7}{2} = 3\dfrac{1}{2}$

   b) $y - \dfrac{3}{4} = 1$. Thus, $y = 1 + \dfrac{3}{4} = \dfrac{7}{4} = 1\dfrac{3}{4}$

   c) $\dfrac{1}{5}a = 10$. Thus, $a = 10(5) = 50$.

2. 6 is in the solution of the inequality $2x > x + 5$. Because $2(6) > 6 + 5$. That is, $12 > 11$.

Answers to Exercise 2. C

1. i. c   ii. d   iii. a   iv. b

2. a) False, Since $16 + 4 = 20 \neq 14$
   b) True.
   c) True.
   d) False. Since $21 \div 7 = 3 \neq 2$

3. a) 4   b) 29   c) 4   d) 60

4. a) $n + 4 = 9$ b) $a - 3 = 16$ c) $6x = 48$ d) $\dfrac{b}{3} = 6$

5. a) $x = 2$ b) $y = 9$ c) $m = 13$ d) $n = 7$

6. let n be the number assumed.
   Then $n - 5 = 7$
        $\therefore n = 12$

7. Let y be the sale price, then the price of Almaz's sweater (after reduction) = y – 30

8. Cost of 5kg of sugar = Birr x

Therefore, the cost of 1kg of sugar

$= \text{Birr } \dfrac{x}{5}$

Answers to Activity 2.6

   a) $4(6+3) < 100$ True. Because $36 < 100$.

   b) $20 – 6 < 4(3+2)$ True. Because $14 < 20$.

   c) $10 – 3 > 24 – 5(3)$ False. Because $7 < 9$.

   d) $3(10\text{-}3) \neq 4(7\text{-}1)$ True. Because $21 \neq 24$.

Answers to Exercise 2. D

1.   a) $n\text{-}2 \geq 10$ b) $2x + 3 < 20$ c) $y^2 \leq 6$

    d) $a^2 > 100$ e) $\dfrac{z}{3} \leq 10$

2. 1. c   2.e   3. a   4. b

3. i) 3 or 5 Because $3 + 7 = 10 < 20$ and

                      $5 + 7 = 12 < 20$

  ii) 200 or 100. Because $200 – 28 = 172 > 30$ and

                  $100 – 28 = 72 > 30$

  iii) 48 or 6. Because $\dfrac{48}{6} = 8 \leq 8$ and

                    $\dfrac{6}{6} = 1 \leq 8$

  iv) 14 or 20. Because $8(14) = 112 \geq 96$ and

                  $8(20) = 160 \geq 96$

e) 2 or 3. Because $\dfrac{108}{2} = 54 \geq 36$ and

$$\dfrac{108}{3} = 36 \geq 36$$

Assessment

In addition to traditional teacher demonstrations and teacher – led discussions, greater opportunities should be provided for small – group work, individual explorations, peer instruction and whole – class discussions in order to check whether students show progress in the discussion of working with variables. You may use a different approach to look at the students performance and other assessment techniques such as class work, group discussion, assignment, and exercise problems and quiz or test.

Selected problems to slow learners

1. Classify each of the following as either an expression or an equation.
   a. The quotient of 6 and y is 2.
   b. x decreased by 10.
   c. The difference of x and 3 is 20.
   d. Two times y is 10.
2. Which of the following given numbers can satisfy the inequality $3+x>8$?
   a) 4   b) 5   c) 6   d) 10
3. Write an equation for "sum of a and 3 is equal to 10"
4. Write an inequality for "x greater than 10".
5. Find x if x + 1 = 10.
6. Find a if 2a = 14
7. If 4 is added to a number the result is 11. What is the number?
8. The product of a number and 3 is equal to 27. What is the number?
9. If a number is divided by 3 it becomes 3. What is the number?

Selected problems to fast learners

1. Find three whole numbers which can satisfy the inequality $2a - 5 < 6$.
2. Solve each of the following equations.
   a) $\frac{x}{30} = 20$
   b) $15y = 300$
3. Twice a number increased by 4 becomes 16. What is the number?
4. There are only 80 fifty-cent coins in a bag. If the bag contains Birr x, then what is the value of x?
5. Sum of two consecutive numbers is 53. What are the numbers?
6. Sum of two consecutive odd numbers is 48. What are the numbers?
7. Sum of two consecutive even numbers is 62. What are the numbers?
8. What is the sum of the first 8 consecutive odd numbers?

Answers to Review Exercise

1. i. g   ii. d   iii. f   iv. i   v. h   vi. g
   vii. b   viii. c   ix. a   x. e

2. a) $\dfrac{x - y}{3} = \dfrac{18 - 9}{3} = \dfrac{9}{3} = 3$

   b) $\dfrac{a + b}{4} = \dfrac{16 + 8}{4} = \dfrac{24}{4} = 6$

   c) $\dfrac{x + y}{2} = \dfrac{6 + 12}{2} = \dfrac{18}{2} = 9$

   d) $\dfrac{a - b}{3} = \dfrac{18 - 6}{3} = \dfrac{12}{3} = 4$

3. a) n + 3 +1 = n + 4   b) m + 2+ 2 = m + 4

4. Rahel's present age = a+ 3

   Rahel's age after 7 years = a+ 3 + 7 = a + 10

5. a) True   b) True   c) True   d) False   e) True

6. a) equation given   b) expression given   c) equation given by

   by $\dfrac{a}{10}$ =7   by w+ 20   3x – 7 = 2

   d) equation given by 5 + 2y = 13

7. b

8. Because Beza spent Birr 2 and left with Birr 5, before she spent
   Birr 2 Beza had Birr 7(that is, 5 + 2).

9. Before shopping, Fatuma had Birr 49 (that is, 32 + 17).

10. a) Let d represents days. Then the given inequality can be
    described by "d ≥ 28".

b) Let t represents temperature. Then the given inequality can be
   described by "t>30".

c) Let p represents people in the show room. Then the given
   inequality can be described by "p ≤ 350".

d) Let m represent people attending the meeting, then the given
   inequality can be described by "m<100".

11. Let a = number of girls.

         b = number of boys.

         a = b + 30 (because there are 30 more) girls than boys.

    and a + b = 120 (total number of students) in grade 8.

    Therefore b + 30 + b = 120 (because a = b + 30)

    or 2b + 30 = 120   or   2b = 90   or   b =45. Thus a = 75

    (because a = b + 30). Therefore, the answer is c.

12. d=15×8=120

UNIT THREE

FRACTIONS, DECIMALS AND THE FOUR OPERATIONS

Introduction

This unit reviews students' previous concepts in regards to fractions. In this unit, students learn to identify compare, order, use simple fractions, express as a percentage, convert a percentage to fractions and decimals and vice- versa. Different approaches to writing fractions as decimals and percentages is also elaborated with descriptive examples. Various activities, group works and exercises, which relate to real life situations, are included. In addition, rules in performing the four basic fundamental operations on fractions and decimals are stated in good detail in the unit. In general, the concepts discussed in this unit enable students to operate and use simple fractions.

Unit outcomes

At the end of this unit, students will be able to:

- know types of fractions.
- understand concept of percentage and principles of conversion of percentage to fraction and decimal.
- know method of comparing fractions.
- perform the four basic fundamental operations on fractions and decimals.

Suggested Teaching Aids in Unit 3

This unit is intended for developing the concepts on fractions and operations on fractions. In order to make students practice about fractions it is essential to include practical examples that relate to students' daily life. Fraction of the day students spend in common activities: school, meals, travel, sleep, sports, etc are among the practical examples that you can use support teaching fractions. Different diagrams that represent fractions are also useful as teaching aids in this unit.

Charts containing representation of fractions like the following are recommended:

Fractions

a)   b)

c)   d)

Proper fraction

$$\frac{5}{2}$$

$$2\frac{1}{2}$$
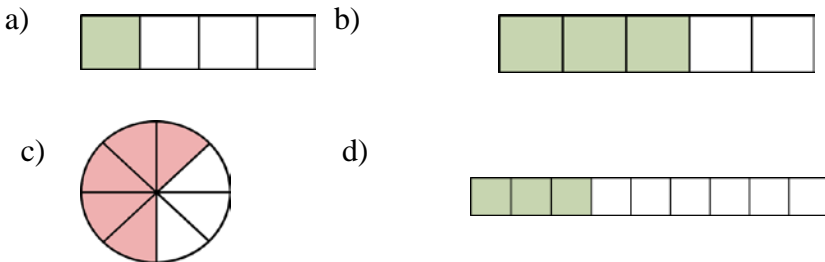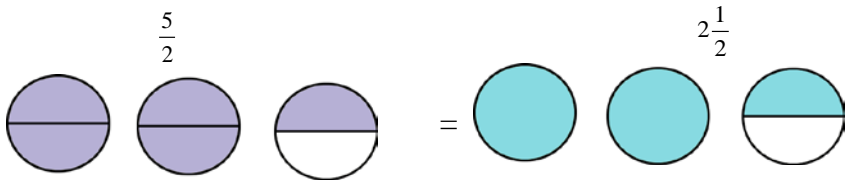
Percentage
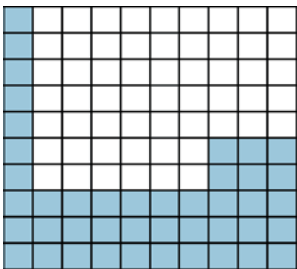
Addition of fractions

- To add $\dfrac{1}{2}$ and $\dfrac{1}{4}$, the common unit of measure is fourths.

$$\dfrac{1}{2} \quad + \quad \dfrac{1}{4} \quad\Longrightarrow\quad \dfrac{2}{4} \quad + \quad \dfrac{1}{4} \quad = \quad \dfrac{3}{4}$$

Multiplication

1

$\dfrac{1}{3}$	$\dfrac{1}{3}$	$\dfrac{1}{3}$

Each rectangle on the diagram represents $\dfrac{1}{3}$.

$\dfrac{1}{2}$ of $\dfrac{1}{3}$ or $\dfrac{1}{6}$
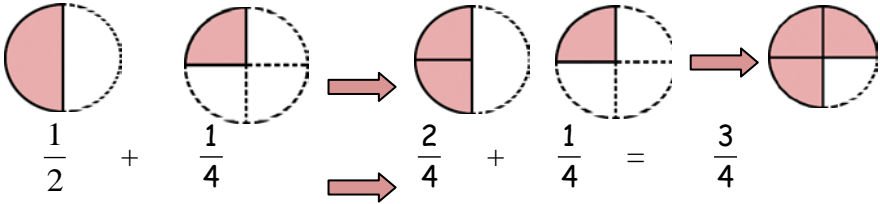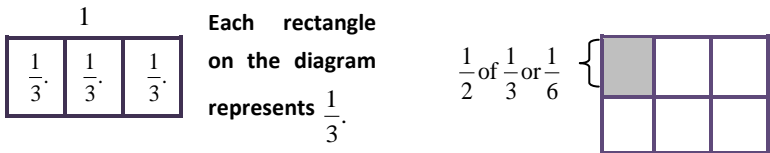
Division

Divide 4 by $1\dfrac{1}{3}$

$1\dfrac{1}{3}$ $1\dfrac{1}{3}$ $1\dfrac{1}{3}$

$$4 \div 1\dfrac{1}{3} = 3$$
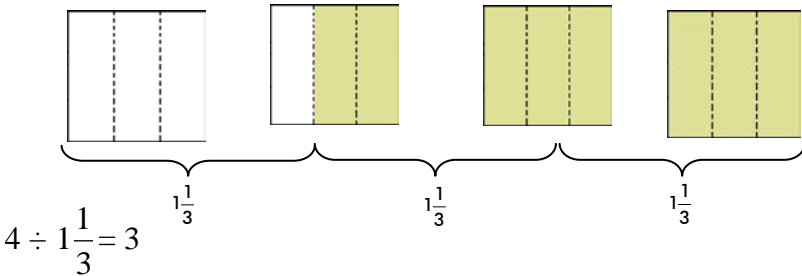
3.1 Types of fractions

Periods allotted: 4 periods

Competencies

At the end of this sub-unit students will be able to:

- identify proper and improper fractions and mixed numbers.
- change improper fractions to mixed numbers and vice versa.

Introduction

This sub- unit begins with reversing what a fraction is and introduce types of fractions.

It provides students with definition of proper fractions, improper fractions and mixed numbers.

Teaching notes

Before introducing types of fractions remind students what they had learnt about fractions and motivate them by asking numerators and denominators of fractions. For the purpose of revision, you may ask students questions like:

1. Tell the numerator and denominator of each fraction.

   a) $\dfrac{5}{8}$　　　　　　b) $\dfrac{3}{7}$　　　　　　c) $\dfrac{9}{4}$

2. Write each fraction in simplest form.

   a) $\dfrac{3}{6}$　　　　　　b) $\dfrac{4}{12}$　　　　　　c) $\dfrac{9}{36}$

You may also use Activity 3.1 for the purpose of revising students' prior knowledge on fractions.

Following the revision, you may continue introducing types of fractions by using simple objects (like 2 and $\dfrac{1}{2}$ oranges or even their test results say $6\dfrac{1}{2}$ out of 10). Let students come to the meaning of proper fractions, improper fractions and mixed numbers. You may use examples like "If a teacher gives $\dfrac{1}{2}$ of an orange to a student as a reward and has $4\dfrac{1}{2}$ oranges, how many students can get the reward? This means how many $\dfrac{1}{2}$ are there in $4\dfrac{1}{2}$?" Assist the

students to convert improper fractions to mixed numbers and vice versa. Lead the students to generalize that

$$a\frac{b}{c} = \frac{(c \times a) + b}{c}$$ for all a, b, c in N and b < c. Group your students

and let them discuss Activity 3.2. Ask students to suggest how they could convert mixed numbers to improper fractions and vice versa and check their work.

Answers to Activity 3.1

1. a) $\frac{1}{4}$ b) $\frac{3}{5}$ c) $\frac{5}{8}$ d) $\frac{3}{9}$ or $\frac{1}{3}$

2. a) $\frac{18}{20} = \frac{18 \div 2}{20 \div 2} = \frac{9}{10}$ b) $\frac{42}{60} = \frac{42 \div 6}{60 \div 6} = \frac{7}{10}$

    c) $\frac{24}{40} = \frac{24 \div 8}{40 \div 8} = \frac{3}{5}$ d) $\frac{42}{56} = \frac{42 \div 14}{56 \div 14} = \frac{3}{4}$

    e) $\frac{65}{75} = \frac{65 \div 5}{75 \div 5} = \frac{13}{15}$

3. a) Because 7> 3, $\frac{7}{8} > \frac{3}{8}$ b) $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$,

                                            8 > 6, and $\frac{8}{10} > \frac{6}{10}$

                                     Thus $\frac{4}{5} > \frac{6}{10}$

    c) $\frac{6}{12} = \frac{4}{8}$ because both simplify to $\frac{1}{2}$

    d) $\frac{10}{15} > \frac{8}{15}$ because 10 > 8

    e) $\frac{4}{9} < \frac{7}{9}$ because 4 < 7

    f) $\frac{7}{16} < 1$ because 1= $\frac{16}{16}$ and 7 < 16

Answers to Activity 3.2

Seven 'one – fourths' are shaded and the remaining one 'one-fourths' is not shaded.

$$1\frac{3}{4} = \text{seven 'one-fourth's} = \frac{7}{4}$$

Answers to Group Work 3.1

a) $\frac{3\times7+5}{7} = \frac{26}{7}$ c) $\frac{8\times2+1}{2} = \frac{17}{2}$

b) $\frac{6\times4+1}{4} = \frac{25}{4}$

Assessment

To make sure that students have properly identified types of fractions, and understood the method of converting improper fractions to mixed numbers and vice versa, you can give them Exercise 3.1 as home work and let selected students present it to class. Check their work and the overall discussion during presentation. You can also give the following problems to students as additional assessment.

Additional Assessment

Compare each pair of fractions.

a) $1 \square \frac{4}{5}$ b) $\frac{3}{7} \square \frac{6}{7}$ c) $2 \square \frac{14}{3}$

d) $\frac{12}{24} \square \frac{5}{10}$ e) $\frac{17}{16} \square 1$

Answers to   Additional Assessment

a) $1 > \frac{4}{5}$ b) $\frac{3}{7} < \frac{6}{7}$ c) $2 < \frac{14}{3}$

d) $\frac{12}{24} = \frac{5}{10}$ e) $\frac{17}{16} > 1$

Answers to Exercise 3.A

1. a) true   b) true   c) True   d) False

   e) True   f) True   g)False,$5\dfrac{1}{6}=\dfrac{31}{6}$

2. a) proper   b) improper   c) improper   d) proper   e) improper

3. a) $\dfrac{21}{10}=\dfrac{2\times10+1}{10}=2\dfrac{1}{10}$ b) $\dfrac{46}{5}=\dfrac{9\times5+1}{5}=9\dfrac{1}{5}$

   c) $\dfrac{18}{7}=\dfrac{2\times7+4}{7}=2\dfrac{4}{7}$ d) $\dfrac{59}{8}=\dfrac{7\times8+3}{8}=7\dfrac{3}{8}$

   e) $\dfrac{29}{6}=\dfrac{4\times6+5}{6}=4\dfrac{5}{6}$ f) $\dfrac{39}{2}=\dfrac{19\times2+1}{2}=19\dfrac{1}{2}$

   g) $\dfrac{69}{9}=\dfrac{7\times9+6}{9}=7\dfrac{6}{9}$ h) $\dfrac{97}{3}=\dfrac{32\times3+1}{3}=32\dfrac{1}{3}$

   i) $\dfrac{101}{10}=\dfrac{10\times10+1}{10}=10\dfrac{1}{10}$ j) $\dfrac{98}{9}=\dfrac{10\times9+8}{9}=10\dfrac{8}{9}$

4. a) $8\dfrac{1}{7}=\dfrac{8\times7+1}{7}=\dfrac{56+1}{7}=\dfrac{57}{7}$

   b) $7\dfrac{1}{9}=\dfrac{7\times9+1}{9}=\dfrac{63+1}{9}=\dfrac{64}{9}$

   c) $6\dfrac{1}{10}=\dfrac{6\times10+1}{10}=\dfrac{60+1}{10}=\dfrac{61}{10}$

   d) $5\dfrac{3}{11}=\dfrac{5\times11+3}{11}=\dfrac{55+3}{11}=\dfrac{58}{11}$

   e) $1\dfrac{2}{11}=\dfrac{1\times11+2}{11}=\dfrac{11+2}{11}=\dfrac{13}{11}$

   f) $4\dfrac{2}{13}=\dfrac{4\times13+2}{13}=\dfrac{52+2}{13}=\dfrac{54}{13}$

   g) $8\dfrac{3}{10}=\dfrac{8\times10+3}{10}=\dfrac{80+3}{10}=\dfrac{83}{10}$

h) $9\frac{4}{11} = \frac{9 \times 11 + 4}{11} = \frac{99 + 4}{11} = \frac{103}{11}$

i) $2\frac{1}{16} = \frac{2 \times 16 + 1}{16} = \frac{32 + 1}{16} = \frac{33}{16}$

j) $9\frac{7}{8} = \frac{9 \times 8 + 7}{8} = \frac{72 + 7}{8} = \frac{79}{8}$

5. $\frac{7}{24}$ 6. $\frac{5}{8}$ 7. $\frac{45}{60}$ or $\frac{3}{4}$

8. a) $\frac{5}{8}$ b) $\frac{3}{8}$

Selected problems to slow learners

1. Classify the given fraction as proper or improper.

     a. $\frac{11}{13}$ b. $\frac{13}{5}$ c. $\frac{3}{3}$ d. $\frac{0}{4}$ e. $\frac{7}{1}$

2. Write the mixed number as an improper fraction.

     a. $7\frac{1}{3}$ b. $11\frac{1}{4}$ c. $12\frac{1}{2}$

     d. $15\frac{1}{2}$ e. $20\frac{1}{7}$

3. Name the denominator of $\frac{4}{7}$.

4. What is the mixed number represented by $\frac{8}{3}$?

5. Are all the fractions $\frac{15}{4}, \frac{4}{3}, \frac{6}{5}$ and $\frac{7}{4}$ improper?

6. Represent 3 parts out of 5 as a fraction.

7. What is the fraction form of the shaded part?
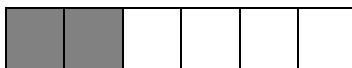
8. Which of the following fractions is the same as $3 + \frac{1}{3}$?

     a) $\frac{31}{3}$ b) $\frac{13}{3}$ c) $\frac{10}{3}$ d) $\frac{8}{3}$

9. A club spent one third of its money and saved the rest. What fraction was saved?

Selected problems to fast learners

1. Which of the following fractions are equal?

   $18\frac{1}{3}$, $\frac{54}{7}$, $\frac{57}{3}$, $100\frac{1}{7}$, $7\frac{5}{7}$, $\frac{701}{7}$

2. A person slept for 35 minutes. What fraction of an hour is that?

3. What is the mixed number represented by $\frac{625}{17}$?

4. If $6 + \frac{x}{3} = \frac{20}{3}$, then find the value of x.

5. In one 30 – gram food, there are 24 grams of carbohydrates. What fraction of the food is made up of carbohydrates?

6. What is the numerator of $3\frac{4}{11}$?

7. What is the fraction form of the shaded part?

8. Which is bigger than one, $\frac{7}{8}$ or $1\frac{1}{8}$?

9. Mekdes has discovered that two-thirds of her class are girls. There are 24 girls in the class. How many students are there in Mekdes' class?
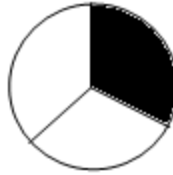
3.2 Percentage as Fractions

Periods allotted: 5 periods

Competencies
At the end of this sub-unit students will be able to:

- express percentage as a fraction
- convert percentage to decimal

Introduction
Once students study types of fractions, it will be essential to convert fractions to different forms. In this sub-units, emphasis will be given to expressing percentage as fractions. Converting percentage to decimal will also be discussed in this sub-unit.

Teaching Notes

You may use a rectangular region that is divided in to 100 equal parts and consider different shaded regions with different colours. By doing this, introduce the concept of percentage as part of a whole that is divided in to 100 parts.

Have students work the Activities with partners. Assist students to express percentage as fraction and decimal. You may use examples like:

$$65\% = \frac{65}{100} = 0.65$$

Answers to Activity 3.3

One way of shading the given percentage is shown as follows:
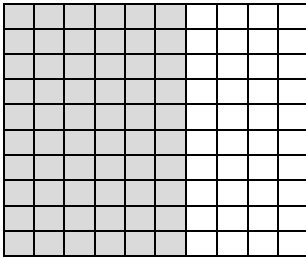
60%
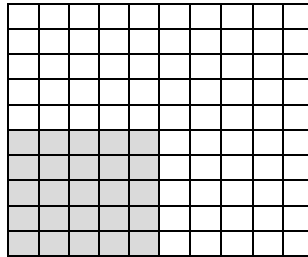
25%

$35\frac{1}{2}$
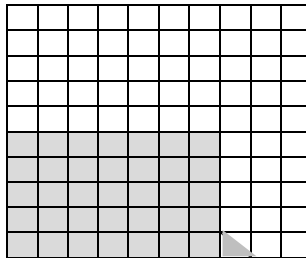
You can find the percent represented by the shaded area by finding the area of the shaded area.

Answers to Group Work 3.2
a) 28%   b) 3.75%   c) 70%   d) 360%

Assessment
You need to assess students in order to be informed about their progress. Making assessment on expressing percentage as a fraction and converting percentage to decimal will enable you to find out what aspect of the lesson is difficult. So you can give class work, group work, discussion, assignments, quiz or test in order to assess students' performance and get a clear picture of students' knowledge and skill.

Answers to Exercise 3B

1) a) $\dfrac{14}{15} = \dfrac{14 \times 100}{15 \times 100} = \left(\dfrac{14 \times 100}{15}\right)\dfrac{1}{100} = \dfrac{1400}{15}\% = \dfrac{280}{3}\% = 93.\dot{3}\%$

b) $\dfrac{23}{30} = \dfrac{23 \times 100}{30 \times 100} = \left(\dfrac{23 \times 100}{30}\right)\dfrac{1}{100} = \dfrac{230}{3}\% = 76.\dot{6}\%$

c) $\dfrac{18}{25} = \dfrac{18 \times 4}{25 \times 4} = \dfrac{72}{100} = 72\%$

d) $\dfrac{13}{20} = \dfrac{13 \times 5}{20 \times 5} = \dfrac{65}{100} = 65\%$

e) $\dfrac{1}{8} = \dfrac{1 \times 100}{8 \times 100} = \left(\dfrac{100}{8}\right)\dfrac{1}{100} = 12.5\%$

f) $\dfrac{5}{8} = \dfrac{5 \times 100}{8 \times 100} = \left(\dfrac{500}{8}\right)\dfrac{1}{100} = 62.5\%$

g) $\dfrac{7}{7} = \dfrac{1 \times 100}{1 \times 100} = \dfrac{100}{100} = 100\%$

h) $\dfrac{19}{20} = \dfrac{19 \times 5}{20 \times 5} = \dfrac{95}{100} = 95\%$

j) $\dfrac{12}{25} = \dfrac{12 \times 4}{25 \times 4} = \dfrac{48}{100} = 48\%$

i) $\dfrac{3}{50} = \dfrac{3 \times 2}{50 \times 2} = \dfrac{6}{100} = 6\%$

2.

a) $55\% = \dfrac{55}{100} = \dfrac{11}{20} = 0.55$

b) $12\% = \dfrac{12}{100} = \dfrac{3}{25} = 0.12$

c) $75\% = \dfrac{75}{100} = \dfrac{3}{4} = 0.75$

d) $10\% = \dfrac{10}{100} = \dfrac{1}{10} = 0.1$

e) $90\% = \dfrac{90}{100} = \dfrac{9}{10} = 0.9$

f) $36\dfrac{2}{3}\% = \left(\dfrac{36 \times 3 + 2}{3}\right)\dfrac{1}{100} = \dfrac{108 + 2}{300} = \dfrac{110}{30} = 0.3\dot{6}$

g) $19\dfrac{1}{2}\% = \left(\dfrac{19 \times 2 + 1}{2}\right)\dfrac{1}{100} = \dfrac{38 + 1}{200} = \dfrac{39}{200} = 0.195$

h) $14\dfrac{1}{3}\% \quad = \left(\dfrac{14 \times 3 + 1}{3}\right)\dfrac{1}{100} = \left(\dfrac{42 + 1}{3}\right)\dfrac{1}{100} = \dfrac{43}{300} = 0.14\dot{3}$

i) $9\dfrac{1}{4}\% = \left(\dfrac{9 \times 4 + 1}{4}\right)\dfrac{1}{100} = \dfrac{37}{400} = 0.0925$

j) $16\dfrac{1}{5}\% = \left(\dfrac{16 \times 5 + 1}{5}\right)\dfrac{1}{100} = \left(\dfrac{80 + 1}{5}\right)\dfrac{1}{100} = \dfrac{81}{500} = 0.162$

3. a) $0.18 = \dfrac{18}{100} = 18\%$ b) $0.01 = \dfrac{1}{100} = 1\%$

c) $0.7 = \dfrac{7}{10} = \dfrac{7 \times 10}{10 \times 10} = \dfrac{70}{100} = 70\%$

d) $0.025 = \dfrac{25}{1000} = \dfrac{2.5}{100} = 2.5\%$

e) $0.375 = \dfrac{375}{1000} = \dfrac{37.5}{100} = 37.5\%$

f) $0.681 = \dfrac{681}{1000} = \dfrac{68.1}{100} = 68.1\%$

g) $0.086 = \dfrac{86}{1000} = \dfrac{8.6}{100} = 8.6\%$

h) $0.0625 = \dfrac{625}{10,000} = \dfrac{625}{100} = 6.25\%$

4. a) 21%   b) 10%   c) 50%   d) 15%   e) 60%

Selected problems to slow learners

1. Express each percentage as a fraction.
    a. 7%   b. 10%   c. 50%
    d. 80%   e. 100%

2. Express each fraction as a percentage.
    a. $\dfrac{11}{100}$ b. $\dfrac{20}{100}$ c. $\dfrac{7}{50}$
    d. $\dfrac{8}{25}$ e. $\dfrac{4}{5}$

3. What percent is 6 hours of a day?
4. What percent is 15 days of a month?
5. If 25% of a number is 4, then what is the number.
6. What is 20% of 100?
7. Find x if $40\% = \dfrac{x}{5}$
8. Convert $10\dfrac{1}{10}$ in to percentage.
9. Express 0.2 as percentage.

10. A club spent one fourth of its money and saved the rest. What percent was saved?
11. What percent of 48 is 16?

Selected problems to fast learners

1. Express each decimal as percentage.
   a. 0.002   b. 0.301   c. 0.0075
   d. 0.00725   e. 0.0001
2. Express each percentage as a fraction in its simplest form and a decimal.

   a. $21\frac{1}{2}$ %   b. $33\frac{1}{5}$%   c. $29\frac{4}{9}$%

   d. $38\frac{3}{8}$ %   e. $57\frac{5}{6}$%
3. What percent of 80 is 10?
4. A man earns Birr 1250 a month and spends Birr 1250 a month and spends Birr 750. What percent of his earning does he save?
5. What is 40% of 200?
6. What is $8\frac{1}{2}$ of 162?
7. $6\frac{1}{4}$% of a number is 75. Find the number.
8. 20% of a number is 40. Find the number.
9. Find the percentage form of
   a) 3 out of 20   b) 4 out of 5
10. 4 litres of water is added to 16 litres of milk. What percentage of milk does the mixture contain?
11. Express 0.004 as percentage.

3.3 Comparison and ordering of fractions

Periods allotted: 8 periods

Competency
At the end of this sub-unit students will be able to:
- compare fractions.

Introduction
The main task of this sub- unit is to enable students compare fractions. Thus, it will be useful to revise comparison of fractions having the same denominators. Rewriting fractions using the same denominator by using equivalent fractions will be considered in this sub-unit.

Teaching Notes
You may start the discussion of this sub-unit by revising comparison of fractions having the same denominators. Conduct a class discussion about equivalent fractions.

Assist students to compare simple fractions with different denominators using equivalent fractions. You may use examples like:

Compare $\dfrac{3}{4}$ and $\dfrac{5}{7}$

Solution. $\dfrac{3}{4} = \dfrac{3 \times 7}{4 \times 7} = \dfrac{21}{28}$, $\dfrac{5}{7} = \dfrac{5 \times 4}{7 \times 4} = \dfrac{20}{28}$

Since $21 > 20$, it follows that $\dfrac{21}{28} > \dfrac{20}{28}$.

That means $\dfrac{3}{4} > \dfrac{5}{7}$.

Let students participate in answering activities included in this sub-unit. Help students to order two or more given fractions, with different denominators, either in an increasing or a decreasing order.

Answers to Activity 3.5

- $\frac{1}{8} < 1$. Thus $\frac{1}{8}$ would be located in region C

- $\frac{2}{3} < 1$, $\frac{1}{2} < \frac{2}{3}$. Thus $\frac{2}{3}$ would be located in region G

- $\frac{7}{4} > \frac{1}{2}$. Thus $\frac{7}{4}$ would be located in region B.

- $\frac{8}{9} < 1$, $\frac{1}{2} < \frac{8}{9}$. Thus $\frac{8}{9}$ would be located in region G.

- $1\frac{1}{2} = \frac{3}{2} > \frac{1}{2}$ and $\frac{3}{2}$ is a fraction whose numerator is 3. Thus $1\frac{1}{2}$ would be located in region D.

- $\frac{3}{5} > \frac{1}{2}$, $\frac{3}{5} < 1$ and $\frac{3}{5}$ is a fraction whose numerator is 3. Thus $\frac{3}{5}$ would be located in region E.

- $\frac{4}{4} = 1$, $\frac{4}{4} > \frac{1}{2}$. Thus $\frac{4}{4}$ would be located in region B.

- $\frac{3}{7} < 1$, and $\frac{3}{7}$ is a fraction whose numerator is 3. Thus $\frac{3}{7}$ would be located in region F.

Answers to Group Work 3.3

$$\frac{4}{7}$$

Answers to Group Work 3.4

$$\frac{7}{8}$$

Assessment

After completing this lesson, you can use any of the following techniques for assessing students learning: class work, group discussion, assignment, quiz or test. You can ask students to answer the following questions as additional assessment.

Additional Assessment

Arrange the fractions in ascending order

a) $\dfrac{3}{2}, \dfrac{4}{5}, \dfrac{5}{8}$
b) $\dfrac{2}{5}, \dfrac{6}{7}, \dfrac{23}{35}$
c) $\dfrac{5}{12}, \dfrac{3}{4}, \dfrac{2}{3}$

Answer to Additional Assessment

a) $\dfrac{5}{8}, \dfrac{4}{5}, \dfrac{3}{2}$
b) $\dfrac{2}{5}, \dfrac{23}{35}, \dfrac{6}{7}$
c) $\dfrac{5}{12}, \dfrac{2}{3}, \dfrac{3}{4}$

Answers to Exercise 3.C

1. a) $5 < 7$. Therefore $\dfrac{5}{18} < \dfrac{7}{18}$

   b) $4 < 5$. Therefore $\dfrac{4}{11} < \dfrac{5}{11}$

   c) $3 > 1$ Therefore $\dfrac{3}{20} > \dfrac{1}{20}$

   d) $\dfrac{7}{12} = \dfrac{7 \times 5}{12 \times 5} = \dfrac{35}{60}$ and $\dfrac{9}{10} = \dfrac{9 \times 6}{10 \times 6} = \dfrac{54}{60}$

   $35 < 54$. Thus $\dfrac{35}{60} < \dfrac{54}{60}$. That is, $\dfrac{7}{12} < \dfrac{9}{10}$

   e) $105 > 96$ Thus $\dfrac{105}{240} > \dfrac{96}{240}$. That is, $\dfrac{7}{16} > \dfrac{6}{15}$

   f) $\dfrac{4}{7} = \dfrac{4 \times 15}{7 \times 15} = \dfrac{60}{105}$, $\dfrac{14}{15} = \dfrac{14 \times 7}{15 \times 7} = \dfrac{98}{105}$

   $60 < 98$. Thus, $\dfrac{60}{105} < \dfrac{98}{105}$. That is, $\dfrac{4}{7} < \dfrac{14}{15}$

g) $\dfrac{7}{6} > 1$ but $\dfrac{9}{10} < 1$. Thus $\dfrac{7}{6} > \dfrac{9}{10}$

h) $\dfrac{5}{14} = \dfrac{5 \times 2}{14 \times 2} = \dfrac{10}{28}$, $10 > 3$. Thus, $\dfrac{10}{28} > \dfrac{3}{28}$ That is, $\dfrac{5}{14} > \dfrac{3}{28}$

i) $\dfrac{5}{8} < 1$ but $\dfrac{11}{10} > 1$ Thus, $\dfrac{5}{8} < \dfrac{11}{10}$

j) $1\dfrac{4}{7} = \dfrac{1 \times 7 + 4}{7} = \dfrac{11}{7}$, $1\dfrac{5}{7} = \dfrac{1 \times 7 + 5}{7} = \dfrac{12}{7}$

$11 < 12$ Thus $\dfrac{11}{7} < 1\dfrac{5}{7}$

k) $6\dfrac{1}{3} = \dfrac{6 \times 3 + 1}{3} = \dfrac{19}{3}, 6\dfrac{2}{5} = \dfrac{6 \times 5 + 2}{5} = \dfrac{32}{5}$

$\dfrac{19}{3} = \dfrac{19 \times 5}{3 \times 5} = \dfrac{95}{15}, \dfrac{32}{5} = \dfrac{32 \times 3}{5 \times 3} = \dfrac{96}{15}$

$\dfrac{95}{15} < \dfrac{96}{15}$. That is, $6\dfrac{1}{3} < 6\dfrac{2}{5}$

l) $11\dfrac{2}{7} = \dfrac{11 \times 7 + 2}{7} = \dfrac{79}{7}$, $11\dfrac{3}{8} = \dfrac{11 \times 8 + 3}{8} = \dfrac{91}{8}$

$\dfrac{79}{7} = \dfrac{79 \times 8}{7 \times 8} = \dfrac{632}{56}$ and $\dfrac{91}{8} = \dfrac{91 \times 7}{8 \times 7} = \dfrac{637}{56}$

$\dfrac{632}{56} < \dfrac{637}{56}$. Thus, $11\dfrac{2}{7} < 11\dfrac{3}{8}$

2. a) $\dfrac{5}{2} = \dfrac{5 \times 6}{2 \times 6} = \dfrac{30}{12}$, $\dfrac{4}{3} = \dfrac{4 \times 4}{3 \times 4} = \dfrac{16}{12}$,

$\dfrac{7}{4} = \dfrac{7 \times 3}{4 \times 3} = \dfrac{21}{12}$

$16 < 21 < 30$

Thus $\dfrac{16}{12} < \dfrac{21}{12} < \dfrac{30}{12}$

That is $\dfrac{4}{3} < \dfrac{7}{4} < \dfrac{5}{2}$ or $\dfrac{4}{3}, \dfrac{7}{4},$ and $\dfrac{5}{2}$ are in ascending order

b) $\dfrac{8}{15} = \dfrac{8 \times 7}{15 \times 7} = \dfrac{56}{105}$, $\dfrac{14}{35} = \dfrac{14 \times 3}{35 \times 3} = \dfrac{42}{105}$ and $\dfrac{11}{21} = \dfrac{11 \times 5}{21 \times 5} = \dfrac{55}{105}$

$42 < 55 < 56$

Thus $\dfrac{42}{105} < \dfrac{55}{105} < \dfrac{56}{105}$

That is $\dfrac{14}{35} < \dfrac{11}{21} < \dfrac{8}{15}$ are in ascending order

c) $\dfrac{5}{6} = \dfrac{5 \times 6}{6 \times 6} = \dfrac{30}{36}$, and $\dfrac{1}{18} = \dfrac{1 \times 2}{18 \times 2} = \dfrac{2}{36}$,

$2 < 23 < 30$

Thus $\dfrac{2}{36} < \dfrac{23}{36} < \dfrac{30}{36}$

Or $\dfrac{1}{18}, \dfrac{23}{36}$ and $\dfrac{5}{6}$ are in ascending order

d) $\dfrac{3}{7} = \dfrac{3 \times 9}{7 \times 9} = \dfrac{27}{63}$, $\dfrac{4}{9} = \dfrac{4 \times 7}{9 \times 7} = \dfrac{28}{63}$ and $\dfrac{15}{21} = \dfrac{15 \times 3}{21 \times 3} < \dfrac{45}{63}$

$27 < 28 < 45$

Thus $\dfrac{27}{63} < \dfrac{28}{63} < \dfrac{45}{63}$

Or $\dfrac{3}{7} < \dfrac{4}{9} < \dfrac{15}{21}$

e) $\dfrac{1}{10} = \dfrac{3}{30}$, $\dfrac{3}{5} = \dfrac{3 \times 6}{5 \times 6} = \dfrac{18}{30}$

$2 < 3 < 18$

Thus $\dfrac{2}{30} < \dfrac{3}{30} < \dfrac{18}{30}$ That is, $\dfrac{2}{30} < \dfrac{1}{10} < \dfrac{3}{5}$ or

$\dfrac{2}{30}, \dfrac{1}{10}$ and $\dfrac{3}{5}$ are in ascending order.

3. a) $\dfrac{2}{3} = \dfrac{2 \times 8}{3 \times 8} = \dfrac{16}{24}$, $\dfrac{5}{6} = \dfrac{5 \times 4}{6 \times 4} = \dfrac{20}{24}$ and $\dfrac{3}{8} = \dfrac{3 \times 3}{8 \times 3} = \dfrac{9}{24}$

$20 > 16 > 9$. Thus, $\dfrac{20}{24} > \dfrac{16}{24} > \dfrac{9}{24}$

That is $\dfrac{5}{6} > \dfrac{2}{3} > \dfrac{3}{8}$ or $\dfrac{5}{6}, \dfrac{2}{3}$ and $\dfrac{3}{8}$ are in descending order

b) $\frac{7}{2} = \frac{7 \times 8}{2 \times 8} = \frac{56}{16}$, and $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16} > \frac{56}{16} > \frac{12}{16} > \frac{5}{16}$

That is, $\frac{7}{2} > \frac{3}{4} > \frac{5}{16}$

Or $\frac{7}{2}, \frac{3}{4}$ and $\frac{5}{16}$ are in descending order

c) $\frac{9}{10} = \frac{9 \times 3}{10 \times 3} = \frac{27}{30}$, $\frac{7}{6} = \frac{7 \times 5}{6 \times 5} = \frac{35}{30}$ and $\frac{11}{15} = \frac{11 \times 2}{15 \times 2} = \frac{22}{30}$

$35 > 27 > 22$, Thus $\frac{35}{30} > \frac{27}{30} > \frac{22}{30}$ that is, $\frac{7}{6}. > \frac{9}{10} > \frac{11}{15}$

Or $\frac{7}{6}, \frac{9}{10}$ and $\frac{11}{15}$ are in descending order

d) $\frac{4}{5} = \frac{4 \times 12}{5 \times 12} = \frac{48}{60}$, $\frac{5}{6} = \frac{5 \times 10}{6 \times 10} = \frac{50}{60}$ and $\frac{7}{12} = \frac{7 \times 5}{12 \times 5} = \frac{35}{60}$

$50 > 48 > 35$, thus $\frac{50}{60} > \frac{48}{60} > \frac{35}{60}$ That is, $\frac{5}{6} > \frac{4}{5} > \frac{7}{12}$

Or $\frac{5}{6}, \frac{4}{5}$ and $\frac{7}{12}$ are in descending order

e) $\frac{3}{7} = \frac{3 \times 6}{7 \times 6} = \frac{18}{42}$, $\frac{5}{14} = \frac{5 \times 3}{14 \times 3} = \frac{15}{42}$ and $\frac{8}{21} = \frac{8 \times 2}{21 \times 2} = \frac{16}{42}$

$18 > 16 > 15$, thus $\frac{18}{42} > \frac{16}{42} > \frac{15}{42}$

That is $\frac{3}{7} > \frac{8}{21} > \frac{5}{14}$ or $\frac{3}{7} = \frac{8}{21}$ and $\frac{5}{14}$ are in descending order

4. $\frac{24}{84} = \frac{2}{7}$ and $\frac{21}{63} = \frac{1}{3}$

$\frac{2}{7} = \frac{6}{21}$ and $\frac{1}{3} = \frac{7}{21}$

$6 < 7$, thus $\frac{6}{21} = \frac{7}{21}$ that is $\frac{24}{84} < \frac{21}{63}$

Selected problems to slow learners

1. Which one is the least?

   $\frac{1}{2}, \frac{1}{4}$ or $\frac{2}{5}$?

2. Arrange $\frac{1}{2}, \frac{3}{4}$ and $\frac{3}{8}$ in an ascending order.

3. Arrange $\frac{1}{5}, \frac{4}{5}$ and $\frac{1}{10}$ in a descending order.

4. Compare $\frac{2}{5}$ and $\frac{5}{12}$.

5. Write $\frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{2}{5}$ in ascending order.

6. Which one is greater, $\frac{2}{9}$ or $\frac{3}{5}$?

7. Which of the following are true?

   a) $\frac{7}{9} < \frac{13}{9}$ c) $\frac{3}{7} > \frac{2}{7}$

   b) $\frac{5}{8} > \frac{5}{10}$ d) $\frac{4}{7} < \frac{1}{3}$

8. Express $\frac{14}{49}, \frac{28}{196}$ and $\frac{85}{119}$ as fractions with denominator 7.

9. Which one is smaller, 2 out of 3 or 3 out of 4?

10. Which of $<$ or $>$ goes in the box?

   a) $\frac{5}{7} \square \frac{6}{7}$ b) $\frac{3}{11} \square \frac{2}{11}$ c) $\frac{2}{3} \square \frac{4}{9}$

Selected problems to fast learners

1. Arrange $\frac{3}{40}, \frac{40}{3}$ and $12\frac{3}{20}$ in an ascending order.

2. Arrange $\frac{1}{3}, \frac{2}{25}$ and $\frac{3}{4}$ in descending order.

3. Compare $\frac{3}{5}, \frac{5}{7}, \frac{7}{10}$ and $\frac{3}{14}$. Write them in descending order.

4. Which of the following are true?

   a) $\frac{2}{7} > \frac{4}{5}$ b) $\frac{3}{4} > \frac{1}{4}$ c) $\frac{2}{7} < \frac{8}{20}$

   d) $\frac{8}{15} < \frac{12}{27}$ e) $\frac{5}{18} < \frac{5}{12}$

5. Which is greater, $\frac{11}{12}$ or $\frac{8}{9}$?

6. Express $\frac{3}{8}, \frac{4}{15}$ and $\frac{11}{20}$ as fractions with the same denominator.

7. Arrange $\frac{4}{15}, \frac{8}{9}, \frac{5}{9}$, and $\frac{1}{3}$ in ascending order.

8. Which of $<$ or $>$ goes in the box?

   a) $\frac{3}{7}$ $\square$ $\frac{1}{3}$ b) $\frac{2}{21}$ $\square$ $\frac{3}{16}$ c) $\frac{3}{10}$ $\square$ $\frac{4}{5}$

3.4 Operations on Fractions

Periods allotted: 13 periods

Competencies

At the end of this sub-unit students will be able to:

- add fractions with different denominators.
- subtract fractions with different denominators.
- multiply fractions.
- divide fractions.

Introduction

This sub-unit is intended to make students familiar with operations on fractions. In the process of doing this, first it will be useful to discuss addition and subtraction of fractions. Fraction models and real life problems are included in this sub-unit to clarify addition and subtraction of fractions. After discussing these multiplying and dividing fractions will be considered.

Teaching Notes

To address this sub-unit, the presentation is classified in to two parts (i.e. Addition and subtraction of fractions, and multiplication and division of fractions) as follows.

3.4.1 Addition and Subtraction of fractions

You can start the discussion of this sub- topic by revising denominators. Revise how to find an equivalent fraction for a given fraction. Remind students the fact that multiplying the numerators and denominator by the same number does not change the value.

Motivate students to estimate what fraction of the day they spend in common activities: School, meals, travel, sleep, sports. When all activities are listed, discuss how students could determine how much "spare time" they have each day. (Add the fractions and subtract the total from one whole day.)

By using simple given fractions, assist students to add and subtract fractions of different denominators (during subtraction the difference should be positive).

Working with different denominators is less straight forward for students. There is the additional step of finding the common denominator. Students need to grasp that when there are different denominators, the fractions can not be directly combined because they are different parts of a whole.

Encourage students to arrive at and use the following principles:

1. $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a \times d}{b \times d} + \dfrac{c \times b}{d \times b} = \dfrac{ad + cb}{bd}$

2. $\dfrac{a}{b} - \dfrac{c}{d} = \dfrac{a \times d}{b \times d} - \dfrac{c \times b}{d \times b} = \dfrac{ad - cb}{bd}, \ (ad - cd > 0)$

Answers to Activity 3.6

a) $\dfrac{2}{7} + \dfrac{3}{7} = \dfrac{2+3}{7} = \dfrac{5}{7}$

b) $\dfrac{6}{13} - \dfrac{2}{13} = \dfrac{6-2}{13} = \dfrac{4}{13}$

c) $\dfrac{5}{9} - \dfrac{5}{9} = \dfrac{5-5}{9} = \dfrac{0}{9} = 0$

d) $\dfrac{8}{12} + \dfrac{7}{12} = \dfrac{8+7}{12} = \dfrac{15}{12} = \dfrac{5}{4}$

e) $\dfrac{8}{9} + \dfrac{5}{9} = \dfrac{8+5}{9} = \dfrac{13}{9}$

f) $\dfrac{6}{15} - \dfrac{4}{15} = \dfrac{6-4}{15} = \dfrac{2}{15}$

g) $\dfrac{1}{10}+\dfrac{2}{10}+\dfrac{3}{10}+...+\dfrac{9}{10}=\dfrac{1+2+3+...+9}{10}=\dfrac{45}{10}=\dfrac{9}{2}=4\dfrac{1}{2}$

Answers to Activity 3.7

To find the sum or difference of two fractions with different denominators, rename the fractions using equivalent fractions. Then add or subtract and simplify.

That is, if $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are two fractions (where b, d ≠ 0), then

i) $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{a \times d}{b \times d}+\dfrac{c \times b}{d \times b}=\dfrac{ad+cb}{bd}$

ii) $\dfrac{a}{b}-\dfrac{c}{d}=\dfrac{a \times d}{b \times d}-\dfrac{c \times b}{d \times b}=\dfrac{ad-cb}{bd}$ (ad – cb > 0)

Assessment

You can evaluate the students' performance on this sub-topic by giving either class work or quiz and giving them immediate feedback. You can select questions from exercise 3. D for this purpose. Check the assignment responses and discuss their problems if there are any, indicating major sources of error. You can ask them the following questions as additional exercise.

Additional Assessment
Add or subtract

a) $\dfrac{2}{3}+\dfrac{1}{3}$

b) $\dfrac{7}{15}+\dfrac{8}{15}$

c) $\dfrac{6}{9}-\dfrac{2}{9}$

d) $\dfrac{13}{20}+\dfrac{7}{20}$

e) $\dfrac{1}{20}+\dfrac{2}{20}+\dfrac{3}{20}+\dfrac{4}{20}+...+\dfrac{19}{20}$

Answers to Additional Assessment

a) 1 b) 1 c) $\dfrac{4}{9}$ d) 1 e) $9\dfrac{1}{2}$

Answers to Group Work 3.5

a) $6\dfrac{1}{4}$ b) $15\dfrac{1}{6}$ c) $45\dfrac{13}{16}$

Answers to Group Work 3.6

3

Answers to Exercise 3.D

1. a) $\dfrac{2}{5}+\dfrac{3}{4}=\dfrac{2\times 4}{5\times 4}+\dfrac{3\times 5}{4\times 5}=\dfrac{8}{20}+\dfrac{15}{20}=\dfrac{8+15}{20}=\dfrac{23}{20}=1\dfrac{3}{20}$

b) $\dfrac{5}{6}+\dfrac{3}{8}=\dfrac{5\times 4}{6\times 4}+.\dfrac{3\times 3}{8\times 3}=\dfrac{20}{24}+\dfrac{9}{24}=\dfrac{29}{24}=1\dfrac{5}{24}$

c) $\dfrac{4}{15}+\dfrac{2}{25}=\dfrac{4\times 5}{15\times 5}+.\dfrac{2\times 3}{25\times 3}=\dfrac{20}{75}+\dfrac{6}{75}=\dfrac{26}{75}$

d) $\dfrac{5}{14}+\dfrac{8}{7}=\dfrac{5}{14}+.\dfrac{8\times 2}{7\times 2}=\dfrac{5}{14}+\dfrac{16}{14}=\dfrac{21}{14}=\dfrac{3}{2}=1\dfrac{1}{2}$

e) $2\dfrac{3}{4}+1\dfrac{7}{8}=\dfrac{11}{4}+\dfrac{15}{8}=\dfrac{11\times 2}{4\times 2}+\dfrac{15}{8}=\dfrac{22}{8}+\dfrac{15}{8}=\dfrac{37}{8}=4\dfrac{5}{8}$

f) $1\dfrac{5}{16}+2\dfrac{3}{8}=\dfrac{21}{16}+\dfrac{19}{8}=\dfrac{21}{16}+\dfrac{19\times 2}{8\times 2}=\dfrac{21}{16}+\dfrac{38}{16}=\dfrac{59}{16}=3\dfrac{11}{16}$

g) $4\dfrac{3}{10}+\dfrac{9}{20}=\dfrac{43}{10}+\dfrac{9}{20}+\dfrac{43\times 2}{10\times 2}=\dfrac{9}{20}=\dfrac{86}{20}+\dfrac{9}{20}=\dfrac{95}{20}=4\dfrac{15}{20}=4\dfrac{3}{4}$

h) $3\dfrac{2}{7}+2\dfrac{5}{14}=\dfrac{23}{7}+\dfrac{33}{14}=\dfrac{23\times 2}{7\times 2}+\dfrac{33}{14}=\dfrac{46}{14}+\dfrac{33}{14}=\dfrac{79}{14}=5\dfrac{9}{14}$

i) $\dfrac{3}{5}+\dfrac{5}{7}+2\dfrac{1}{35}=\dfrac{3\times 7}{5\times 7}+\dfrac{5\times 5}{7\times 5}+\dfrac{71}{35}+=\dfrac{21}{35}+\dfrac{25}{35}+\dfrac{71}{35}=\dfrac{117}{35}=3\dfrac{12}{35}$

j) $4 + 3\dfrac{5}{18} + 2\dfrac{1}{9} = \dfrac{4}{1} + \dfrac{59}{18} + \dfrac{19}{9}$

$$= \dfrac{4 \times 18}{18} + \dfrac{59}{18} + \dfrac{19 \times 2}{9 \times 2}$$

$$= \dfrac{72}{18} + \dfrac{59}{18} + \dfrac{38}{18}$$

$$= \dfrac{72 + 59 + 38}{18} = \dfrac{169}{18} = 9\dfrac{7}{18}$$

k) $1\dfrac{7}{10} + 2\dfrac{1}{20} + 3\dfrac{4}{40} = \dfrac{17}{10} + \dfrac{41}{20} + \dfrac{124}{40} = \dfrac{17 \times 4}{10 \times 4} + \dfrac{41 \times 2}{20 \times 2} + \dfrac{124}{40}$

$\quad = \dfrac{68}{40} + \dfrac{82}{40} + \dfrac{124}{40} = \dfrac{274}{40} = \dfrac{137}{20} = 6\dfrac{17}{20}$

2. a) $\dfrac{3}{4} - \dfrac{1}{8} = \dfrac{3 \times 2}{4 \times 2} - \dfrac{1}{8} = \dfrac{6}{8} - \dfrac{1}{8} = \dfrac{6 - 1}{8} = \dfrac{5}{8}$

b) $\dfrac{7}{5} - \dfrac{3}{10} = \dfrac{7 \times 2}{5 \times 2} - \dfrac{3}{10} = \dfrac{14}{10} - \dfrac{3}{10} = \dfrac{14 - 1}{10} = \dfrac{11}{10} = 1\dfrac{1}{10}$

c) $\dfrac{5}{12} - \dfrac{7}{36} = \dfrac{5 \times 3}{12 \times 3} - \dfrac{7}{36} = \dfrac{15}{36} - \dfrac{7}{36} = \dfrac{15 - 7}{36} = \dfrac{8}{36} = \dfrac{2}{9}$

d) $7\dfrac{1}{2} - 5 = \dfrac{15}{2} - \dfrac{5}{1} = \dfrac{15}{2} - \dfrac{5 \times 2}{1 \times 2} = \dfrac{15}{2} - \dfrac{10}{2} = \dfrac{15 - 10}{2} = \dfrac{5}{2} = 2\dfrac{1}{2}$

e) $\dfrac{11}{4} - 2\dfrac{1}{3} = \dfrac{11}{4} - \dfrac{7}{3} = \dfrac{11 \times 3}{4 \times 3} - \dfrac{7 \times 4}{3 \times 4} = \dfrac{33}{12} - \dfrac{28}{12} = \dfrac{33 - 28}{12} = \dfrac{5}{12}$

f) $3\dfrac{1}{5} - \dfrac{3}{8} = \dfrac{16}{5} - \dfrac{3}{8} = \dfrac{16 \times 8}{5 \times 8} - \dfrac{3 \times 5}{8 \times 5} = \dfrac{128}{40} - \dfrac{15}{40} = \dfrac{113}{40} = 2\dfrac{33}{40}$

g) $4\dfrac{1}{6} - 2\dfrac{1}{5} = \dfrac{25}{6} - \dfrac{11}{5} = \dfrac{25 \times 5}{6 \times 5} - \dfrac{11 \times 6}{5 \times 6} = \dfrac{125}{30} - \dfrac{66}{30} = \dfrac{59}{30} = 1\dfrac{29}{30}$

3. a) $\dfrac{7}{4} + \dfrac{5}{6} - \dfrac{1}{12} = \dfrac{7 \times 3}{4 \times 3} + \dfrac{5 \times 2}{6 \times 2} - \dfrac{1}{12} = \dfrac{21}{12} + \dfrac{10}{12} - \dfrac{1}{12} = \dfrac{21 + 10 - 1}{13} = \dfrac{30}{12}$

$\quad = \dfrac{5}{2} = 2\dfrac{1}{2}$

b) $\dfrac{3}{4}+\dfrac{7}{2}-\dfrac{1}{8}=\dfrac{3\times 2}{4\times 2}+\dfrac{7\times 4}{2\times 4}-\dfrac{1}{8}=\dfrac{6}{8}+\dfrac{28}{8}-\dfrac{1}{8}$

$\quad=\dfrac{6+28-1}{8}=\dfrac{33}{8}=4\dfrac{1}{8}$

c) $\dfrac{5}{3}+\dfrac{3}{4}-\dfrac{1}{2}=\dfrac{5\times 4}{3\times 4}-\dfrac{3\times 3}{4\times 3}-\dfrac{1\times 6}{2\times 6}=\dfrac{20}{12}-\dfrac{9}{12}-\dfrac{6}{12}=\dfrac{20-9-6}{12}=\dfrac{5}{12}$

d) $\dfrac{4}{15}+\dfrac{7}{9}-\dfrac{1}{3}=\dfrac{4\times 3}{15\times 3}+\dfrac{7\times 5}{9\times 5}-\dfrac{1\times 15}{3\times 15}=\dfrac{12}{45}+\dfrac{35}{45}-\dfrac{15}{45}$

$\quad=\dfrac{12+35-15}{45}=\dfrac{32}{45}$

e) $\dfrac{7}{12}+\dfrac{5}{6}-\dfrac{3}{4}=\dfrac{7}{12}+\dfrac{5\times 2}{6\times 2}-\dfrac{3\times 3}{4\times 3}=\dfrac{7}{12}+\dfrac{10}{12}-\dfrac{9}{12}$

$\quad=\dfrac{7+10-9}{12}=\dfrac{8}{12}=\dfrac{2}{3}$

f) $\dfrac{1}{4}+\dfrac{1}{7}-\dfrac{1}{28}=\dfrac{1\times 7}{4\times 7}+\dfrac{1\times 4}{7\times 4}-\dfrac{1}{28}=\dfrac{7}{28}+\dfrac{4}{28}-\dfrac{1}{28}=\dfrac{7+4-1}{28}$

$\quad=\dfrac{10}{28}=\dfrac{5}{14}$

4. $\dfrac{3}{4}+\dfrac{5}{8}-\dfrac{5}{6}=\dfrac{3\times 6}{4\times 6}+\dfrac{5\times 3}{8\times 3}-\dfrac{5\times 4}{6\times 4}=\dfrac{18}{24}+\dfrac{15}{24}-\dfrac{20}{24}=$

$\quad\dfrac{18+15-20}{24}=\dfrac{13}{24}$

$\quad\dfrac{5}{8}+\dfrac{5}{6}-\dfrac{3}{4}=\dfrac{5\times 3}{8\times 3}+\dfrac{5\times 4}{6\times 4}-\dfrac{3\times 6}{4\times 6}=\dfrac{15}{24}+\dfrac{20}{24}-\dfrac{18}{24}=$

$\quad\dfrac{15+20-18}{24}=\dfrac{17}{24}$

Thus $\dfrac{3}{4}+\dfrac{5}{8}-\dfrac{5}{6}\neq\dfrac{5}{8}+\dfrac{5}{6}-\dfrac{3}{4}$

5. $1\dfrac{1}{2}-\dfrac{1}{4}=\dfrac{3}{2}-\dfrac{1}{4}=\dfrac{6}{4}-\dfrac{1}{4}=\dfrac{6-1}{4}=\dfrac{5}{4}=1\dfrac{1}{4}$

Therefore, $1\dfrac{1}{4}$ liter of water left

6. $\dfrac{1}{2} - \dfrac{3}{10} = \dfrac{1\times 5}{2\times 5} - \dfrac{3}{10} = \dfrac{5}{10} - \dfrac{3}{10} = \dfrac{5-3}{10} = \dfrac{2}{10} = \dfrac{1}{5}$

To get $\dfrac{1}{2}$, we must add $\dfrac{1}{5}$ to $\dfrac{3}{10}$

7. $\dfrac{7}{12} + \dfrac{1}{4} = \dfrac{7}{12} + \dfrac{1\times 3}{4\times 3} = \dfrac{7}{12} + \dfrac{3}{12} = \dfrac{7+3}{12} = \dfrac{10}{12} = \dfrac{5}{6}$

Therefore, $\dfrac{5}{6}$ must be subtracted from $\dfrac{7}{12}$ to get . $\dfrac{1}{4}$

8. $9 - 4\dfrac{1}{6} = \dfrac{9}{1} - \dfrac{25}{6} = \dfrac{9\times 6}{1\times 6} - \dfrac{25}{6} = \dfrac{54}{6} - \dfrac{25}{6} = \dfrac{29}{6} = 4\dfrac{5}{6}$

Therefore, the length of the other piece $= 4\dfrac{5}{6}$ metres

9. $1 - \left(\dfrac{1}{4} + \dfrac{1}{2} + \dfrac{1}{8}\right) = 1 - \left(\dfrac{2}{8} + \dfrac{4}{8} + \dfrac{1}{8}\right) = 1 - \dfrac{2+4+1}{8} = 1 - \dfrac{7}{8} = \dfrac{8}{8} - \dfrac{7}{8}$

$= \dfrac{8-7}{8} = \dfrac{1}{8}$

Therefore $\dfrac{1}{8}$ of the money remained.

3.4.2 Multiplication and division of fractions

To introduce students to multiplication of fractions, motivate them by asking multiplication facts. Use specific objects in the multiplication review (e.g. 7 rows of 8 chairs equals 56 chairs).

Let students practice multiplication of fractions by using the principle:

$\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{a\times c}{b\times d} = \dfrac{ac}{bd}$

Remind students that whole numbers can be written as fractions with a denominator of 1. When all numbers have been expressed as fractions, simplify completely, then multiply numerators, and then multiply denominators.

Reading the multiplication sign as "of" may help students understand the process. For example, $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, and $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{5}$.

To introduce students to dividing fractions, ask them to use mental math to divide first with whole numbers and then with fractions (eg. How many 4's are in 24? How many $\frac{1}{3}$'s are in 2?). Point out that dividing by a fraction less than one produces a quotient greater than the dividend.

Let students find reciprocal of a given fraction. Assist students to practice division of fractions by using the principle:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Answers to Activity 3.8

a) $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5}$

b) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{1} \times \frac{1}{3} = \frac{4}{3}$

c) $\frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} = 5 \times \frac{3}{7} = \frac{15}{7}$

Assessment

Give exercises on multiplication of fractions and check whether students follow the principles or not. You can ask fast learners or interested students   to answer the following questions as additional exercise.

Additional Assessment

Multiply

a) $\frac{3}{2} \times \frac{4}{9}$

b) $\frac{18}{7} \times \frac{14}{36}$

c) $\frac{5}{7} \times \frac{21}{10} \times \frac{35}{63}$

Answers to Additional Assessment

a) $\frac{2}{3}$

b) 1

c) $\frac{5}{6}$

Answers to Group Work 3.7

a) $1\frac{1}{3}$ b) 9 c) $\frac{45}{64}$

Answers to Exercise 3.E

1. a) $\frac{3}{5} \times \frac{10}{21} = \frac{30}{105} = \frac{2}{7}$

   b) $\frac{5}{9} \times \frac{27}{35} = \frac{135}{315} = \frac{3}{7}$

   c) $\frac{3}{4} \times \frac{8}{15} = \frac{24}{60} = \frac{2}{5}$

   d) $\frac{2}{7} \times \frac{21}{6} = \frac{42}{42} = 1$

   e) $\frac{9}{5} \times \frac{35}{36} = \frac{7}{4} = \frac{7}{4} = 1\frac{3}{4}$

   f) $\frac{20}{3} \times \frac{9}{40} = \frac{3}{2} = 1\frac{1}{2}$

   g) $\frac{4}{5} \times \frac{2}{4} \times \frac{4}{6} = \frac{32}{120} = \frac{4}{15}$

   h) $6\frac{1}{8} \times \frac{8}{9} = \frac{49}{8} \times \frac{8}{9} = \frac{49}{9} = 5\frac{4}{9}$

   i) $3\frac{1}{8} \times 3\frac{4}{5} = \frac{25}{8} \times \frac{19}{5} = \frac{95}{8} = 11\frac{7}{8}$

2. $ab = 1\frac{5}{7} \times 2\frac{5}{8} = \frac{12}{7} \times \frac{21}{8} = \frac{9}{2} = 4\frac{1}{2}$

3. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{98}{99} \times \frac{99}{100} = \frac{1}{100}$

4. a) $\frac{1}{4} \times 100 = \frac{100}{4} = 25$

   b) $\frac{1}{7} \times 98 = \frac{98}{7} = 14$

   c) $\frac{1}{2} \times 64 = \frac{64}{2} = 32$

   d) $\frac{3}{5} \times 80 = \frac{240}{5} = 48$

   e) $\frac{7}{6} \times 120 = \frac{840}{6} = 140$

5. $\frac{3}{10} \times 100 = \frac{300}{10} = 30$

   $100 - 30 = 70$. Therefore, 70 pages are left to read

6. a) $\dfrac{7}{3}$ b) $\dfrac{1}{4}$ c) $\dfrac{5}{14}$ d) $\dfrac{6}{29}$

7. a) $\dfrac{2}{5} \div \dfrac{1}{4} = \dfrac{2}{5} \times \dfrac{4}{1} = \dfrac{8}{5} = 1\dfrac{3}{5}$

   b) $\dfrac{3}{14} \div \dfrac{2}{7} = \dfrac{3}{14} \times \dfrac{7}{2} = \dfrac{21}{28} = \dfrac{3}{4}$

   c) $5 \div \dfrac{1}{6} = 5 \times 6 = 30$

   d) $8 \div 2\dfrac{1}{2} = 8 \div \dfrac{5}{2} = 8 \times \dfrac{2}{5} = \dfrac{16}{5} = 3\dfrac{1}{5}$

   e) $2\dfrac{1}{4} \div \dfrac{2}{3} = \dfrac{9}{4} \div \dfrac{2}{3} = \dfrac{9}{4} \times \dfrac{3}{2} = \dfrac{27}{8} = 3\dfrac{3}{8}$

   f) $2\dfrac{2}{3} \div 5\dfrac{1}{3} = \dfrac{8}{3} \div \dfrac{16}{3} = \dfrac{8}{3} \times \dfrac{3}{16} = \dfrac{24}{48} = \dfrac{1}{2}$

   g) $1\dfrac{1}{9} \div 1\dfrac{2}{3} = \dfrac{10}{9} \div \dfrac{5}{3} = \dfrac{10}{9} \times \dfrac{3}{5} = \dfrac{30}{45} = \dfrac{2}{3}$

   h) $4\dfrac{1}{2} \div 6\dfrac{3}{4} = \dfrac{9}{2} \div \dfrac{27}{4} = \dfrac{9}{2} \times \dfrac{4}{27} = \dfrac{36}{54} = \dfrac{2}{3}$

   i) $5\dfrac{1}{4} \div 3 = \dfrac{21}{4} \div \dfrac{3}{1} = \dfrac{21}{4} \times \dfrac{1}{3} = \dfrac{21}{12} = \dfrac{7}{4} = 1\dfrac{3}{4}$

8. $5\dfrac{3}{8} \div 6\dfrac{3}{4} = \dfrac{43}{8} \div \dfrac{27}{4} = \dfrac{43}{8} \times \dfrac{4}{27} = \dfrac{43}{54}$ proper.

9. $2000 \times \dfrac{3}{5} = \dfrac{6000}{5} = 1200$

Number of boys $= 2000 - 1200 = 800$.

Selected problems to slow learners

1. Perform the indicated operation.

   a. $\dfrac{1}{5} + \dfrac{3}{25}$ c. $\dfrac{3}{4} \times \dfrac{8}{27}$

   b. $\dfrac{7}{8} - \dfrac{1}{4}$ d. $\dfrac{15}{16} \div \dfrac{3}{8}$

2. Evaluate ab if

   (i) $a = \frac{1}{3}$ and $b = \frac{6}{7}$

   (ii) $a = \frac{1}{9}$ and $b = \frac{3}{4}$

3. Add $\frac{3}{5}, \frac{7}{10}, \frac{5}{7}$ and $\frac{3}{14}$.

4. Subtract $3\frac{1}{2}$ from $6\frac{1}{4}$.

5. Find the value of $4\frac{1}{3} - 1\frac{5}{6} + 1\frac{7}{12} - \frac{2}{3}$.

6. Simplify $\frac{1}{2} \times \frac{4}{6} \times \frac{6}{8}$.

7. Divide 4 by $\frac{1}{2}$.

8. The product of two fraction is one.
   If one of them is $\frac{3}{4}$, then find the other.

9. Simplify $\frac{1}{4}$ of $\left(\frac{1}{2} \div \frac{3}{2} \right)$

10. Simplify $4\frac{3}{4} - \left(2\frac{1}{4} - 1\frac{1}{2} \right)$

Selected problems to fast learners

1. Perform the indicated operation.

   a. $\frac{1}{5} + \frac{3}{4} + \frac{2}{7}$ c. $11\frac{1}{8} \times \frac{16}{89}$

   b. $\frac{22}{3} - 4\frac{1}{4}$ d. $1\frac{35}{49} \div \frac{23}{6}$

2. Evaluate xy if
   $x = 2\frac{1}{7}$ and $y = 4\frac{2}{5}$

3. Subtract $2\frac{3}{5}$ from $5\frac{7}{12}$.

4. Find the value of $4\frac{1}{2} - 3\frac{5}{6} + 1\frac{1}{4} - 1\frac{7}{12}$

5. Simplify $1\frac{3}{5} \times 2\frac{4}{7} \times \frac{5}{18}$

6. Divided 8 by $5\frac{2}{5}$.

7. The product of two fractions is 53. If one of them is $15\frac{1}{7}$, then find the other.

8. Simplify $\frac{4}{5}$ of $\frac{3}{2} \times \frac{2}{5}$

9. Simplify $1 - \left[\frac{1}{2} - \left(\frac{1}{3} + \left(\frac{1}{4} - \frac{1}{8}\right)\right)\right]$

10. Robel has Birr $75\frac{1}{4}$. He spent Birr $59\frac{1}{2}$ in a party. How much is left with him?

3.5 Operations on Decimals

Periods allotted: 12 periods

Competencies

At the end of this sub-unit students will be able to:

- find the sum of decimals (up to 3 decimal places).
- find the difference between two decimals (having up to 3 decimal places).
- find the product of two decimals.
- determine quotient of simple decimals.

Introduction

This sub-unit deals with operations on decimals (i.e. addition and subtraction of decimals, multiplication of decimals and division of decimals). Activities are given to encourage group work and revise previous mathematics lessons. Real life problems (e.g. prices for clothing or other items) are included to clarify operations on decimals. Rules for adding, subtracting, multiplying and dividing decimals will be discussed in this sub-unit.

Teaching notes

In order to address this sub-unit, the presentation is classified in to three topics (i.e. addition and subtraction of decimals, multiplication of decimals, and division of decimals) as follows.

3.5.1 Addition and subtraction of decimals
(up to 3 decimals places)

You may start the discussion of this sub-unit by revising addition of decimals up to 2 decimal places. Motivate students to describe situations in which they might need to add or subtract decimals (eg., to find the total cost of a number of purchases, calculating the number of kilometers a car has traveled over several days). Group your students and let them discuss the Activity given.

Ask students how using zero as a place holder helps them to find the correct sum.

Assist students to add decimals with1 decimal, 2 decimal and 3 decimal places to 3 decimal places. (With and without carrying).

You can use vertical addition and make sure that students have correctly placed the decimal points (line up the decimal points), write zeros so that both number have the same number of decimal places. you may use examples like:

$$3.215$$
$$+\ 6.4$$

Assist students to subtract decimals up to 3 decimal places with and without borrowing so that the result is not negative. Again, make sure that students have correctly placed the decimal points.

Encourage students to check the correctness of the answer.

Answers to Activity 3.9

1. a) $3.42 = 3$ ones   4 tenths   2   hundredths
    b) $4.5,1 = 4$ ones   5 tenths   1 hundredths
    c) $0.345 = $ 3 tenths   4 hundredths   5   thousandths
    d) $15.27 = $ 1 tens   5   ones   2 tenths   7   hundredths
2. a) $0.3 < 0.5$ d) $5.\ 0.8 < 5.8$
    b) $0.04 > 0.01$ e) $0.9 > 0.09$
    c) $1.31 > 1.13$ f) $0.7\ \ < 0.71$

3. $3.67 < 3.79 < 3.8 < 3.81$

Answers to Group Work 3.8
a) 853.67
b) 1631.95

Answers to Group Work 3.9
Sum of speeds $=40.001+17.601=57.602$ km/hr

Answers to Group Work 3.10
a) 2.12   b)320.31

Assessment
Give exercises on addition and subtractions of decimals in the form of class work, group work and quiz or test. Make sure that students have placed the decimal points correctly. You can ask students to answer the following questions as additional exercise.

Additional Assessment
a)	4.351	b)	9.012	c)	6.743	d)	23.964
	+ 2.043		+ 2.723		- 0.231		- 1.032

Answers to Additional Assessment
a) 6.394   b) 11.735   c) 6. 512   d) 22.932

Answers to Exercise 3.F
a)	3.210	b)	0.040	c) 25.002	d) 10.134
	+ 4.015		+ 2.132	+40.115	+ 9.021
	7.225		4.013	13.101	120.412
			6.185	78.218	139.567

2.   a) 5.623   b) 18.451   c)17.591
       - 3.210   - 7.341   - 4.300
          2.413   11.110   13.291

3. 3.26 million – 2.15 million = 1.11 million

   Therefore, 1.11 million more people visited this year.

4.   215.355   Therefore, his friend's house is 201.255km

       - 14.100   far from his house.

        201.255

5. 125. 30m + 40.45 m = 165. 75m therefore, the second building is
      165.75m high.

6. 13.25m + 21.40m + 18.30m = 52.95m

    80m – 52.95 = 80.00 – 52.95 = 27. 05 m

    Therefore, 27.05M long rope is left.

3.5.2 Multiplication of decimals

You may start the discussion of this sub-unit by asking students to multiply decimals by 1 digit whole number. You may use the Activity given in order to involve student in the discussion.

Assist students to multiply decimals up to 3 decimal places by powers of 10 (such as by 10, 100 and 1000). Encourage students to multiply two decimals between 0 and 1. You may use examples like:

0.4 x 0.3, 0.13 x 0.25, …

Some students write zeroes as place holders at the wrong side of the number in exercises. Remind students to always count from right to left when placing the decimal point. Let students follow the procedure of multiplications of decimals explained in the students text.

Assist the students to multiply two decimals that are greater than 1. You may use examples like:

11.2 x 3.6;   3.24 x 2.7,

Encourage students to make sure that products have the same number of decimal places as the sum of the decimal places in the multipliers.

Answers to Activity 3.10

a)
$$
\begin{array}{r}
35 \\
\times\ 21 \\
\hline
35 \\
70 \\
\hline
735
\end{array}
$$

b)
$$
\begin{array}{r}
47 \\
\times 82 \\
\hline
94 \\
376 \\
\hline
3854
\end{array}
$$

c)
$$
\begin{array}{r}
124 \\
\times\ 35 \\
\hline
620 \\
372 \\
\hline
4340
\end{array}
$$

d)
$$
\begin{array}{r}
853 \\
\times\ 46 \\
\hline
5118 \\
3412 \\
\hline
39238
\end{array}
$$

e)
$$
\begin{array}{r}
236 \\
\times\ 103 \\
\hline
708 \\
000 \\
236 \\
\hline
24308
\end{array}
$$

f)
$$
\begin{array}{r}
343 \\
\times 59 \\
\hline
3087 \\
1715 \\
\hline
20237
\end{array}
$$

Answers to Activity 3.11

a) 1.2 x 10 = 12

1.2 x 100= 120

1.2 x 1000= 1200

b) 0.37 x 10 = 3.7

0.37 x 100= 37

0.37 x 1000 = 370

c) 0.048x 10 = 0.48

0.048 x 100 = 4.8

0.048 x 1000 = 48

d) 3.65 x 10 = 36.5

3.65 x 100 = 365

3.65 x 1000 = 3650

Assessment

Give problems on multiplication of decimals as a form of class activity, group work and quiz or test. Make sure that students understood how the decimals of the product changes. You can ask students to answer the following questions as additional exercise.

Additional Assessment

a) 2.3×10

2.3×100

2.3×1000

b) 4.36×10

4.36×100

4.36×1000

c) 0.0128×10 d) 0.235×4

   0.0128×100

   0.0128×1000

Answers to Additional Assessment

a) 23   b) 43. 6   c) 0.128   d) 0.94

  230   436   1.28

  2300   4360   12.8

Answers to Exercise 3.G

1. a) 0.12 x 3= 0.36   b) 0.17 x 4 = 0.68

   c) 3.4 x 8 = 27.2

d) 8.3	e) 7.6	f) 4.25	g) 0.47
× 1.4	× 5.6	× 2.3	×0.32
332	456	1275	94
83	380	850	141
11.62	42.56	9.775	0.1504

h) 1.23	i) 5.31
× 4.8	× 0.48
984	4248
492	2124
5.904	2.5488

2. 0.32   Therefore, the total length of

 × 12   12 such pieces of string is 3.84 cm

  64

  32

  3.84

3. $9 \times 36. 75 = 330 .75$

Therefore, the charge to rent this car for 9 hours is Birr 330.75.

4. a) $1.5 \times 1.2 = 1.8$ and $3.6 \times 0.5 = 1.8$

    Therefore, $1.5 \times 1.2 = 3.6 \times 0.5$

  a) $3.2 \times 2.4 = 7.68$ and $5.1 \times 1.2 = 6.12$

    Therefore, $3.2 \times 2.4 > 5.1 \times 1.2$

  b) $0.34 \times 1.3 = 0.442$ and $0.4 \times 1.2 = 0.48$

    Therefore, $0.34 \times 1.3 < 0.4 \times 1.2$

  c) $7.75 \times 1.5 = 11.625$ and $77.5 \times 2.5 = 193.75$

    Therefore $7.75 \times 1.5 < 77.5 \times 2.5$

  d) $0.86 \times 0.8 = 0.688$ and $8.6 \times 0.1 = 0.86$

    Therefore $0.86 \times 0.8 < 8.6 \times 0.1$

5. 1.35 Therefore, Alemu runs 4.725 km in 3.5 games

$$
\begin{array}{r}
1.35 \\
\times\ 3.5 \\
\hline
675 \\
405\ \ \\
\hline
4.725
\end{array}
$$

3.5.3 Division of Decimals

Discuss the steps of division of simple decimal by one digit natural number or the division of one digit natural number by simple decimal. You may use examples like.

$6 \div 0.2$, $0.6 \div 2$, …

$6 \div 0.2 = \dfrac{6}{0.2} \times \dfrac{10}{10} = \dfrac{60}{2} = 30$

Discuss how to place the decimal point in the quotient when they divide a decimal by a non – zero whole number that they should first place the decimal point in the dividend, then divide as usual. If the divisor has a decimal point, students could complete the steps discussed in the students' text. Assist students to divide decimals (having up to 3 decimal places) by powers of 10. You may use examples like $0.2 \div 10$, $23.4 \div 100$, … Let students do Activity 3.12

in group so that they share ideas on division of decimals. Help students to practice division of decimals and let them conclude how the decimal points shifts in the quotient.

Answers to activity 3.12

a) $4 \div 0.5 = 4 \div \dfrac{1}{2} = 4 \times 2 = 8$

b) $2 \div 0.1 = 2 \div \dfrac{1}{10} = 2 \times 10 = 20$

Answers to Activity 3.13

$2.13 \div 10 = 0.213$

$2.13 \div 100 = 0.0213$

$2.13 \div 1000 = 0.00213$

$2.13 \div 10,000 = 0.000213$

Assessment

You can give exercises on division of decimals in the form of class work, group work, assignment, quiz or test and check either students follow the steps or not. You can ask students to answer the following questions as additional exercise.

Additional Assessment

Divide

a) $100 \div 0.5$ c) $12.5 \div 0.001$

b) $1000 \div 0.01$ d) $0.625 \div 2.5$

Answer to Additional Assessment

a) 200   b) 100,000   c) 12, 500   d) 0.25

Answers to Exercise 3.H

1. a) $5 \div 0.1 = 5 \div \dfrac{1}{10} = 5 \times 10 = 50$

b) $80 \div 0.02 = 80 \div \dfrac{2}{100} = 80 \times \dfrac{100}{2} = 4000$

c) $12 \div 0.06 = 12 \div \dfrac{6}{100} = 12 \times \dfrac{100}{6} = 200$

d) $12.8 \div 0.64 = \dfrac{128}{10} \div \dfrac{64}{100} = \dfrac{128}{10} \times \dfrac{100}{64} = 20$

e) $2.25 \div 1.5 = \dfrac{225}{100} \div \dfrac{15}{10} = \dfrac{225}{100} \times \dfrac{10}{15} = 1.5$

f) $3 \div 0.04 = 3 \div \dfrac{4}{100} = 3 \times \dfrac{100}{4} = 75$

g) $19.6 \div 0.14 = \dfrac{196}{10} \div \dfrac{14}{100} = \dfrac{196}{10} \times \dfrac{100}{14} = 140$

h) $25.6 \div 0.16 = \dfrac{256}{10} \div \dfrac{16}{100} = \dfrac{256}{10} \times \dfrac{100}{16} = 160$

i) $10 \div 0.001 = 10 \div \dfrac{1}{1000} = 10 \times 1000 = 10,000$

2. a) $4.27 \div 10 = 0.427$

b) $4.27 \div \boxed{} = 0.427$

Implies that $4.27 \div 0.427 = \boxed{}$

$4.27 \div 0.427 = \dfrac{427}{100} \div \dfrac{427}{1000} = \dfrac{427}{100} \times \dfrac{1000}{427} = 10$

Therefore $4.27 \div \boxed{10} = 0.427$

c) $4.27 \div 100 = \dfrac{427}{100} \div \dfrac{100}{1} = \dfrac{427}{100} \times \dfrac{1}{100} = \dfrac{427}{10000} = 0.0427$

Therefore $4.27 \div 100 = 0.0427$

d) $4.27 \div 1000 = 0.00427$

e) $0.56 \div 0.056 = \dfrac{56}{100} \div \dfrac{56}{1000} = \dfrac{56}{100} \times \dfrac{1000}{56} = 10$

Therefore $0.56 \div \boxed{10} = 0.056$

f) $5.6 \div \square\, 0.56$

Thus $\square = 5.6 \div 0.56 = \dfrac{56}{10} \div \dfrac{56}{100} = \dfrac{56}{10} \times \dfrac{1000}{56} = 10$

g) $14.28 \div \square = 0.1428$

Thus $14.28 \div 0.1428 = \dfrac{1428}{100} \div \dfrac{1428}{10000} = \dfrac{1428}{100} \times \dfrac{10000}{1428}$

Therefore $\square = 100$

Assessment

Remember that the students are expected to solve real life problems using fractions, decimals and the four operations. We hope that you have used a number of formal and   informal assessment techniques like class work, group work, home work, oral and written questions, assignments, quizzes, tests, etc during each period. It is also necessary to conduct a summative assessment to measure the students' level of understanding of this unit. Make sure that students understood the concept 'fractions and decimals', and the method of using the four operations.

If most of the students seem not to have understood a given specific topic; retouching of this topic by using a different approach is recommended.

Selected problems to slow learners

1. Perform the indicated operation.

    a. $3.2 + 5.4$ b. $16.54 - 2.31$

    c. 4.7×2.4 d. $6.4 \div 1.6$

2. Which one is greater?

    $4 \div 0.1$ or $4 \div 0.01$?

3. Which of the following is equal to 1.5?

    a) 0.5×3 b) 0.3×0.5 c) 5×0.03

4. Add 7.23, 7.04 and 7.34

5. Subtract 82.34 from 99.76

6. 97.32 ÷ 12 is equal to

   a) 7.81   b) 8.11   c) 9.11

7. Divide 178.92 by 7

8. A book is 8cm thick (excluding the cover). Each page of it is 0.1cm thick. How many pages are there in the book?

9. Find the value of 0.5 ÷ (0.2 ÷ 0.25).

Selected problems to fast learners

1. Perform the indicated operation.

   a. 135.73
       + 486.97

   b. 581.01
       − 499.98

   c. 532.67
       × 0.78

   d. 120.0 ÷ 15.6

2. Divide 0.17892 by 7.

3. A car needs 1 litter of petrol to run 35.2 km. How many kilometers can it run in 5.7 litter of petrol?

4. Divide 2.04 by 87.605.

5. Wheat costs Birr 12.50 per kg. How much is the cost of 18kg wheat?

6. Abaynesh needs 2.5 meters of cloth for a shirt. She buys cloth costing Birr 63.75 per meter. What is the cost of the shirt piece which she buys?

7. Tea costs Birr 14.50 per packet. How many packets of tea could be purchased for Birr 174?

8. A man went to buy coffee which was costing Birr 27.50 per bottle. How many bottles could he buy for Birr 220?

Answers to Review Exercise

1. a) iv   b) v   c) ii   d) i   e) iii
    f) vii   g) vi

2. a) $\dfrac{50}{100} \times 80 = 40$

    Therefore, 50% of 80 is 40

    b) 35% of 60 $= \dfrac{35}{100} \times 60 = 21$

    c) $\dfrac{1}{4}$ of $100 = \dfrac{1}{4} \times 100 = 25$

    d) $\dfrac{2}{5}$ of $120 = \dfrac{2}{5} \times 120 = 48$

    e) $\dfrac{4}{3}$ of $450 = \dfrac{4}{3} \times 450 = 600$

3. a) $\dfrac{11}{2} = 5\dfrac{1}{2}$ b) $\dfrac{15}{8} = 1\dfrac{7}{8}$ c) $\dfrac{24}{27} = 3\dfrac{3}{7}$

    d) $\dfrac{31}{6} = 5\dfrac{1}{6}$ e) $\dfrac{14}{3} = 4\dfrac{2}{3}$ f) $\dfrac{42}{14} = 3$

    g) $\dfrac{13}{12} = 1\dfrac{1}{12}$ h) $\dfrac{27}{8} = 3\dfrac{3}{8}$ i) $\dfrac{47}{5} = 9\dfrac{2}{5}$

4. a) $2\dfrac{3}{4} = \dfrac{2 \times 4 + 3}{4} = \dfrac{11}{4}$ d) $5\dfrac{2}{5} = \dfrac{5 \times 5 + 2}{5} = \dfrac{27}{5}$

    b) $3\dfrac{5}{8} = \dfrac{3 \times 8 + 5}{8} = \dfrac{29}{8}$ e) $6\dfrac{1}{3} = \dfrac{6 \times 3 + 1}{3} = \dfrac{19}{3}$

    c) $7\dfrac{2}{5} = \dfrac{7 \times 5 + 2}{5} = \dfrac{37}{5}$ f) $12\dfrac{3}{5} = \dfrac{12 \times 5 + 3}{5} = \dfrac{63}{5}$

5. a) $\dfrac{1}{8} + \dfrac{2}{3} = \dfrac{1 \times 3}{8 \times 3} + \dfrac{2 \times 8}{3 \times 8} = \dfrac{3}{24} + \dfrac{16}{24} = \dfrac{3 + 16}{27} = \dfrac{19}{24}$

b) $\dfrac{4}{9}+\dfrac{3}{4}=\dfrac{4\times4}{9\times4}+\dfrac{3\times9}{4\times9}=\dfrac{16}{36}+\dfrac{27}{36}=\dfrac{16+27}{36}=\dfrac{43}{36}=1\dfrac{7}{36}$

c) $2\dfrac{1}{5}+1\dfrac{5}{6}=\dfrac{11}{5}+\dfrac{11}{6}=\dfrac{11\times6}{5\times6}+\dfrac{11\times5}{6\times5}=\dfrac{66}{30}+\dfrac{55}{30}=\dfrac{66+55}{30}=\dfrac{121}{30}=4\dfrac{1}{30}$

d) $5\dfrac{3}{4}+2\dfrac{7}{8}=\dfrac{23}{4}+\dfrac{23}{8}=\dfrac{46}{8}+\dfrac{23}{8}=\dfrac{69}{8}=8\dfrac{5}{8}$

e) $5\dfrac{1}{2}-2\dfrac{4}{5}=\dfrac{11}{2}-\dfrac{14}{5}=\dfrac{11\times5}{2\times5}-\dfrac{14\times2}{5\times2}=\dfrac{55}{10}-\dfrac{28}{10}=\dfrac{27}{10}=2\dfrac{7}{10}$

f) $6\dfrac{1}{3}+1\dfrac{5}{8}=\dfrac{19}{3}+\dfrac{13}{8}=\dfrac{19\times8}{3\times8}+\dfrac{13\times3}{8\times3}=\dfrac{152}{24}+\dfrac{29}{24}=\dfrac{191}{24}=7\dfrac{23}{24}$

g) $8-1\dfrac{2}{3}=8-\dfrac{5}{3}=\dfrac{8\times3}{1\times3}-\dfrac{5}{3}=\dfrac{24}{3}-\dfrac{5}{3}=\dfrac{24-5}{3}=\dfrac{19}{3}=6\dfrac{1}{3}$

h) $4\dfrac{1}{6}-2\dfrac{1}{3}=\dfrac{25}{6}-\dfrac{7}{3}=\dfrac{25}{6}-\dfrac{7\times2}{3\times2}=\dfrac{25}{6}-\dfrac{14}{6}$

$=\dfrac{25-14}{6}=\dfrac{11}{6}=1\dfrac{5}{6}$

i) $3\dfrac{7}{8}-2\dfrac{3}{4}=\dfrac{31}{8}-\dfrac{11}{8}=\dfrac{31-11}{8}=\dfrac{20}{8}=\dfrac{5}{2}$

j) $18\dfrac{2}{5}-9\dfrac{1}{2}=\dfrac{92}{5}-\dfrac{19}{2}=\dfrac{92\times2}{5\times2}-\dfrac{19\times5}{2\times5}=\dfrac{184}{10}-\dfrac{95}{10}$

$=\dfrac{89}{10}=8\dfrac{9}{10}$

k) $8\dfrac{5}{8}-4\dfrac{3}{5}=\dfrac{69}{8}-\dfrac{23}{5}=\dfrac{69\times5}{8\times5}-\dfrac{23\times8}{5\times8}=\dfrac{345}{40}-\dfrac{184}{40}=\dfrac{345-184}{40}$

$=\dfrac{161}{40}=4\dfrac{1}{40}$

l) $17\dfrac{1}{3}+9\dfrac{4}{9}+2\dfrac{6}{7}=\dfrac{52}{3}+\dfrac{85}{9}+\dfrac{20}{7}=\dfrac{52\times21}{3\times21}+\dfrac{85\times7}{9\times7}+\dfrac{20\times9}{7\times9}$

$=\dfrac{1092}{63}+\dfrac{595}{63}+\dfrac{180}{63}=\dfrac{1092+595+180}{63}=\dfrac{1867}{63}=29\dfrac{40}{63}$

6. $957\dfrac{3}{5}=\dfrac{957\times5+3}{5}=\dfrac{4785+3}{5}=\dfrac{4788}{5}$

Therefore, the answer is (b)

7. a) $2\dfrac{1}{3} \times 6\dfrac{2}{5} = \dfrac{7}{3} \times \dfrac{32}{5} = \dfrac{224}{15} = 14\dfrac{14}{15}$

   b) $9 \times 2\dfrac{1}{2} = \dfrac{9}{1} \times \dfrac{5}{2} = \dfrac{45}{2} = 22\dfrac{1}{2}$

   c) $1\dfrac{7}{8} \times \dfrac{5}{6} = \dfrac{15}{8} \times \dfrac{5}{6} = \dfrac{25}{16} = 1\dfrac{9}{16}$

8. a) $\dfrac{5}{9} \div \dfrac{1}{2} = \dfrac{5}{9} \times \dfrac{2}{1} = \dfrac{10}{9} = 1\dfrac{1}{9}$

   b) $\dfrac{6}{11} \div \dfrac{5}{6} = \dfrac{6}{11} \times \dfrac{6}{5} = \dfrac{36}{55}$

   c) $2\dfrac{1}{4} \div 1\dfrac{2}{3} = \dfrac{9}{4} \div \dfrac{5}{3} = \dfrac{9}{4} \times \dfrac{3}{5} = \dfrac{27}{20} = 1\dfrac{7}{20}$

   d) $5\dfrac{5}{6} \div 2\dfrac{2}{5} = \dfrac{35}{6} \div \dfrac{12}{5} = \dfrac{35}{6} \times \dfrac{5}{12} = \dfrac{175}{72} = 2\dfrac{31}{72}$

9. The remaining amount of water $= 20\dfrac{1}{2} - 8\dfrac{1}{4}$

$$= \dfrac{41}{2} - \dfrac{33}{4} = \dfrac{82}{4} - \dfrac{33}{4} = \dfrac{49}{4} = 12\dfrac{1}{4} \, litres$$

10. Each person's share $= \dfrac{82.5}{3} =$ Birr 27.50

11. $0.4 = \dfrac{2}{5} = \dfrac{2 \times 40}{5 \times 40} = \dfrac{80}{200}, \ \dfrac{5}{8} = \dfrac{5 \times 25}{8 \times 25} = \dfrac{125}{200},$ and

$0.57 = \dfrac{57}{100} = \dfrac{52 \times 2}{100 \times 2} = \dfrac{114}{200}$

$80 \ < 125 \ < \ 144$

Thus $\dfrac{80}{200} < \dfrac{125}{200} < \dfrac{144}{200}$

That is $\dfrac{2}{5} < \dfrac{5}{8} < 0.57$

The greatest amount of rain is measured on Wednesday and the least amount of rain is measured on Sunday.

12. a) 3.42   b) 2.3   c) 5.12   d) 4.68

$$\begin{array}{r}3.42\\ \underline{\times 7.2}\\ 684\\ \underline{2394}\\ \underline{24.624}\end{array} \qquad \begin{array}{r}2.3\\ \underline{\times 4.1}\\ 2.3\\ \underline{92}\\ \underline{9.43}\end{array} \qquad \begin{array}{r}5.12\\ \underline{\times 0.3}\\ \underline{1.536}\end{array} \qquad \begin{array}{r}4.68\\ \underline{\times 5.8}\\ 3744\\ \underline{2340}\\ \underline{27.144}\end{array}$$

  e) $2.8 \times 0.05 = 0.14$

  f) $1.45 \times 0.7 = 1.015$

13. a) $4 \div 0.01 = 4 \div \dfrac{1}{100} = 4 \times 100 = 400$

     b) $0.3 \div 0.03 = \dfrac{3}{10} \div \dfrac{3}{100} = \dfrac{3}{10} \times \dfrac{100}{3} = 10$

     c) $3.5 \div 0.7 = \dfrac{35}{10} \div \dfrac{7}{10} = \dfrac{35}{10} \times \dfrac{10}{7} = 5$

     d) $3 \div 0.003 = 3 \div \dfrac{3}{1000} = \dfrac{3 \times 1000}{3} = 1000$

     e) $11 \div 0.001 = 11 \div \dfrac{1}{1000} = 11 \times 1000 = 11,000$

14. His wife's share $= \dfrac{120,000}{2} = \text{Birr}60,000$

   The share of each child $= \dfrac{60,000}{3} = \text{Birr}20,000$

15. 10 such pairs weigh $= 10 \times 1.213 = 12.13$ kilograms

    100 pairs weigh $= 100 \times 1.213 = 121.3$ kilograms

    1000 pairs weigh $= 1000 \times 1.213 = 1213$ kilograms

UNIT FOUR

DATA HANDLING

Introduction

It is very important for students to have the knowledge of data handling because data handling is used in almost every area of human life. Students must be able to collect simple data from their environment and construct bar graphs. They are also expected to find the average of a given data. In this unit, students will get familiarized with introductory ideas of data handling. They will also practice constructing bar graphs, interpreting data and finding average of numbers.

The activities and exercises given in each sub unit are designed to encourage students' participation in constructing bar graphs, interpreting the constructed bar graphs and finding average of numbers.

Unit Outcomes
After completing this unit, students will be able to:
- understand simple graphical representation of data.
- know and calculate average of a given data.

Suggested Teaching Aids In Unit 4
Tables, bar graphs and charts are useful teaching aids in this unit because data handling requires presentations and representations of different data. Practical examples that relate to students daily life such as coins, cards, balls, diagrams of insects, etc are included so that students can best appreciate and understand representations of data.

Charts containing sample bargraphs like the following are recommended:
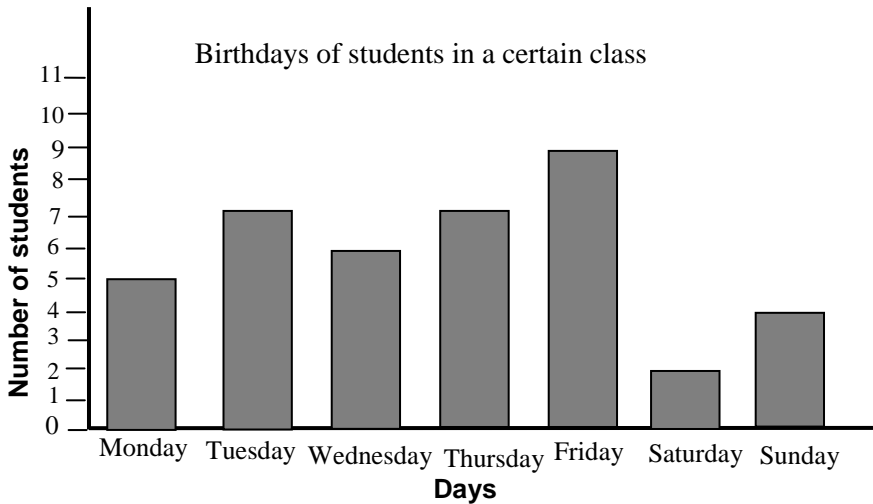
Figure 4.1
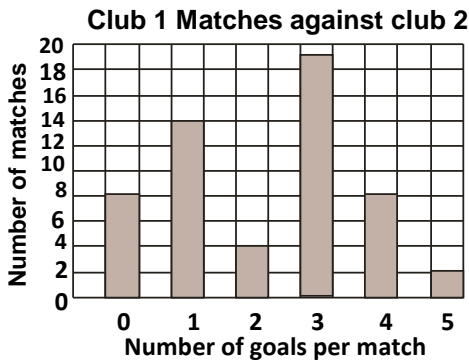
Figure 4.2

4.1 Further on Construction and Interpretation of Bar Graphs

Periods allotted: 9 periods

Competencies

At the end of this sub-unit students will be able to:

- construct bar graphs from a given data.
- interpret the constructed bar graphs.

Introduction

This sub-unit begins with discussion on a useful way of recording data and constructing bar graphs. It provides students with collecting simple data from their lives, organizing, constructing bar graphs and interpreting the constructed bar graphs.

Teaching Notes

This sub-unit is intended to introduce data handling by dealing with collecting simple data, constructing bar graphs and interpreting the constructed bar graphs.

In order to start from the background of the students and to help them relate to their daily life. It is advisable to involve students in the activities outlined in this sub-unit.

Encourage students to construct bar graphs by collecting simple data from their lives. You may take examples like: e.g. the number of people living in each student's home, the number of students in your school, etc. Assist students in reading and interpreting bar graphs.

Answers to Activity 4.1
  1. Mango   2.Poam   3.40

Answers to Activity 4.2
1. Friday   2. Saturday   3. Tuesday and Thursday

Answers to Activity 4.3
 1. 4   2.6

Answers to Group 4.1
1.30	2. 28	3. 26	4. 30	5. 28
6. 27	7. 24	8. 27	9. Grade 4	
10. Grade 4	11. Grade 5	12. Grade 4		
13. 5	14. 2	15. 5	16. 3	

Assessment

To make sure that students have understood graphical representation of data, you can give them problems in the form of class work, home work, group work, assignment, quiz or test and let students present it to class. Check their work and the overall discussion during presentation. You can also ask fast learners or interested students to answer the following problems as additional assessment.

Additional Assessment

Read the bar graph given below and complete the corresponding table.
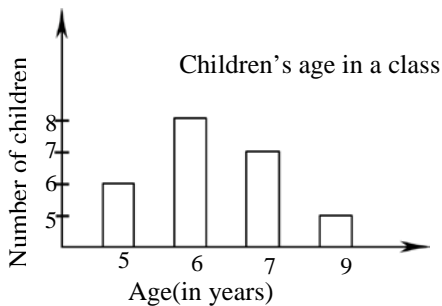
Figure 4.3

Age (in years)	5	6	7	9
Number of children	6			

Answers to Additional Assessment

Age (in years)	5	6	7	9
Number of children	6	8	7	5

Answers to Exercise 4. A

1. a)

Types of Trees	Oak	beech	Birch	elm
Number of Trees planted	24	36	40	32

b) Total number of tree planted= 24+36+40+32=132

c) birch

d) oak

e) oak=20, beech=8, birch = 4 and elm = 12

2. (i)

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of eggs	45	25	35	25	10	40	25

ii) a) 45   b) Tuesday, Thursday and Sunday

c) Monday   d) Friday   e) 40-10=30

f) 45+25+35+25+10+40+25=205 That is, total number of

eggs collected in one week = 205

3.

Figure 4.4

4 (a)

Marks out of ten	0	1	2	3	4	5	6	7	8	9	10
Number of students	3	3	3	5	2	3	3	7	5	3	3

(b)

Figure 4.5

5. a) i) 32   ii) 17   iii) 29

    b) i) 37-17=20   ii) 37-24=13   iii) 37-29=8

    c) 22+37+32+17+24+29=161

6. a) 4   b) 8+2 = 10   c) 3

    d) 8   e) 8+14+4+19+8+2=55

7.

Marks scored in Mathematics test											
Score	0	1	2	3	4	5	6	7	8	9	10
Number of students	0	0	1	0	2	4	9	6	3	3	2

Scores of students in a mathematics test
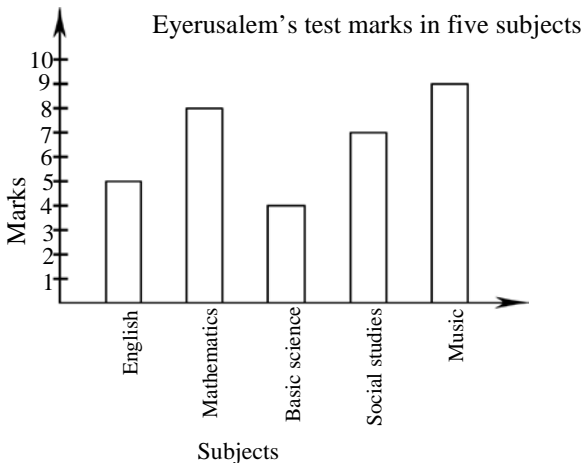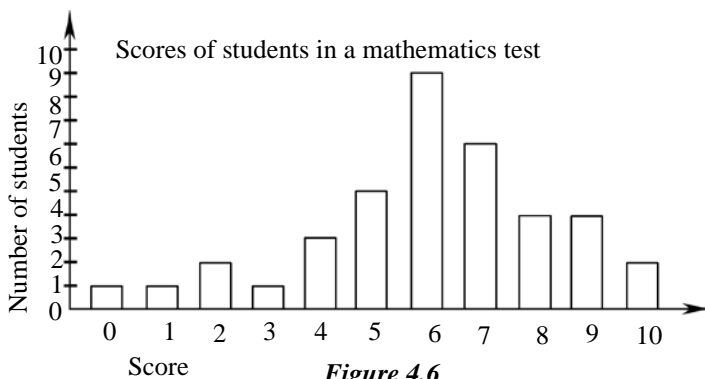
Figure 4.6

Selected problem to slow learners

1. Alemnesh's test mark (out of ten) in five subjects were:
    - English   7
    - Mathematics   6
    - Basic science   5
    - Social studies 10
    - Music   8

    Draw a bar graph showing the subjects on the horizontal axis and marks on the vertical axis.

2. Draw a bar graph for the following table of data on age of students in a certain class.

Age in years	10	11	12	13
Number of students	8	4	6	2

3. Draw a bar graph for the following table of data on weights of family members.

Numbers of family members	Weight in kg
Abayneh	72
Almaz	68
Wondu	62
Haregua	58
Genet	50

4. Draw a bar graph for the following table which shows the number of glasses of water consumed in one day by 8 girls.

Water consumption in a day							
Name	Hana	Jemila	Loza	Fikir	Merima	Sara	Fozia	Nunu
Glass	4	12	3	1	4	7	5	4

5. The bar graph shows the students watch TV in one week. Find the total number of hours students take for watching TV in a week.
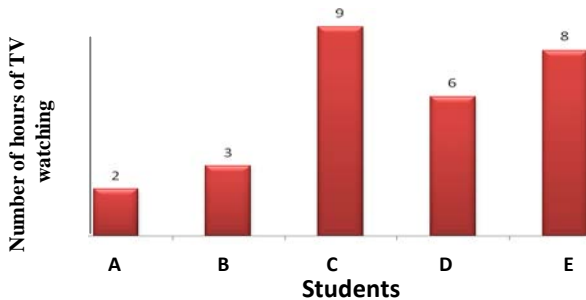
Figure 4.7

Selected problem to fast learners

1. Draw a bar graph for the following table of data on test score of 6 students out of ten.

Name of students	Marta	Birtukan	Chala	Ujulu	Meseret	Abdi
Test score	7	8	10	9	10	8

2. Draw a bar graph for the following table of data on ages of students in a certain class.

Age (in years)	14	15	16	17
Number of students	6	5	4	5

3. Draw a bar graph showing goal scores of club A in a football match.

Goal Scores				
Year	1990	1991	1992	1993
Score	86	90	96	92

4. Use the bar graph to answer each question.
   (a) Which language has the quest native speakers?
   (b) About how many more people speak Hindi than Spanish?

Most widely Spoken Languages
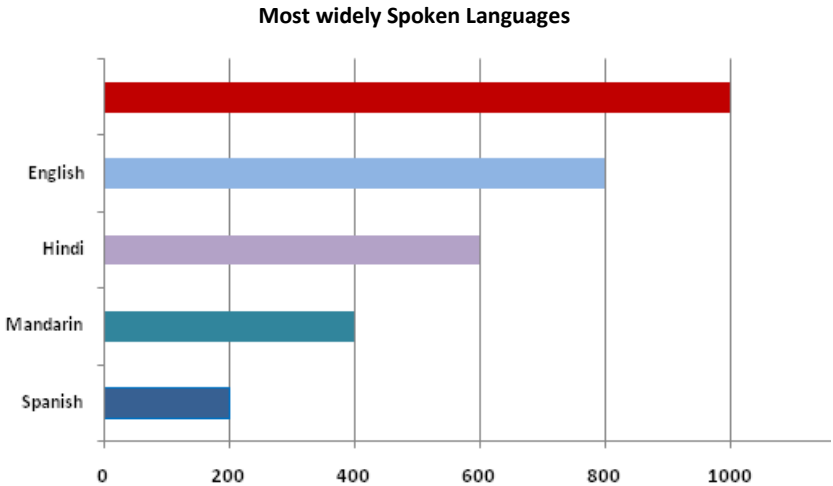
Figure 4.8

5. The bar graph shows school A's students by gender and band membership.
   a) How many of the schools students are band members?
   b) How many of the school's students are not band members?
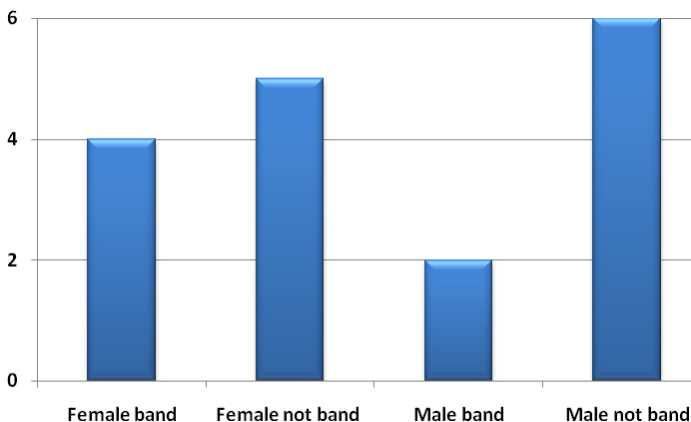
Figure 4.9

4.2 The average of numbers

Periods allotted: 5 periods

Competencies:

At the end of this sub-unit students will be able to:

- explain what is meant by the average of a set of numbers.
- calculate the average of a given data.

Introduction

This sub-unit aims at discussing the idea of the average of numbers. In order to initiate the discussion on calculating the average of numbers, this sub-unit begins with Activity. The first Activity given is intended to involve students (in adding marks, dividing the total mark by the number of subjects) and introduce them with finding the average of numbers. This sub-unit also contains other activities and examples which relate to students' daily life. Thus, it is advisable to involve students in each Activities. Tables, diagrams and bar graphs are discussed to clarify ideas on finding the average of numbers.

Teaching Notes

Motivate students by asking questions how to read data presented in tables and interpret bar graphs. Point out that data handling is used to collect data, present facts in a definite form and to make comparisons.

Ask students to tell what an average is in their own word and let students come to the definition that the average of numbers is the sum of values, divided by the total number of values.

Encourage students to participate in doing Activities outlined in this sub-unit. Let students find the average of any given data.

Answers to Activity 4.4

$$\frac{70 + 80 + 60 + 90 + 50 + 70}{6} = \frac{420}{6} = 70$$

Average mark = 70

Answers to Group Work 4.2

1. $\frac{2+3+4+8+13}{5} = \frac{30}{5} = 6$

2. $\frac{1+1+2+4+6}{5} = \frac{14}{5} = 2.8$

Assessment

You can give exercises on finding averages of numbers in the form of class work, group work, home work, assignment, quiz or test and check whether students accurately find averages of numbers. You can also ask students to answer the following questions as additional exercises.

Additional Assessment

The scores of a student in five subjects out of 100 is given as: 60, 80, 70, 70, 90 and 100.

Find the average score of the student.

Answer to Additional Assessment

$$\text{Average} = \frac{60 + 80 + 70 + 90 + 100}{5}$$

$$\text{Average} = 80$$

Answers to Exercise 4. B

1.

Ages	10	11	12	13	14	15	16
Number of students	3	1	6	4	3	2	1

Average= $\dfrac{3 \times 10 + 1 \times 11 + 6 \times 12 + 4 \times 13 + 3 \times 14 + 2 \times 15 + 1 \times 16}{3 + 1 + 6 + 4 + 3 + 2 + 1}$

$= \dfrac{30 + 11 + 72 + 52 + 42 + 30 + 16}{20}$

$= \dfrac{253}{20} = 12.65$

2. a) Alexander's average test score= $\dfrac{8 + 7 + 6 + 9}{4} = \dfrac{30}{4} = 7.5$

   b) Bosena's average score $= \dfrac{6 + 4 + 5 + 6}{4} = \dfrac{21}{4} = 5.25$

   c) Merima's average test score $= \dfrac{10 + 8 + 9 + 9}{4} = \dfrac{36}{4} = 9$

   d) Average test score of students in Test 1=

   $\dfrac{8 + 9 + 6 + 4 + 6 + 10}{6} = \dfrac{43}{6} = 7.1\dot{6}$

   e) Average test score of students in Test 3

   $= \dfrac{6 + 7 + 7 + 6 + 5 + 9}{6} = \dfrac{40}{6} = 6.\dot{6}$

3. $\dfrac{2 + 4 + 6 + 5 + x}{5} = 10$

   Thus, $\dfrac{17 + x}{5} = 10$ implies that $= \dfrac{17}{5} + \dfrac{x}{5} = 10$

   or $\dfrac{x}{5} = 10 - \dfrac{17}{5} = \dfrac{100}{10} - \dfrac{34}{10} = \dfrac{100 - 34}{10} = \dfrac{66}{10}$

   That is $\dfrac{x}{5} = \dfrac{66}{10}$

   $5 \times \dfrac{x}{5} = \cancel{5}^{1} \times \dfrac{66}{\cancel{10}_{2}}$ (multiplying both sides by 5)

   x = 33

4. Average rainfall $= \dfrac{10 + 15 + 5 + 10 + 20 + 20 + 15 + 10}{8}$

   $= \dfrac{105}{8} = 13.125\text{mm}$

5. Shewaye's average speed $= \dfrac{9\text{km}}{3\text{hr}} = 3\,{}^{\text{km}}\!/_{\text{hr}}$

6. a) Average number of visitors per day

   $= \dfrac{64 + 73 + 70 + 80 + 84 + 90 + 120}{7} = \dfrac{581}{7} = 83$

   b) Friday, Saturday and Sunday

7. a) Three (Abate, Birhanu and Dinkessa)

   b) Average points scored $= \dfrac{16 + 22 + 10 + 22 + 10}{5} = \dfrac{80}{5} = 16$

   c) Two (Chala and Equbay)

Assessment

Remember that the students are expected to handle data by collecting, organizing in a table, draw bar graphs and also find averages of data. We hope that you have used a number of continuous assessment techniques like class work, group work, home work, oral and written questions, assignments, quizzes, tests, etc during each period. It is also necessary to conduct a summative assessment to measure the students' level of understanding of this unit. Make sure that students understood simple graphical representation of data and know and calculate average of a given data.

If most of the students seem not to have understood a given topic; re-teaching of the topic by using a different approach is recommended.

Selected problems to slow learners

1. Find the average of the following set of numbers.
   - a) 2,4,6,8 and 10
   - c) 2,5,8,11 and 14
   - b) 3,5,7,9 and 11
   - d) 10,20,30,40,50 and 60
2. What is the mean of the numbers 8,20,21,24,22 and 25?
3. If the mean of 4,8,12,16 and x is found to be 12, then find the value of x.
4. Find the mean of 44,68,48,61,59,48,63 and 49.
5. The ages of the participants in a bicycle race are 14, 23, 20, 24, 26, 17, 21, 31, 27, 25, 14 and 28. Find the average age of the participants.
6. Find the average monthly low temperature from the given table

Monthly low temperature					
Month	Jun	July	Aug.	Sept.	Oct.
Temperature (^{0}F)	44	41	47	42	36

Selected problems to fast learners

1. Find the average mark of the six students from the given bar graph.
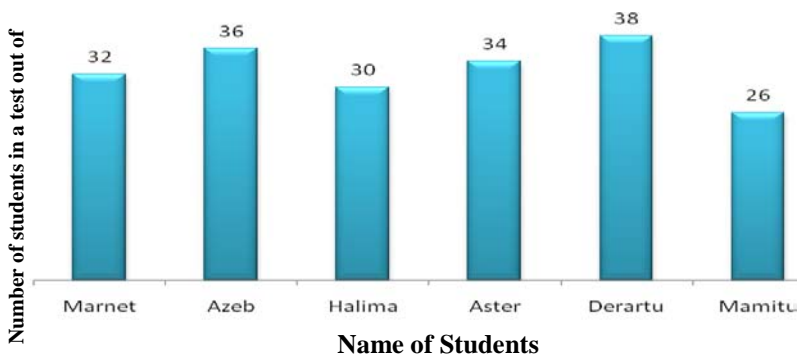
Name of Students

Figure 4.10

2. What is the mean of the numbers 40,43,41,48,47,51,58,59 and 62?

3. The table shows the heights of students in cm in a certain class.

Height (in cm)	
Males	100, 75, 80, 95, 90, 110
Females	95, 100, 110, 120, 90, 100

Find (a) the average height of male students.

(c) The average height of female students.

4. If the mean of 5,30,35,20,5,25 and x is found to be 20, then find the value of x.

5. Find the mean of 2.0, 4.4, 6.2, 3.2, 4.4, 6.2 and 3.7.

6. Find the average monthly rainfall from the table which shows the monthly rainfall in inches for six months.

Month	Rainfall (in)
Jan.	4.33
Feb.	1.62
Mar.	2.17
Apr.	0.56
May.	3.35
Jun.	1.14

Answers to Review Exercise

1. a) True b) False c) True

2.

Number of insects found
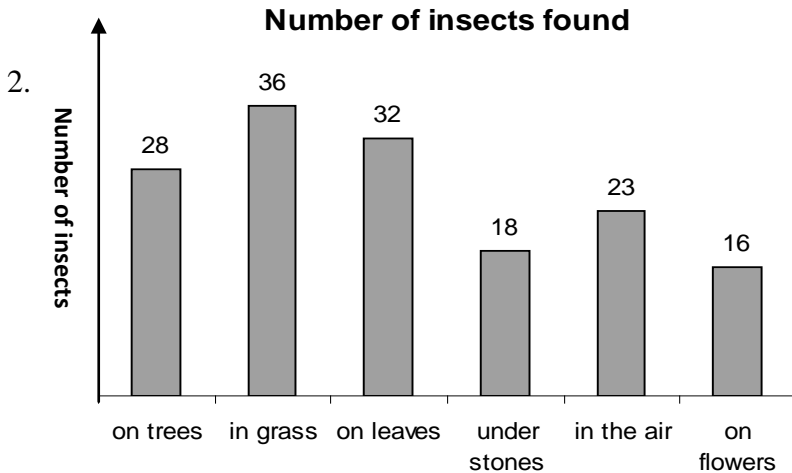
Figure 4.11

3. Average score =

$$\frac{98 + 100 + 84 + 88 + 92 + 96 + 90 + 78 + 50 + 61 + 89 + 85 + 75}{13}$$

$= \frac{1086}{13} = 83.54$ (approximately to two decimal places)

4. Total number of teachers = 13+18+8+11=50

Out of 50 teachers, 8 teachers ordered coffee

$$\frac{8}{50} = \frac{8}{50} \times 100\% = 16\%$$

Therefore, the answer is (A)

UNIT FIVE

GEOMETRIC FIGURES AND MEASUREMENT

Introduction

This Unit is intended to extend and deepen the students' knowledge and capability about the basic concepts of geometric figures and measurement. The unit is divided in to 5 sub-units. The sub-units include: lines, Angles and the measurement of angles, classification of triangles, lines of symmetry and measurement. The sub-unit 'measurement' discusses about the perimeter and area of rectangles and squares, and volumes of solids in everyday life like cubes, and rectangular prisms. Geometric ideas are elaborated with descriptive examples. In order to motivate students, various activities, group works and exercises, which relate to real life situations, are included.

Unit Outcomes

At the end of this unit, students will be able to:

- know important properties of axial symmetry and are able to use this knowledge for carrying out construction.
- bisect line segments and angles.
- know the unit "degree" and are able to measure the size of a given angle.
- understand and apply the formulas used to compute the areas of rectangles and squares.

Suggested Teaching Aids in Unit 5

Angles and parts of angles and geometric figures requires mathematical tools to construct and describe them. Therefore, in addition to the student's textbook and the teacher's guide, you are advised to prepare and bring in to the class the following materials whenever necessary.

Mathematical tools: ruler, pair of compass, set square, protractor, scissors, thin card boards.

Chart containing geometric figures like the following are recommended:

Triangles

Figure 5.1

Symmetric Figures

Figure 5.2

Prisms

Rectangular prism (cuboid)

Cube

Triangular prism

Cylinder

Figure 5.3

Nets of Cubes

Figure 5.4
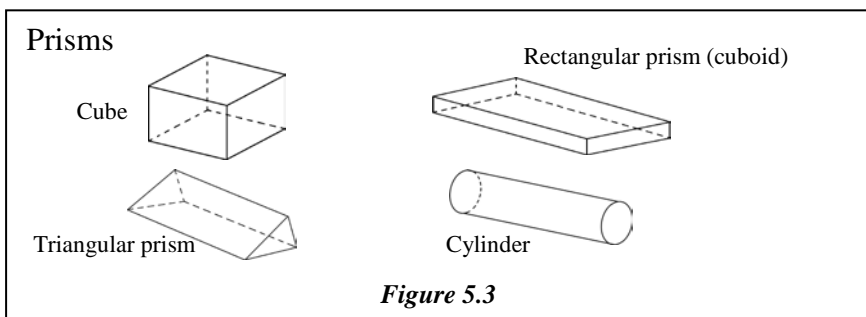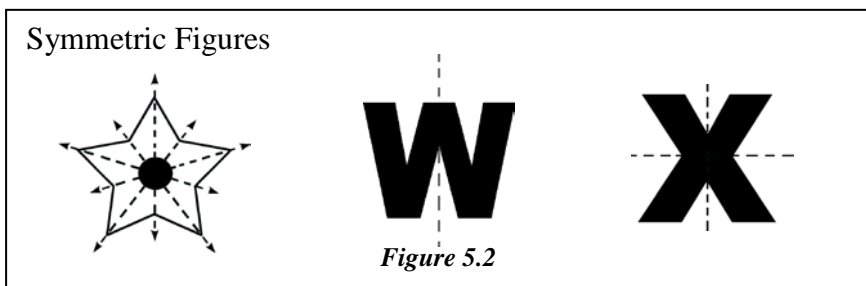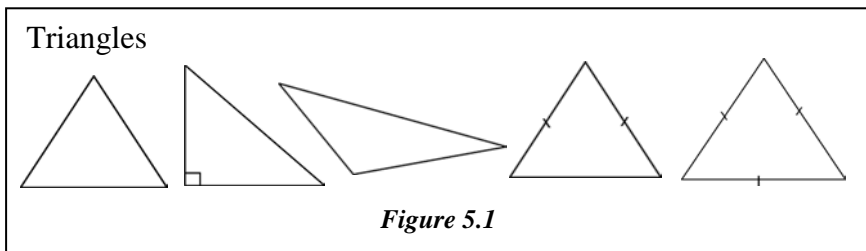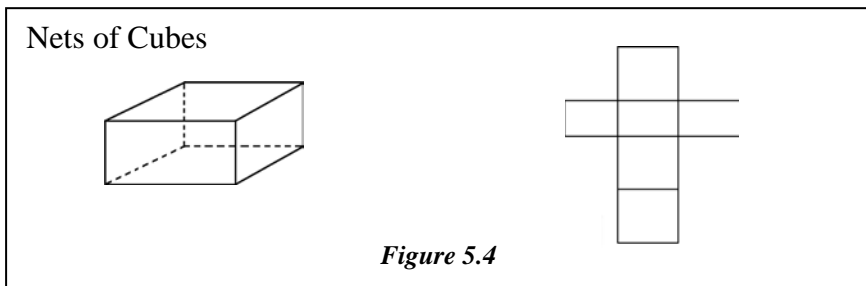
5.1 Lines

Periods allotted: 8 periods

Competencies

At the end of this sub-unit students will be able to:

- draw intersecting lines
- draw parallel lines
- bisect a line segment
- construct a perpendicular line to a given line through a point on the line
- construct a perpendicular line to a given line through a point not on the given line.

Introduction

An important topic in geometry is construction. This sub-unit deals with construction of intersecting and parallel lines, bisecting a given line segment, and construction of perpendicular line to a given line. Various Activities are included to encourage group work and to make sure that each student can construct intersecting and parallel lines, construct perpendicular line to a given line and bisect a given line segment.

Teaching notes

In order to address this sub-unit, the presentation is classified in to three sub topics: construction and parallel lines, bisecting a given

line segment, and construction of perpendicular line to a given line. The teacher is advised to bring a ruler, set square, a pair of compasses and a sharp pencil in the class.

5.1.1 Construction of intersecting and parallel lines

You may start the discussion of this sub-unit by explaining the idea of geometric construction and use Activity 5.1 to encourage participation of students in identifying lines and intersections. Then demonstrate how to draw parallel line AB through another point D not on the given line as follows:

Step 1. Slide   the set square along AB until the short side passes through D.

Step 2. Draw along the short side of the set square then slide the set square up the line you have just drawn using your ruler, until it reaches D. Now draw along the long side of the set square.
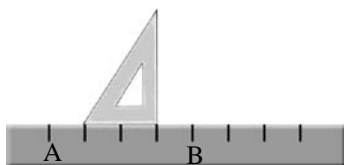
Figure 5.5
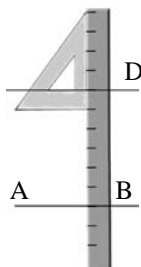
Figure 5.6

Encourage students practice drawing

- Intersecting lines using ruler
- Parallel lines using ruler and set square

Answers to Activity 5.1

a) One line and six points of intersections
b) One line and two points of intersections
c) Three lines and three points of intersections

d) Three lines and two points of intersections

e) Two lines and five points of intersections

Assessment

For assessing students' learning apart from class work, group discussions and assignments, you can give them quiz or test.

Answers to Exercise 5.A

1. a) $\overline{ZW}$, $\overline{ZX}$, $\overline{ZM}$, $\overline{MX}$, $\overline{XY}$ or $\overline{ZY}$

   b) W, Z, M, X or Y

   c) $\overleftrightarrow{WZ}$ and $\overleftrightarrow{WX}$, $\overleftrightarrow{WZ}$ and $\overleftrightarrow{WZ}$, $\overleftrightarrow{WZ}$ and $\overleftrightarrow{YZ}$, $\overleftrightarrow{WX}$ and $\overleftrightarrow{ZX}$, $\overleftrightarrow{ZX}$ and $\overleftrightarrow{XY}$, $\overleftrightarrow{ZY}$ and, $\overleftrightarrow{ZX}$ $\overleftrightarrow{XY}$ and $\overleftrightarrow{YZ}$

5.1.2 Bisecting a given line segment

Encourage students to practice drawing (i) the bisector of a line segment using ruler and set square and (ii) the bisector of a line segment using ruler and compass. Lead students to conclude that there are infinitely many bisectors through exercises.

Answers to Group Work 5.2

You can conclude that $\overline{ML} \equiv \overline{LN}$. Thus L is the midpoint of $\overline{MN}$.

Answers to Activity 5.2

The segment you drew through Z is the bisector of XY.

Assessment

Assessing students' understanding on bisecting a given line segment is essential to pass in to discussion of construction of perpendicular line to a given line. For this purpose you can give problems in the form of class work, home work, assignment, group work, test or quiz.

Hint to Exercise 5.B

Repeat the steps stated in Activity 5.2 in order to bisect the segments given in Exercise 5.B For example, in (a) you may assume XY = 8cm

5.1.3 Construction of perpendicular line to a given line

Encourage students to practice drawing:

i. The perpendicular to a line from a point on the line using ruler and compass.
ii. The perpendicular to a line from a point outside the line using ruler and compass.

Hint to Activity 5.4

Observe that four right angles are formed by perpendicular lines.

Assessment

It is important and essential to pay close attention to the students and check their work. Give students problems on construction of perpendicular line to a given line in the form of class work, home work, quiz or test in order to find out how students are following the lesson.

Hint to Exercise 5. C

1. For (a) and (b) repeat the steps stated in Activity 5.5, number, to construct a line perpendicular to a given line through a point not on the given line.
2. You may repeat the steps stated in Activity 5.5, number 2 9 where RS = 4 cm)
3. Repeat the steps stated in Activity 5.5, number 2
   (where RS= 3   cm, SU= 2cm)

Selected problems to slow learners

1. Draw

   a) a line segment.

   b) two intersecting lines.

   c) two parallel lines.

2. Bisect segment AB, which is 8cm long.

3. Identify intersecting lines in
   the figure. What is their
   point of intersection?
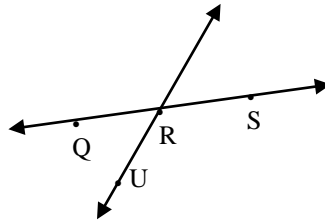
4. Which geometry term might you associate with the tip of a pen?

   a) Line segment   b) point   c) plane

5. Identify the line segments that are congruent in the figure.
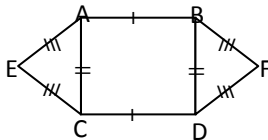
6. Which ray is perpendicular to line AB?
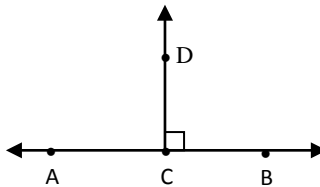
Selected problems to fast learners

1. Draw a square of side 6cm.

2. Draw a rectangle of sides 3cm and 4cm.

3. Give an example of a line segment from everyday life.

4. Which geometry term might you associate with one edge of a cardboard box?

   a) a plane   b) a line segment

5. Which geometry term might you associate with the floor?

   a) a plane or rectangle   b) a lien segment

6. If two distinct lines intersect, then they intersect in exactly one point. (True or False)?

5.2 Angles and measurement of angles

Periods allotted: 8 periods

Competencies

At the end of this sub-unit students will be able to:

- give definition of an angle.
- identify the vertex and arms of an angle.
- draw given angles using protractor.
- classify angles according to their magnitude.
- bisect a given angle.

Introduction

This sub-unit deals with defining an angle, drawing angles, bisecting angles and classifying angles according to their magnitude. The subunit is sub divided in to three subtopics. The

first topic deals with angles. In this subtopic you will discuss how to name angles and denote their degree measures. The second subtopic deals with measurement and classification. You are expected to encourage students to bring geometric instruments to the class room for practicing. The third subtopic deals with bisecting an angle. In the third subtopic you will assist students to bisect a given angle using ruler and compass.

Various activities are included in order to motivate and involve students.

Teaching notes

Because this topic deals with angles and measurement of angles it is necessary that you bring geometric instruments such as ruler, and compass, to the classroom. We expect the students to actively participate practicing the drawing and measuring angles.

5.2.1 Angles

You may start the discussion of this sub-topic by explaining what is meant by an angle, its vertex and arms and how to measure an angle using protractor. Assist students to name an angle and denote its degree measure. You may use example like the following.
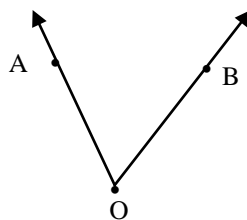
Figure 5.7

Names of the angle: $< AOB,\ A\hat{O}B, \hat{O}, \widehat{O}$

Its degree measure: m $(<AOB)$, m $m(A\hat{O}B)$, $m(\hat{O})$

Encourage students to identify angles and parts of angles. Help students grasp the difference between <B and Point B. Emphasize

that the point is the place where the rays meet, but the angle includes the point and the rays.

Point out that when students are naming angles such as $< ABC$, the vertex of the angle, B, should be written as the middle letter.

Assessment
Collect students exercise book and give corrections on Activity 5.4 and Activity 5.5. You can also give students problems as a class work, homework, assignment, quiz or test. This can help you follow the performances of the students.

Answers to Exercise 5.D
1. (a) $< ABC$, $A\hat{B}C$, $\hat{B}$
   (b) $< RQP$, $R\hat{Q}P$, $\hat{Q}$
   (C) $< UTS$, $U\hat{T}S$, $\hat{T}$
2. m ($<DBA$) $= 100^0$, m ($<ABC$) $= 60^0$, m ($<DBC$) $= 160^0$

5.2.2 Measurement and classification
Have students work in pairs to identify and list objects in the class room that have right, acute, obtuse, and straight angles. Encourage pairs to share and compare their lists with the class. Possible answers: The corner of a text book forms a right angle, the chalkboard the chalk ledge form a right angle, a closed empty binder form an acute angle, the pages of an open book form an obtuse angle.

Group Work 5.3
Use a protractor and ruler. Let students practice measuring an angle.

Group Work 5.4
This activity enables you to discuss with the students about forming a right angle, an actual angle and an obtuse angle.

Assessment

You need to assess students in order to make sure they have understood the idea of measurement and classification and to find out what aspect of the lesson is difficult. Assessment will also give you a clear picture of the knowledge and skill of the students. So, at the end of this lesson, you can give problems on measurement and classification in the form of class work, group work, home work, assignment, quiz or test.

Answers to Exercise 5.E
1. A) 90^0 b) 110^0 c) 245^0 d) 50^0 e) 300^0 f) 180^0
   Use a protractor to find the measures of the angles given
2. b) acute   b) right   c) reflex   d) obtuse
3. a) obtuse   b) acute   c) right   d) straight
   e) reflex   f) acute   g) obtuse
4. Obtuse (because $90.5 > 90$)
5. Use a protractor and check whether students have drawn the required angles or not.
6. 30^0
7. Branches A, C and D need to be spread
8. M $(< MZN) = 50^0$, m $(<NZO)= 45^0$, m $(<PZO) = 55^0$

5.2.3 Bisecting an Angle
Motivate students to bring geometric instruments to the class room and practice bisecting an angle you have drawn. You can see that the measure of the larger angle is twice the measure of one of the smaller angles.

Answers to Activity 5.5
Make sure that students follow the steps stated, use protractor to measure $< QPW$ and $< WPR$, and find that m($<QPR$) = 2m($<WPR$), m($< QPR$) = 2m $(< QPW$) or m($<QPW$) = m $(< WPR$).

Also that m (<CAT) = ½ m (< CAN)

Assessment

Motivate students to practice Group work 5.5 and Activity 5.5 repeatedly. Identify the slow, medium and fast learner and assist them accordingly. Check the class work, home work and assignment you ordered. You may give them a quiz to find out how they are following the lesson.

Answers to Exercise 5.F

1. Use a ruler and compass to make sure that each student understood bisecting the given angles.

2. A) m (<ABC) = 90^0 make sure, after bisecting, that each of the two smaller angles measures 45^0.

   b) m (<DEF) = 150^0. Make sure, after bisecting, that each of the two smaller angles measures 75^0.

   c) m (< GHI) = 30^0. Make sure, after bisecting, that each of the two smaller angles measures 15^0.

3. $\overrightarrow{XZ}$ bisects <WXY. Thus, m (<YXZ) = ½ m (<WXY). That is, a = ½ x124^0 = 62^0.

Selected problems to slow learners

1. If $\overrightarrow{xz}$ is the bisector of <wxy, where m(<wxy) =80^0, then
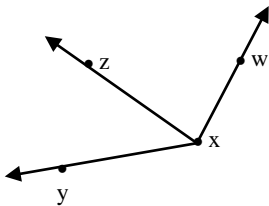
Figure 5.8

What is the value of m(<wxz)?

2. If m(<DBA) = 50^0, and $\overrightarrow{BA}$ is bisector of <DBC, then find m(<DBC).
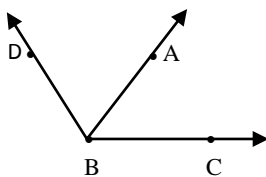
Figure 5.9

3. Tell whether each angle is acute, right, obtuse, straight or reflex.

a)

b)

c)

d)
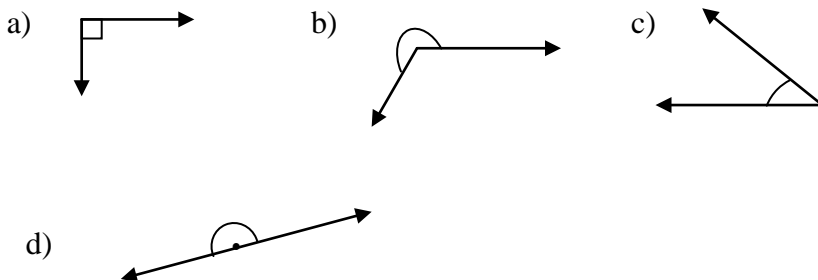
Figure 5.10

4. In the figure what is
    a) m(<DXE)
    b) m(<CXD)
    c) m(<BXC)
    d) m(<AXB)
    e) m(<BXE)
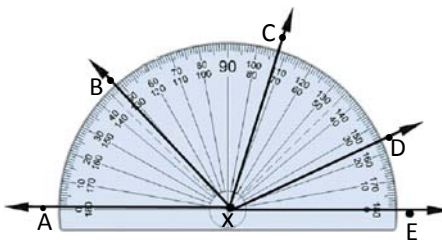
Figure 5.11

5. Draw an angle of 88^0 using a ruler and compass. Then bisect it.
6. Use ruler and compass to bisect the following angle.

7. If m(<ABC) $= 120^0$, and $\overrightarrow{BD}$ is an angle bisector then find m(<ABD)
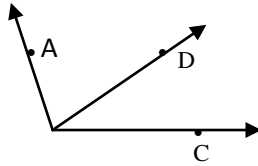
Figure 5.12

Selected problems to fast learners

1. Draw the angle with the given measurement. Then use a ruler and compass to bisect each angle.
1. 82^0 b) 128^0 c) 172^0
2. Use a ruler and compass to bisect the angle shown below.
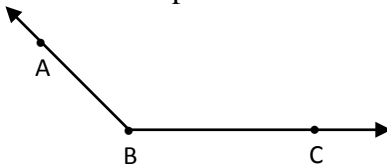
Figure 5.13

3. In the figure, the sum of the measures of the three angles is 180^0. Is angle x acute?
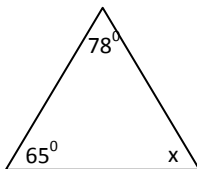
Figure 5.14

4. In the figure, the sum of the measures of the four angles is 360^0. Is angle y obtuse?
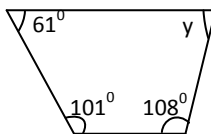
Figure 5.15

5. In the figure, m(<AOB) = 80^0, and $\overrightarrow{OC}$ is the bisector of angle AOB. If m(<AOC) = 2x, then find the value of x.
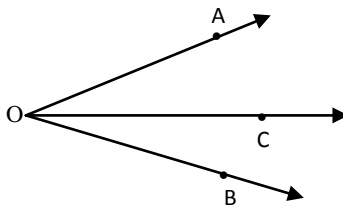
Figure 5.16

6. Use ruler and compass to bisect an angle whose measure is 130^0.

5.3 Classification of Triangles

Periods allotted: 5 periods

Competency

At the end of this sub-unit students will be able to:

- classify triangles according to the magnitude of their angles and sides.

Introduction

This sub-unit deals with classification of triangles according to the magnitude of their angles. Students are required to identify triangles and their parts. Definitions on classification of triangles are given and activities are included to encourage group work in classifying triangles and to ensure participation of students.

Teaching Notes

The students are not new to triangles, therefore you can assist students to classify triangles according to their sides and angles as follows:
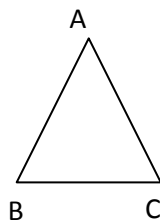
Figure 5.17

Scalene   (AB ≠AC, AB≠BC and AC≠BC)

Isosceles

(DE =DF)
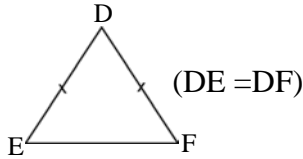
Figure 5.18

Equilateral

(HI = HG =GI)
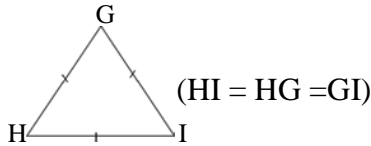
Figure 5.19

Acute angled

(m∠J), m (∠k) and m(∠L) are between) 0^0 and 90^0.
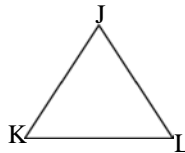
Figure 5.20

Right angled

(m(∠N) = 90^0)
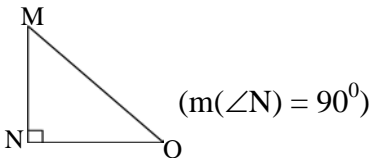
Figure 5.21

Obtuse Angled

(m(∠Q) is between 90^0 and 180^0)
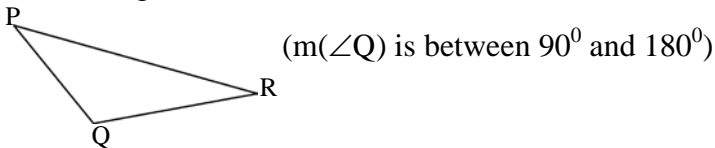
Figure 5.22

Equiangular

$(m (\angle S) = m (\angle T) = m (\angle U))$
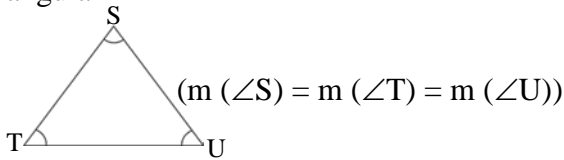
Figure 5.23

Let students draw triangles and classify them according to their sides and angles.

Regarding Group Work 5.6

You are expected to make students practice drawing triangles and sort them in to three groups according to the measure of their sides and angles.

Assessment

You are expected to assess the students' performance on the presented lesson by using appropriate techniques and by giving them problems as class work, home work, assignment, quiz or test. Give comments on their attempts.

Answers to Exercise 5.G

1. a) Isosceles   b) Right angled   c) Obtuse angled
   d) Scalene   e) Equilateral. Equiangular
   f) Isosceles, Right angled

2. Here is one possible answer

a)

b)

c)

d)

e)

Figures 5.24

f)
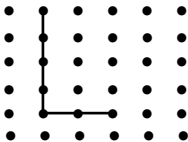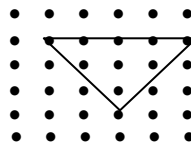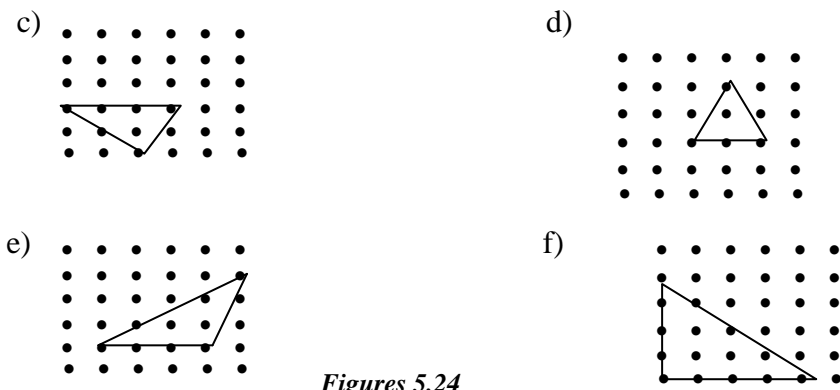
Selected problems to slow learners

1. Classify each of the following triangles as equilateral, isosceles, obtuse or right.

a)

60^0
60^0 60^0

b)

70^0
70^0 40^0

c)

30^0
60^0

d)

120^0
40^0 20^0
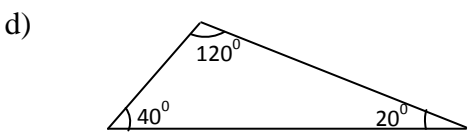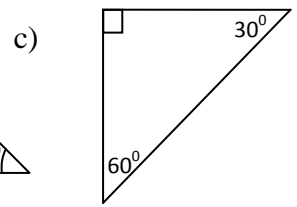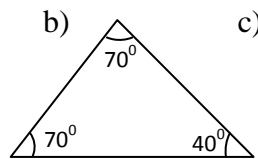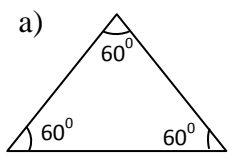
Figure 5.25

2. In the figure, the measures of all the angles are equal. What kind of triangle is ΔABC?

A

B 60^0 C
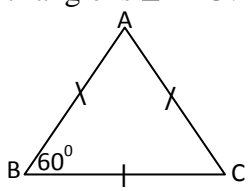
Figure 5.26

3. What kind of triangle is ΔDEF?
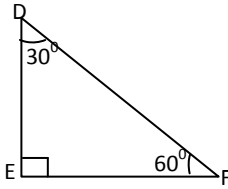
Figure 5.27

4. What kind of triangle is ΔGHI?
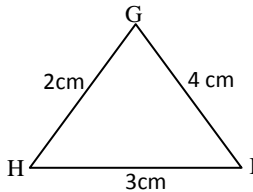
Figure 5.28

5. What is the name given to the triangle shown at the right?
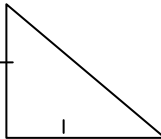
Figure 5.29

6. How many sides are congruent in a scalene triangle?

Selected problems to fast learners

1. Construct

   a) a right angled triangle.

   b) an equilateral triangle.

   c) an obtuse angled triangle.

2. In the figure shown, ΔABC is isosceles, where m(<B) = m(<C). Find the value of x.
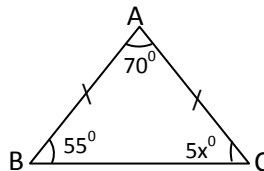
Figure 5.30

3. In the figure shown

   $m(\angle D) + m(\angle E) + m(\angle F) = 180^0$.

   What kind of triangle is ΔDEF?
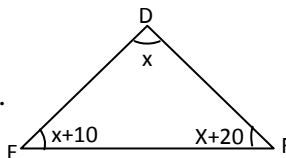
   Figure 5.31

4. Identify the different
   types of triangles
   in the figure.
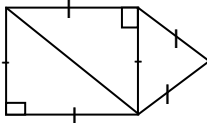
   Figure 5.32

5. Based on the angle measures given, which triangle is not
   acute?

   a) $20^0, 80^0, 80^0$ b) $90^0, 45^0, 45^0$
   c) $64^0, 64^0, 52^0$ d) $70^0, 65^0, 45^0$

6. Which of the following describes the triangle?

   a) Scalene, right triangle

   b) Isosceles, obtuse triangle

   c) Isosceles, acute triangle

   d) Equilateral, acute triangle
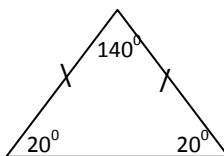
   Figure 5.33

5.4 Lines of symmetry

Periods allotted: 8 periods

Competencies

At the end of this sub- unit students will be able to:

- determine lines of symmetry of given plane figures.
- explain properties of symmetry.

Introduction

This sub-unit deals with introducing the concept' line of symmetry'.
Explanations which enable to understand the lines of symmetry are
given with the aid of diagrams. Activities and examples are
included so as to ensure students' participation.

Teaching Notes

In order to understand the concept 'line of symmetry', the students need to fold isosceles triangles across the line of symmetry and find out the line of symmetry of equilateral triangles and square likewise.

Let students in pairs draw symmetrical figures showing the line of symmetry and find out the properties. Make sure that students can construct simple symmetrical figures and line of symmetry of the given figures.

Regarding of Activity 5.6

You are expected to make students practice the way how to find the line of symmetry by following the steps stated in this activity.

Answers to Exercise 5.H

1. a) True   b) True   c) True   d) True   e) False
2. a) yes   b) No   c) yes   d) yes   e) No
      f) yes   g) yes   h) No   i) No   j) No
      k) yes   l) No

3. Trace each figure. Draw all lines of symmetry.
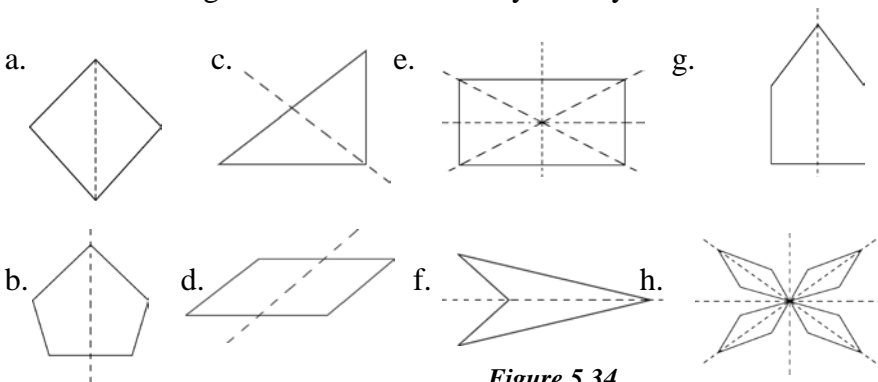
Figure 5.34

4.   a) one   b) three

5. Two

6. The cut out is symmetric along the fold line

7. Infinite (any line through the centre of the circle can be a line of symmetry)

Selected problems to slow learners

1. How many lines of symmetry will you find in
   a) an equilateral triangle?
   b) a square?

2. Is the red line a line of symmetry? Write yes or no.
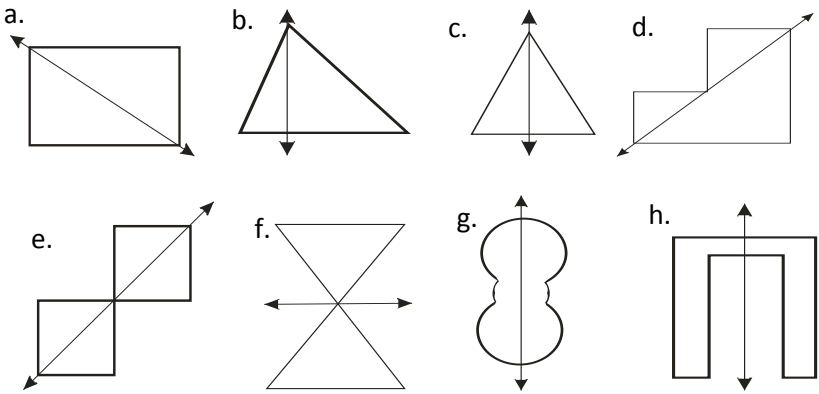
Figure 5.35

Which line is not a line of symmetry?
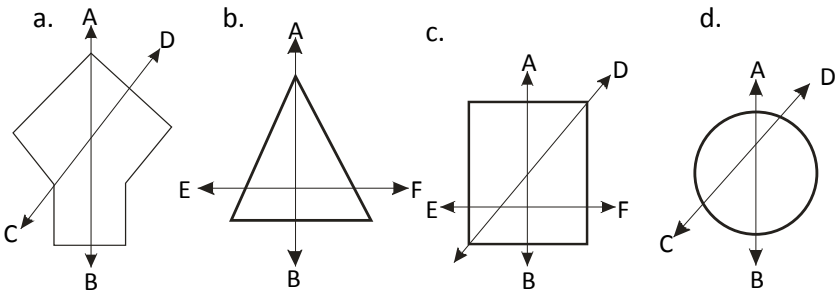
Figure 5.36

Selected problems to fast learners

1. How many lines of symmetry will you find in the following future where the measure of all of its sides are congruent and all of its angles are congruent?
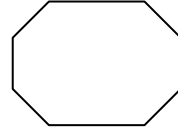
Figure 5.37

2. Copy the figure and draw one lien of symmetry.
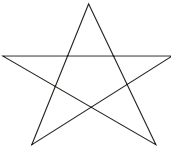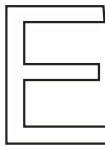
a.　　　　b.　　　　c.　　　　d.
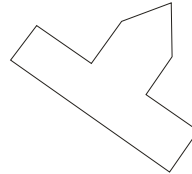
Figure 5.38

3. In each of the following patterns, the red line is the line of symmetry. Copy and complete the drawing so that the figure becomes symmetrical.
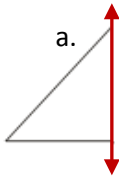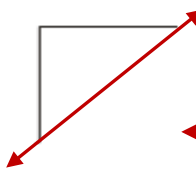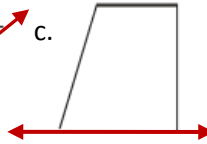
a.　　b.　　c.　　d
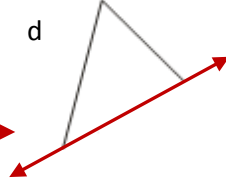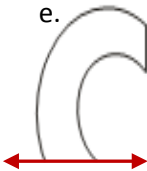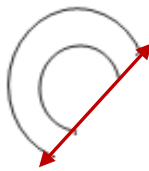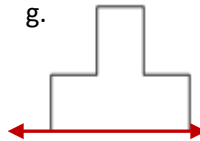
e.　　f.　　g.　　h
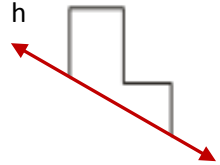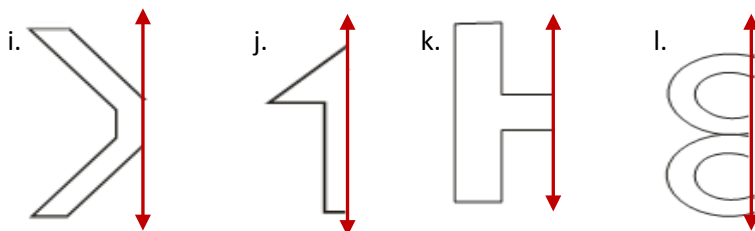
Figure 5.39

5.5 Measurement

Periods allotted: 13 periods

Competencies

At the end of this sub-unit students will be able to:

- calculate the perimeter of squares and rectangles.
- calculate the area of squares and rectangles.
- find the volumes of cubes and rectangular prisms by counting centimeter cubes.
- make nets of cubes and rectangular prisms.
- find the volumes of cubes and rectangular prisms by counting centimeter cubes.

Introduction

This sub-unit is intended to discuss the perimeter and area of squares and rectangles, also the method of finding the volumes of cubes and rectangular prisms. For better understanding of perimeter and area of squares and rectangles, Activity 5.12 and Activity 5.13 are useful.

Activity 5.14 and Activity 5.15 are useful in the discussion of nets of cubes and rectangular prisms, and in determining the volumes of cubes and rectangular prisms. Definition, formulas and diagrammatic explanations are included so as to involve students in the discussion.

Teaching notes

Students should be introduced to the perimeter and areas of rectangles and squares, and solids in everyday life like cubes, cuboids, cylinders, cones and spheres. Let students practice the activities given. You are expected to ensure active participation of students.

5.5.1 The perimeters and areas of squares and rectangles

Introduce the idea of perimeter of squares and rectangles. Let students find perimeter of squares and rectangles by drawing and measuring. Then introduce areas of squares and rectangles by using square centimeters. Lead students to reach to the fact that (i) area of a rectangle = A= length x width and
(ii) Perimeter of a rectangle = P=2 (length + width)

Answers to Activity 5.7

   I)   1) 25   2) 5^2

   II)   1) 24   2) 6x4

Answers to Group Work 5.7

1. P= 1+6+1+6,   P= 2+5+2+5
   P= 14cm   P= 14cm
2. A= $6cm^2$,   A= $10cm^2$
3. Even if two rectangles have equal perimeter they may not have equal area.

Assessment

You may give students a set of problems on finding areas of squares and rectangles by dividing them in to unit squares in the form of class work, homework, assignment, quiz or test. Ask students to verify the result by multiplication in each case.

Answers to Exercise 5.I

1. a) p= 2(l+w) = 2(12+4.5)= 2(16.5) = 33cm

   A= lw= (12cm) (4.5cm) = 54 cm^2

   b) p= 2 (2.5 + 7.4) = 2(9.9) = 19.8cm

   A= (2.5) (7.4) = 18.5 cm^2

   c) p= 2(9+9) =   2(18) = 36cm

   A= (9) (9) = 81 cm^2

d)
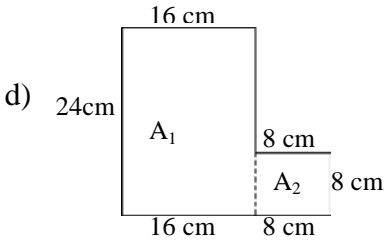
P= 16+8+8+8+16+24=80cm

A= A$_1$ + A$_2$

A= (24x16) + (8x8)

A= 384+64

A= 448cm^2

Figure 5.40

e)
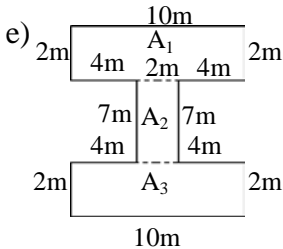
P= 10+2+4+7+4+2+10+2+4+7+4+2

P= 58m

A= A$_1$+ A$_2$ + A$_3$

= (10)(2)+(2)(7)+2(10)

= 20+14+20

A= 54

Figure 5.41

f)
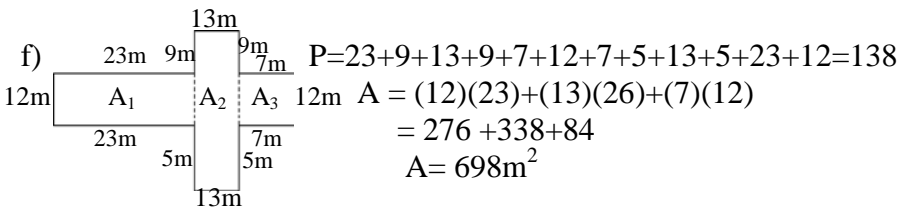
P=23+9+13+9+7+12+7+5+13+5+23+12=138

A = (12)(23)+(13)(26)+(7)(12)

= 276 +338+84

A= 698m^2

Figure 5.42

2. Perimeter of the rectangle = 2(200 +85) = 2(285) = 570m

$$6 \times 570 = 3420m$$

Therefore, the cyclist covered a total of 3,420m

3. i)

12m

1.5m   1.5m

9m

ii)

2m

9m   9m

2m

12m

iii)

6m

3m   6m   3m

9m

12m
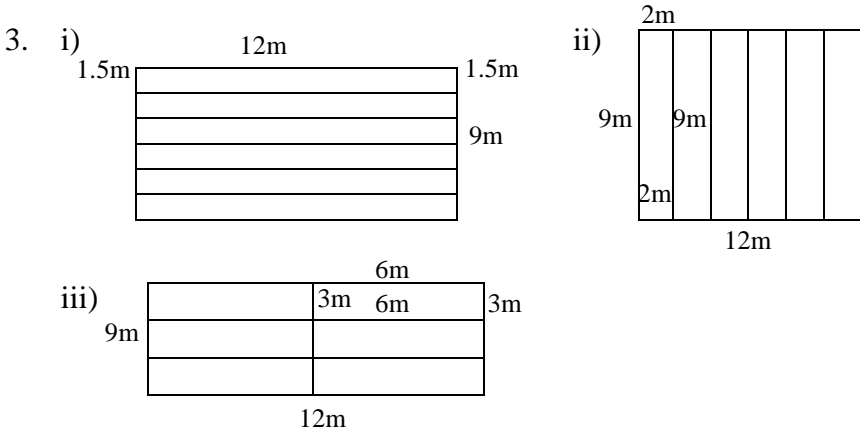
Figure 5.43

Thus, three possible perimeters for one section of the garden can be 27m, 22m and 18m.

4. 16m   x 10m   = 160m^2

   Thus, 160 tiles are needed

5. 100m x 73m = 7300m^2

   Thus, 7300 m^2 of sod will be needed

6. Area of walk = 17 x 30 – 12 x 25

   = 510 – 300

   = 210m^2

5.5.2 Nets of cubes and rectangular prisms

If you remove the surface from the three- dimensional figure and lay it out flat, the pattern you make is called a **net**. Nets allow you to see all the surfaces of a solid at one time. You can use nets to help you find the surface area of a three – dimensional figure. Surface area is the sum of the areas of all of the surfaces of a figure.

You can use nets to write formulas for the surfaces area of prisms. The surface area S of a prism is the sum of the area of faces of the prism for the rectangular prism shown:
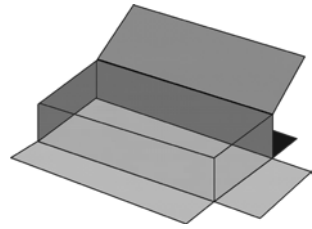
$S = 2\ell w + 2\ell h + 2wh$

Figure 5.44

Let students fold the net in to a figure. Let students tell the solid figure formed.
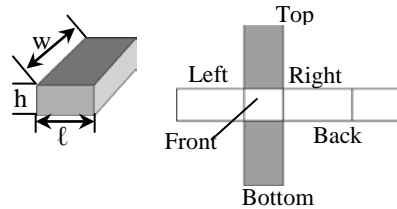
Figure 5.44

Answers to Group Work 5.8

	Dimensions	Area
Front	10 by 5	50 cm^2
Back	10 by 5	50 cm^2
Top	10 by 4	40 cm^2
Bottom	10 by 4	40 cm^2
Left side	10 by 5	20 cm^2
Right side	10 by 5	20 cm^2
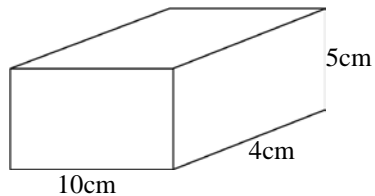
Figure 5.46

Assessment

You can ask students to make nets from different cubes and regular prisms in the form of class work, home work, assignment, quiz or test so as to check their progress. You can also ask students the following problems as additional assessment.

Additional Assessment
Make a net for the rectangular prism shown.

Figure 5.47

Answers to exercise 5.J

1. a.

b.

c.
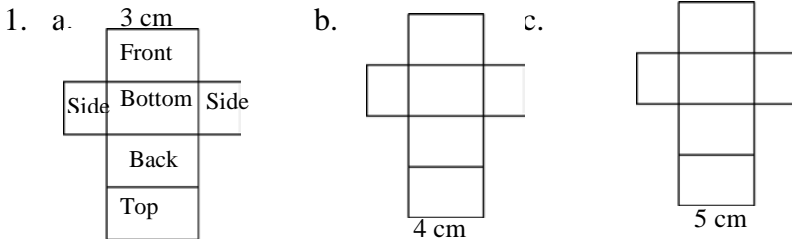
Figure 5.48

For questions 2-5 you are expected to motivate students to make nets as required. Make sure they have understood making nets from different cubes.

5.5.3 The volumes of cubes and Rectangular prisms
Solids such as cubes and cuboids have faces, vertices and edges.

When we examine the cube, we find that it has:

- 6 faces – ABCD is the bottom face.
- 12 edges – AB, BF are edges.
- 8 vertices – A, B, C, D are vertices.
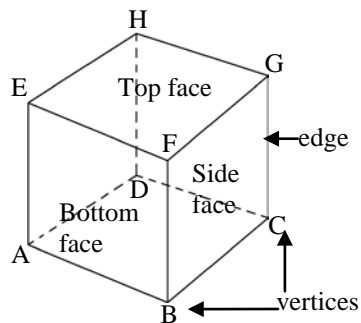
Figure 5.49

We can also see that:

- Opposite sides and faces are parallel.
- Adjacent faces are perpendicular to each other.
- Adjacent faces meet in an edge.

In order to find the volume of a solid figure, we compare it with another solid figure, usually a smaller one. Then we attempt to fill the given solid figure with unit space figures and count how many are required to fill it.
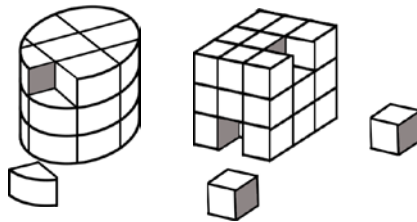
Figure 5.50

Activity 5.8 will help students to clarify the way to find the volume of a solid. Let students exercise finding volumes of cubes and rectangular prisms by filling them with unit cubes.

Answers to Activity 5.8

a) 12   b) 8   c) 11   d) 10   e) 6

Answers to Group Work 5.9

12 cubes will fill her trunk

Assessment

You can ask students different problems on determination of volumes by filling and counting in the form of class work, home work, assignment, quiz or test. You can also ask students to answer the following questions as additional assessment.

Additional Assessment

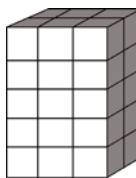Find how many cubes the prism holds

a)
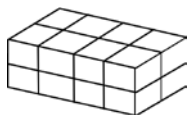
b)
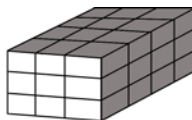
c)

Figure 5.51

Answer to Additional Assessment

   a)   16   b) 36   c) 30

Assessment

As this is the end of the unit you are expected to conduct a summative students level of understanding. If most of the students seem not to have understand a given specific topic retouching this topic by using a different approach is recommended.

Answers to Exercise 5. k

1. a) 4   b) 20
2. a) 12 cubic units   c) 15 cubic units
   b) 16 cubic units   d) 18 cubic units   e) 27 cubic units
3. 48 cubic units   (That is, $4 \times 6 \times 2$)

Selected problems to slow learners

1. Find the perimeter and area of a rectangle with length 3cm and width 2cm.
2. Find the perimeter and area of a square whose sides measure 20cm.
3. Make a net for the following cube.

4cm
4cm
4cm
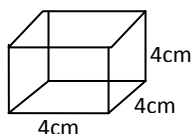
Figure 5.52

4. Find the perimeter of each rectangle.

a)

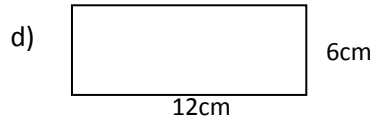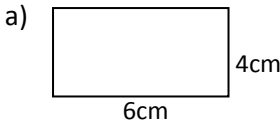4cm
6cm

b)

3cm
9cm

c)

5cm
5cm

d)

6cm
12cm

Figure 5.53

5. Alemu is building a dog pen that is 6metere by 8metre. How much fencing material will he need to go all the way around the pen?

6. If the perimeter of a square is 36cm, then what is the length of one side of the square?

7. If the area of a square is $4cm^2$, then what is its perimeter?
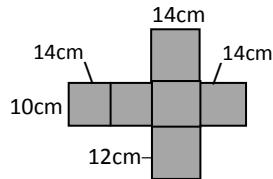
8. Find the surface area of the prism formed by the net shown.

14cm
14cm   14cm
10cm
12cm

Figure 5.54

9. The length of a garden is $12\frac{1}{4}$km and width is $8\frac{1}{2}$km. Find its perimeter.

Selected problems to fast learners

1. Find the perimeter of a square whose area is $36cm^2$.
2. Find the length of a side of a cube if it has a capacity of 64 cubic units.
3. Jundi and Lidya are fencing a rectangular area of the yard for their dog. The width of the dog yard is 4.5m. Its area is $67.5m^2$. What is the length of the dog yard?
4. The area of a rectangle is $84cm^2$. If the length is 5cm longer than the width, what is the length of the rectangle?
5. If the area of a square is $144cm^2$, then what is its perimeter?

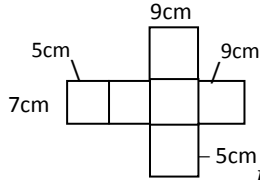6. Find the surface area of the prism formed by the net shown.

5cm
9cm
9cm
7cm
5cm

Figure 5.55
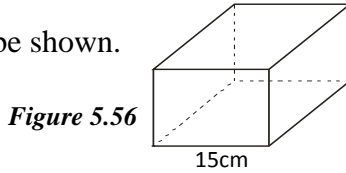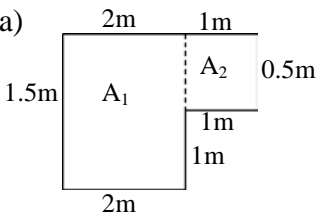
7. Find the volume of the cube shown.

Figure 5.56

15cm

8. The length of a garden is $12\frac{2}{5}$km and width is $8\frac{1}{5}$km. Find its perimeter.

Answers to Review Exercise

1. a) straight   b) obtuse   c) right   d) reflex
2. c
3. a) $150°$ b) $210°$
4. a) parallel   b) parallel   c) perpendicular d) perpendicular
5. b
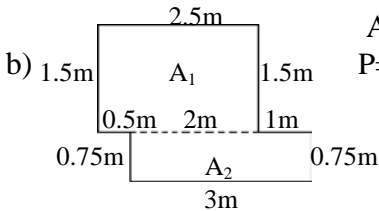6. Check that the lines drawn are lines of symmetry
7. a)

2m   1m
1.5m   A₁   A₂   0.5m
1m
1m
2m

$P = 3+1+0.5+1+1+2+1.5$
$P = 10m$
$A= A1+A2= 2× 1.5 + 1× 0.5$
    $= 2+ ½$
$A= 3.5 \text{ m}^2$

b)

2.5m
1.5m   A₁   1.5m
0.5m   2m   1m
0.75m   A₂   0.75m
3m

$P= 2.5+ 1.5 +1+0.75 +3+ 0.75+ 0.5+ 1.5$
$P= 11.5m$
$A= A1+A2 = 1.5 ×2.5 + 3× 0.75$
    $= 3.75 + 2.25$
$A= 6m^2$

c)
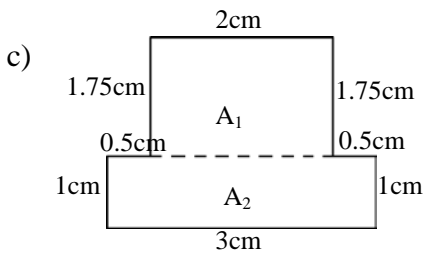
P= 2+1.75+0.5+1+3+1+0.5+1.75

P = 11.5 cm

A= $A_1 + A_2$

= 1.75×2+3×1

= 3.5 +3

= 6.5cm^2

Figure 5.57

8. a) 30   b) 80   c) 48

V= 30 cubic units   v= 80 cubic units   C= 48 cubic units

Bibliography

📖 Aklilu Cherinet, Afework Awlachew, Yalew Zeleke, Samson Bogale (Mathematics Student's text book for grade 6) Addis Ababa Education Bureau for Grade 6, 1997 E.C.

📖 Aseged Atnafu, (Mathematics Supplementary Book for Grade 5) Mega Publishing, 1994E.C

📖 Bennett, Burger, Chard, Kennedy, Renfro, Scheer, Waits (Holt Mathematics, Teacher's Edition, Course 2) 2007, Holt, Rinehart and Winston.

📖 Glencoe (Mathematics, applications and connections) McGraw-Hill, 1998.

📖 Heath (Heath Mathematics, Connections) D.C. Heath and Company, 1992.

📖 Juliet Donker, Daniel Apronti, Michel E.Amppiah, Bryn Roberts, Derek Mc Monayle (Maths Grade 5, Maths Grade 6 Puipl's Book) (Ghana), Linimax Mac Millan Ltd. 2005

📖 Juliet Donker, Daniel Apronti, Michel E.Amppiah Bryn Roberts, Derek Mc Monayle (Maths Grade 5, Maths Grade 6 Teacher's Guide) Ghana,

📖 Kidnemarim Argawi, Gebrie Bekure, Michel Tesfay, Kidan Akalu, (Mathematics Student's textbook for Grade 5) Region 14 Education Bureau, 1987.

📖 Leo Okol, Fred Kiggundu (Oxford Primary Mathematics (Books 5) Oxford University Press Kampala 1999.

📖 Igancio Bello, (Basic College Mathematics, a real world approach) McGraw-Hill, 2006.

📖 Unimax Macmillan Ltd 2005

📖 http://www.aaa math.com

📖 http://www.word math.com

📖 http://www.homepage.mac.com