Quadratic Functions for Projectile Motion

Total Estimated Time

two 45-minute class periods

Learning Goals

Day 1
Students will:

- describe the relationship between initial conditions and projectile motion using appropriate vocabulary such as launch angle, initial speed, and initial height
- describe defining characteristics of the function and defend their reasoning

Day 2

- write quadratic equations in vertex form to model projectile launches
- use any point on the parabola to confirm the equation

Standards

Day 1

- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). (CCSS.8.F.B.5)
- Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: HSF.IF.C.7.a)
- Make observations and/or measurements of an object's motion to provide evidence that a pattern can be used to predict future motion. (NGSS-3-PS2-2)

Day 2

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (CCSS: A-REI.D.10)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8; 2.1.c.vi)
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS.HSF.IF.A.2)

Curriculum Alignment

Sullins Academy Curriculum Framework: Analyze the characteristics and graph quadratic functions (A.EQR.13)

This lesson also corresponds to St Vrain Algebra I Unit 7: *Polynomial Operations, Quadratic Functions, Quadratic Equations and Models.* Textbook resource is HMH Algebra I: Modules 19, 21-23).

Prior Knowledge

This lesson is an application of functions and quadratics. Students should be familiar with vocabulary such as increasing and decreasing, positive and negative, minima and maxima, intercepts, and vertex. They should have also learned about vertex and standard forms of quadratics and the basics of graphing parabolas.

Materials

Device with internet access for each student
Pencil
Projectile Motion PhET Simulation
Student Activity Sheet Day 1
KEY for Student Activity Sheet Day 1
Exit Ticket Day 1
KEY for Exit Ticket Day 1
Student Activity Sheet Day 2
Exit Ticket Day 2

Day 1

Warm-Up (10 min)

Teacher will...	Students will...
• Have the day 1 learning goals on the board or on a slide. Ask a student or students to read the goals for the day aloud. Have students explain terms and meaning of goals. Have them identify terms they don't know and make a guess about what those terms mean.	• Read the learning goals. • Explore the simulation. • Make at least 5-10 trajectories. • Record and/or prepared to share two discoveries and one question or wondering. • Share their discoveries and question/wondering

• Have the warm-up task below written on the board, on a slide, or on Google classroom.
• Encourage students to explore the Projectile Motion PhET simulation.
• Model appropriate vocabulary such as initial height, initial speed, launch angle, trajectory, projectile height, and projectile distance.
• Circulate the Room and ask students:
    ○ What happens when you change the initial height? Initial speed? Launch angle?
    ○ What type of equation do you think would model this graph and why?
• Ask students to briefly share their discoveries and question/wondering, and discuss the questions above.	

The Task

1. Open the Phet Simulation Projectile Motion

Explore the simulation. Be prepared to share at least two discoveries you have made and one question/wondering you have.

Discoveries

1.

2.

Question/wondering:

Guided Exploration and Discussion: Making Target Projectile Paths (5-7 min)

Teacher will...	Students will...
• Distribute activity sheets • Direct students to complete #1-3 on the activity sheet. • Circulate the room to be available for questions and ask probing/pushing questions such as	• Complete #1-3 on the activity sheet. • Respond to teacher questions. • Ask questions or ask for help as needed. • Share observations with partners or whole group as teacher decides is

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- ○ Is there more than one way to hit the target?
- ○ What does a negative launch angle look like? Can you do a negative launch if the starting height is zero?
- ○ Is there more than one way to reach a height of 14 m and still hit the target?
- ○ What forces are at work that cause the projectile to go forward? To go up? To go down?

- As you circulate, make note of students with interesting answers so they can share out later.
- Facilitate a class discussion or partner discussions that address any of the questions above.
    - ○ Use responses from the above questions to seed this discussion.
    - ○ Strategically direct the conversation by calling on students with interesting answers to share their ideas.
- Have students demo their settings if they were able to find a solution that others find challenging.

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beneficial.

- Demo their simulation with classmates if necessary.

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Guided Exploration and Discussion: How Initial Conditions Affect the Trajectory (7-10 min)

Teacher will...	Students will...
• Direct students to complete #4-5 on the activity sheet and then share answers with a partner or partners. • Circulate the room to be available for questions • Facilitate a class discussion that particularly focuses on ○ why changing the object does not change the trajectory.	• Complete #4-5 on the activity sheet. • Ask questions or ask for help as needed. • Share observations with partners and then the whole group. • Demo their simulation with classmates if necessary.

Then turn on air friction and show that it does. Briefly explain gravity and air friction without allowing this to derail the more relevant math conversations. ○ why changing the angle creates a max distance at 45 degrees and decreases before and after that. Use student answers on the activity sheet to create dissonance before revealing the answer. For example, if one student said that increasing the angle increases the distance and another said decreases, have them model their simulations and reasoning and have the group decide who is right. (They both are depending on which angles they used to draw their conclusions.) Have students formalize the relationship between launch angle and distance as a class.	

Guided Exploration and Discussion: Assessing Projectile Motion Using Function Characteristics (10 - 15 min)

Teacher will...	Students will...
● Direct students to complete #6 on the activity sheet. (Ideally, you would allow each student to screenshot their own example and answer the questions for theirs! Feel free to edit that on the activity sheet.) ● Circulate the room to be available for questions and ask additional probing/pushing questions such as ○ What is the difference between where the function is positive	● Complete #6 on the activity sheet. ● Use previous notes on function characteristics to help answer questions. ● Ask questions or ask for help as needed after an attempt has been made to use previous notes. ● Share answers and reasoning with classmates.

and where it is increasing? ○ If you were going to graph this on a coordinate plane, which quadrant(s) would you use and how do you know? ● Facilitate a class discussion that addresses the activity sheet questions and any of the questions above. Ideally, the image used in #6 is projected and students can mark on it in front of the classroom as they explain their answers and reasoning. ● Focus on making sense of the physical scenario using function characteristics. (parts b,c, and g)	

Informal Assessment: Exit Ticket (10 min)

Teacher will...	Students will...
● Distribute the exit ticket. ● Clarify instructions if necessary.	● Complete the exit ticket.

Day 2: Warm-Up (10 min)

Teacher will...	Students will...
● Have the Day 2 learning goals on the board or on a slide. Ask a student or students to read the goals for the day aloud. ● Have the warm-up task below written on the board, on a slide, or on Google classroom. ● Encourage students to explore the Projectile Motion PhET simulation. ● Circulate the room and ask students questions to create discussion: ○ What information would you need to write a Vertex Form equation? ○ How would the Vertex Form equation for this launch look?	● Explore the simulation. ● Make at least 5-10 trajectories. ● Record and/or be prepared to share conjectures and questions about how “a” values in Vertex Form equations might differ based on what launches look like.

○ How might the “a” value in your equations be different for different launches? ● Ask several students to briefly share their conjectures about how “a” values in Vertex Form equations might differ based on what launches look like.	

Warm-Up Task: Open the Phet Simulation Projectile Motion
Create several different launches. Use the crosshairs tool and move it around a projectile’s path. What does the crosshair tool tell you about points on the graph?

Notice the vertex points. Discuss what the Vertex Form equations for these launches will look like. How might the “a” coefficient in each equation change depending on what the launch looks like?

Guided Exploration: Writing Vertex Form Equations (10 min)

Teacher will...	Students will...
● Have the students begin working on #2 - #7. ● Circulate the room to be available for questions and ask probing/pushing questions such as: ○ What information would you need to write a Vertex Form equation? ○ What are some points on your projectile’s path besides the vertex? ○ Are any particular points on the graph more important or useful than others? ○ What do the x and y axis represent in this simulation? ○ Does this seem like a realistic launch? ○ Why did you choose the initial conditions that you did?	● Complete #2 - #7 on the activity sheet. ● Work individually or with a desk partner but utilize table groups for collaboration, comparing, and questions. ● Respond to teachers questions. ● Ask questions or ask for help as needed.

Discussion: (7-10 min)

After all students/groups have created a Vertex Form equation (#7), pause and discuss findings.

Teacher will...	Students will...

• Have several students show their launch simulation noting their initial conditions along with their Vertex Form equation.
• Facilitate a class discussion by identifying students to share about how different launch paths generate different “a” values in equations. Have selected students answer questions such as:
  ○ Which parabolas have “a” coefficients with the greatest absolute value?
  ○ How can you roughly predict the “a” coefficient based on what you see?
  ○ Will the “a” coefficient always be negative?
  ○ What type of launch path might have the smallest “a” value? The biggest “a” value?
  ○ Can you ever create a launch that doesn’t make a parabola?
  ○ Where does the vertex point show up in all these equations?
  ○ Can you determine the initial launch height, initial speed, or launch angle by knowing the equation?
	• Be prepared to share their simulated launch and its equation.
	• Be prepared to share conjectures about how “a” values in Vertex Form equations might differ based on what launches look like.
	• Be prepared to discuss any unexpected findings.
	• Based on class discussion students will answer question #8 on their activity sheet.

Guided Exploration Part Two: (10 min)

This part of the lesson corresponds with questions #9 - #10 on the activity sheet. If time is an issue this part of the lesson can be skipped, used on a subsequent day, or used as enrichment. All students should complete question #11 as it does not depend on questions #9 and #10.

Teacher will...	Students will...
• Have the students complete #9 - #10 on the activity sheet. • Circulate the room to be available for questions and ask probing/pushing	• Complete #9 - #10 on the activity sheet, if time. • Work individually or with a desk partner but utilize table groups for

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questions such as:

What do Standard Form equations tell you about a projectile’s path? How is that different from what you see in a Vertex Form equation?
Which form of a quadratic function equation makes it easier to find key points during the flight of a projectile’s launch?
How will the value of the “a” coefficient compare between Vertex Form and Standard Form?

Have the students complete #11 on the activity sheet. Direct students to answer #11 for their Vertex Form equation even if they ran out of time to change into Standard Form.
Circulate the room to be available for questions and ask probing/pushing questions such as

How can you prove that your equation is accurate?
Is there more than one way to confirm it?
Can you use other tools (Desmos, graphing calculator) to help you?
How might rounding be impacting your answer?

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collaboration, comparing, and questions.

Respond to teachers questions.
Ask questions or ask for help as needed.

Complete #11 on the activity sheet to confirm that the Vertex Form equation they created is accurate (all students should complete this question even if they weren’t able to complete #9 and #10).
Work individually or with a desk partner but utilize table groups for collaboration, comparing, and questions.
Respond to teachers questions.
Ask questions or ask for help as needed.
Be prepared to justify results to others.

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Summary and Informal Assessment: Exit Ticket (10 min)

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Regroup the class and facilitate a brief discussion about ways to confirm the accuracy of an equation.
Have several students share justifications for their equations and take questions from other students.
Distribute the exit ticket.
Clarify instructions if necessary.

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Answer discussion questions and be able to justify the accuracy of their equations to others.

Complete the exit ticket.

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