MATHEMATICS

Student Textbook

Grade 8

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Table of Contents

UNIT 1

SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

Unit outcomes

After completing this unit, you should be able to:

- understand the notion square and square roots and cubes and cube roots.
- determine the square roots of the perfect square numbers.
- extract the approximate square roots of numbers by using the numerical table.
- determine cubes of numbers.
- extract the cube roots of perfect cubes.

Introduction

What you had learnt in the previous grade about multiplication will be used in this unit to describe special products known as squares and cubes of a given numbers. You will also learn what is meant by square roots and cube roots and how to compute them. What you will learn in this unit are basic and very important concepts in mathematics. So get ready and be attentive!

1.1 The Square of a Number

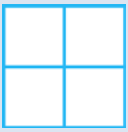
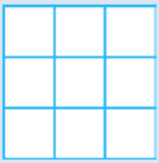
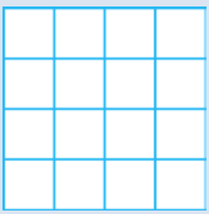
1.1.1 Square of a Rational Number

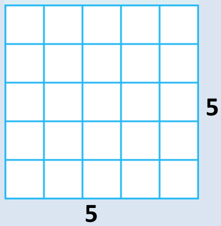
Addition and subtraction are operations of the first kind while multiplication and division are operation of the second kind. Operations of the third kind are **raising to a power** and **extracting roots**. In this unit, you will learn about raising a given number to the power of "2" and power of "3" and extracting square roots and cube roots of some perfect squares and cubes.

Group Work 1.1

Discuss with your friends

1. Complete this Table 1.1. Number of small squares

	Standard Form	Factor Form	Power Form
a) 1  1	1	1×1	1^2
b) 2  2	4	2×2	2^2
c) 3  3	________	________	________
d) 4  4	________	________	________

e)  Figure 1.1	________	________	________

2. Put three different numbers in the circles so that when you add the numbers at the end of each – line you always get a square number.

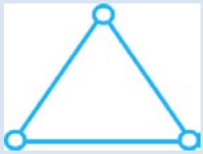


Figure 1.2

3. Put four different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.

Figure 1.3

Definition 1.1: The process of multiplying a rational number by itself is called squaring the number.

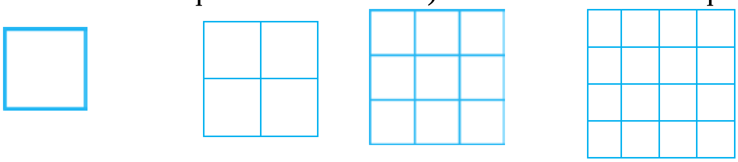
For example some few square numbers are:

a) $1 \times 1 = 1$ is the 1st square number.

b) $2\times 2 = 4$ is the 2nd square number.

c) $3\times 3 = 9$ is the 3rd square number.

d) $4\times4 = 16$ is the 4th square number.



a) b) c) d)

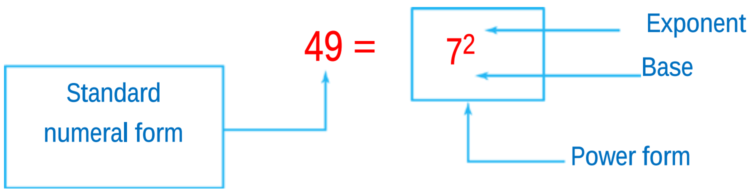
Figure 1.4 A square number can be shown as a pattern of squares

If the number to be multiplied by itself is 'a', then the product (or the result $a \times a$) is usually written as a^2 and is read as:

- ✓ a squared or
- ✓ the square of a or
- ✓ a to the power of 2

In geometry, for example you have studied that the area of a square of side length 'a' is $a \times a$ or briefly a^2.

When the same number is used as a factor for several times, you can use an exponent to show how many times this numbers is taken as a factor or base.



Note: 7^2 is read as

- ✓ 7 squared or
- ✓ the square of 7 or
- ✓ 7 to the power of 2

Example 1: Find the square of each of the following.

a) 8 b) 10 c) 14 d) 19

Solution

a) $8^2 = 8 \times 8 = 64$

b) $10^2 = 10 \times 10 = 100$

c) $14^2 = 14 \times 14 = 196$

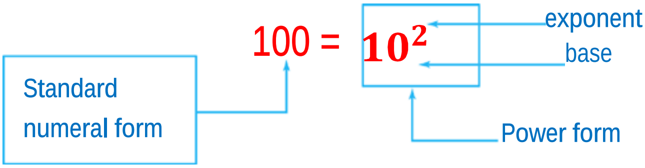
d) $19^2 = 19 \times 19 = 361$

Example2: Identify the base, exponent, power form and standard form of the following expression.

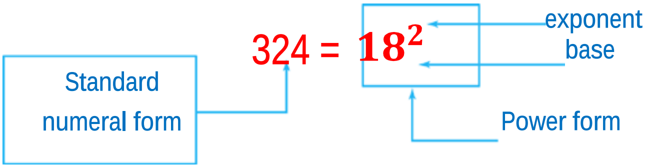
a) 10^2 b) 18^2

Solution

a)



b)



Note: There is a difference between a^2 and $2a$. To see this distinction consider the following examples of comparison.

Example3: a) $30^2 = 30 \times 30 = 900$ while $2 \times 30 = 60$

b) $40^2 = 40 \times 40 = 1600$ while $2 \times 40 = 80$

c) $52^2 = 52 \times 52 = 2704$ while $2 \times 52 = 104$

Hence from the above example; you can generalize that $a^2 = a \times a$ and $2a = a + a$, are quite different expressions.

Definition 1.2: A rational number x is called a perfect square, if and only if $x = n^2$ for some $n \in \mathbb{Q}$.

Example 4: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$. Thus 1, 4, 9, 16 and 25 are perfect squares.

Note: A perfect square is a number that is a product of a rational number times itself and its square root is a rational number.

Example5: In Table 1.2 below some natural numbers are given as values of x. Find x^2 and complete table 1.2.

x	1	2	3	4	5	10	15	20	25	35
x^2										

Solution

When $x = 1$, $x^2 = 1^2 = 1 \times 1 = 1$

When $x = 2$, $x^2 = 2^2 = 2 \times 2 = 4$

When $x = 3$, $x^2 = 3^2 = 3 \times 3 = 9$

When $x = 4$, $x^2 = 4^2 = 4 \times 4 = 16$

When $x = 5$, $x^2 = 5^2 = 5 \times 5 = 25$

When $x = 10$, $x^2 = 10^2 = 10 \times 10 = 100$

When $x = 15$, $x^2 = 15^2 = 15 \times 15 = 225$

When $x = 20$, $x^2 = 20^2 = 20 \times 20 = 400$

When $x = 25$, $x^2 = 25^2 = 25 \times 25 = 625$

When $x = 35$, $x^2 = 35^2 = 35 \times 35 = 1225$

x	1	2	3	4	5	10	15	20	25	35
x^2	1	4	9	16	25	100	225	400	625	1225

You have so far been able to recognize the squares of natural numbers, you also know that multiplication is closed in the set of rational numbers. Hence it is possible to multiply any rational number by itself.

Example 6: Find x^2 in each of the following where x is rational number given as:

a) $x = \frac{4}{3}$ b) $x = \frac{1}{3}$ c) $x = \frac{3}{5}$ d) $x = 0.26$

Solution

a) $x^2 = \left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9}$

b) $x^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

c) $x^2 = \left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{3 \times 3}{5 \times 5} = \frac{9}{25}$

d) $x^2 = (0.26)^2 = \left(\frac{26}{100}\right)^2 = \frac{26}{100} \times \frac{26}{100} = \frac{26 \times 26}{100 \times 100} = \frac{676}{10{,}000}$

Note:

i. The squares of natural numbers are also natural numbers.

ii. $0 \times 0 = 0$ therefore $0^2 = 0$

iii. We give no meaning to the symbol 0^0

iv. If $a \in \mathbb{Q}$ and $a \neq 0$, then $a^0 = 1$

v. For any rational number 'a', $a \times a$ is denoted by a^2 and read as "a squared" or "a to the power of 2" or "the square of a".

Exercise 1A

1. Determine whether each of the following statements is true or false.

a) $15^2 = 15 \times 15$ d) $81^2 = 2 \times 81$ g) $x^2 = 2^x$

b) $20^2 = 20 \times 20$ e) $41 \times 41 = 41^2$ h) $x^2 = 2^{2x}$

c) $19^2 = 19 \times 19$ f) $-(50)^2 = 2500$ i) $(-60)^2 = 3600$

2. Complete the following.

a) $12 \times$ _____ $= 144$ d) $(3a)^2 =$ ___ $\times$ ___

b) $51 \times$ _____ $= 2601$ e) $8a =$ ___ $+$ ____

c) $60^2 =$ ____ $\times$ ____ f) $28 \times 28 =$ ______

3. Find the square of each of the following.

a) 8 b) 12 c) 19 d) 51 e) 63 f) 100

4. Find x^2 in each of the following.

a) $x = 6$ c) $x = -0.3$ e) $x = \frac{-50}{3}$ g) $x = 0.07$

b) $x = \frac{1}{6}$ d) $x = -20$ f) $x = 56$

5. a. write down a table of square numbers from the first to the tenth.

b. Find two square numbers which add to give a square number.

6. Explain whether:

a. 441 is a square number. c. 1007 is a square number.

b. 2001 is a square number.

Challenge Problems

7. Find
   a) The 8^{th} square number.
   b) The 12^{th} square number.
   c) The first 12 square numbers.
8. From the list given below indicate all numbers that are perfect squares.
   a) 50 20 64 30 1 80 8 49 9
   b) 10 21 57 4 60 125 7 27 48 16 25 90
   c) 137 150 75 110 50 625 64 81 144
   d) 90 180 216 100 81 75 140 169 125
9. Show that the difference between any two consecutive square numbers is an odd number.
10. Show that the difference between the 7^{th} square number and the 4^{th} square number is a multiple of 3.

Theorem1.1: Existence theorem
For each rational number x, there is a rational number y ($y \geq 0$) such that $x^2 = y$.

Example 7: By the existence theorem, if

a) $x = 9$, then $y = 9^2 = 81$

b) $x = 0.5$, then $y = (0.5)^2 = 0.25$

c) $x = -17$, then $y = (-17)^2 = 289$

d) $x = \frac{7}{11}$, then $y = \left(\frac{7}{11}\right)^2 = \frac{49}{121}$

Rough calculation could be carried out for approximating and checking the results in squaring rational numbers. Such an approximation depends on rounding off decimal numbers as it will be seen from the following examples.

Examples 8: Find the approximate values of x^2 in each of the following:

a) $x = 3.4$

b) $x = 9.7$

c) $x = 0.026$

Solution

a) $3.4 \approx 3$ thus $(3.4)^2 \approx 3^2 = 9$

b) $9.7 \approx 10$ thus $(9.7)^2 \approx 10^2 = 100$

c) $0.026 \approx 0.03$ thus $(0.026)^2 \approx \left(\frac{3}{100}\right)^2 = 0.0009$

Exercise 1B

1. Determine whether each of the following statements is true or false.
   a) $(4.2)^2$ is between 16 and 25
   b) $0^2 = 2$
   c) $11^2 > (11.012)^2 > 12^2$
   d) $(9.9)^2 = 100$
   e) $(-13)^2 = -169$
   f) $81 \times 27 = 9^2 \times 9 \times 3$

2. Find the approximate values of x^2 in each of the following.
   a) $x = 3.2$
   b) $x = 9.8$
   c) $x = -12.1$
   d) $x = 2.95$
   e) $x = 0.086$
   f) $x = 8.80$

3. Find the square of the following numbers and check your answers by rough calculation.
   a) 0.87
   b) 16.45
   c) 12.12
   d) 42. 05
   e) 25. 14
   f) 28. 23
   g) 38.9
   h) 54. 88

1.1.2 Use of Table of Values of Squares

Activity 1.1

Discuss with your friends / partners/

Use table of square to find x^2 in each of the following.

a) $x = 1.08$
b) $x = 2.26$
c) $x = 9.99$
d) $x = 1.56$
e) $x = 5.48$
f) $x = 7.56$

- ✓ To find the square of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work a table of squares is prepared and presented in the "Numerical tables" at the end of this book.
- ✓ In this table the first column headed by x lists numbers starting from 1.0. The remaining columns are headed respectively by the digits 0 to 9.

Now if you want to determine the square of a number for example 2.54 proceed as follows.

Step i. Under the column headed by x, find the row with 2.5.

Step ii. Move to the right along the row until you get the column under 4, (or find the column headed by 4).

Step iii. Then read the number at the intersection of the row in (i) and the column (ii), (see the illustration below).

Hence $(2.54)^2 = 6.452$

X	0	1	2	3	4	5	6	7	8	9
1.0										
⋮										
2.0										
2.5					6.452					
3.0										
⋮										
4.0										
⋮										
5.0										
⋮										
6.0										
⋮										
7.0										
⋮										
8.0										
9.0										

Figure 1.5 Tables of squares

Note that the steps (i) to (iii) are often shortened by saying "2.5 under 4".

- ✓ Mostly the values obtained from the table of squares are only approximate values which of course serves almost for all practical purposes.

Group work 1.2

Discuss with your group.

Find the square of the number 8.95

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.
d) compare your answer from "a" to "c".
e) write your generalization.

Example9: Find the square of the number 4.95.

Solution: Do rough calculation and compare your answer with the value obtained from the table.

i. Rough calculation

$4.95 \approx 5$ and $5^2 = 25$

$(4.95)^2 \approx 25$

ii. Value obtained from the table

i) Find the row which starts with 4.9.
ii) Find the column headed by 5.
iii) Read the number, that is $(4.95)^2$ at the intersection of the row in (i) and the column in (ii) ; $(4.95)^2 = 24.50$.

iii. Exact Value

Multiply 4.95 by 4.95

$4.95 \times 4.95 = 24.5025$

Therefore $(4.95)^2 = 24.5025$.

This example shows that the result obtained from the "Numerical table" is an approximation and more closer to the exact value.

Exercise 1C

1. Determine whether each of the following statements is true or false.

a) $(2.3)^2 = 5.429$ c) $(3.56)^2 = 30.91$ e) $(5.67)^2 = 32$

b) $(9.1)^2 = 973.2$ d) $(9.90)^2 = 98.01$ f) $(4.36)^2 = 16.2$

2. Find the squares of the following numbers from the table.

a) 4. 85 c) 88.2 e) 2. 60 g) 498 i) 165

b) 6.46 d) 29. 0 f) $\frac{3}{2}$ h) 246

1.2 The Square Root of a Rational Number

Group Work 1.3

Discuss with your Friends

Find the square root of each of the following numbers.

a) 81 **b)** 324 **c)** $\frac{1}{4}$ **d)** $\frac{64}{49}$ **e)** $\frac{4}{49}$ **f)** $\frac{25}{625}$

✓ So far you have studied the meaning of x^2 when x is a rational number. It is now logical to ask, whether you can go in the reverse (or in the opposite direction) or not. In this sub unit you will answer this and related questions in a more systematized manner.

Definition 1.3: Square roots

For any two rational numbers a and b if $a^2 = b$, then a is called the square root of b.

Example 10:

a) 4 is the square root of 16, since $4^2 = 16$.

b) 5 is the square root of 25, since $5^2 = 25$.

c) 6 is the square root of 36, since $6^2 = 36$.

Example 11: The area of a square is 49m^2. What is the length of each side?

Solution:

$\ell \times w = A$

$s \times s = 49 \text{ m}^2$

$s^2 = 49 \text{ m}^2$

$s = 7\text{m}$

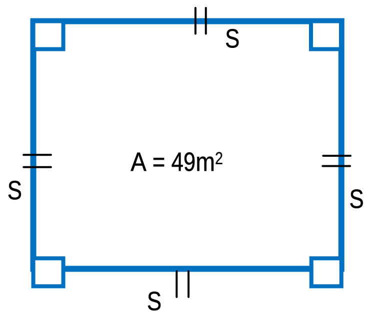


Figure 1.6

The length of each side is 7 meters. This is one way to express the mathematical relationship "7 is the square root of 49" because $7^2 = 49$.

Note:

i. The notion "*square root*" is the inverse of the notion "*square of a number*".

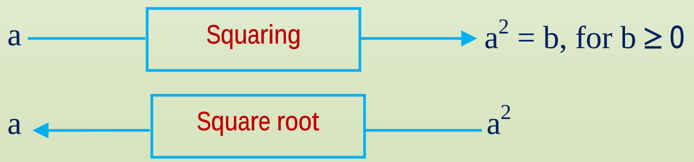
ii. The operation "*extracting square root*" is the inverse of the operation "*squaring*".

iii. In extracting square roots of rational numbers, first decompose the number into a product consisting of two equal factors and take one of the equal factors as the square root of the given number.

iv. The symbol or notation for square root is "$\sqrt{\ }$" it is called radical sign.

v. For $b \geq 0$, the expression $\sqrt{b}$ is called radical b and the number b is called a radicand.

vi. The relation of squaring and square root can be expressed as follows:



vii. a is the square root of b and written as $a = \sqrt{b}$.

Example 12: Find the square root of x, if x is:

a) 100 b) 125 c) 169 d) 256 e) 625 f) 1600

Solution

a) $x = 100 = 10 \times 10$

$x = 10^2$, thus the square root of 100 is 10.

b) $x = 225 = 15 \times 15$

$x = 15^2$, thus the square root of 225 is 15.

c) $x = 169 = 13 \times 13$

$x = 13^2$, thus the square root of 169 is 13.

d) $x = 256 = 16 \times 16$

$x = 16^2$, thus the square root of 256 is 16.

e) $x = 625 = 25 \times 25$

$x = 25^2$, thus the square root of 625 is 25.

f) $x = 1600 = 40 \times 40$

$x = 40^2$, thus the square root of 1600 is 40.

Exercise 1D

1. Determine whether each of the following statements is true or false.

a) $\sqrt{0} = 0$

b) $\sqrt{25} = \pm 5$

c) $\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

d) $-\sqrt{121} = -11$

e) $-\sqrt{\frac{36}{324}} = \frac{1}{3}$

f) $\sqrt{\frac{324}{625}} = \frac{18}{25}$

g) $-\sqrt{\frac{900}{961}} = -\frac{30}{31}$

2. Find the square root of each of the following numbers.

a) 121 c) 289 e) 400 g) 484

b) 144 d) 361 f) 441 h) 529

3. Evaluate each of the following.

a) $\sqrt{\frac{1}{25}}$

b) $\sqrt{\frac{1}{81}}$

c) $-\sqrt{\frac{36}{144}}$

d) $-\sqrt{576}$

e) $\frac{\sqrt{529}}{\sqrt{625}}$

f) $-\sqrt{676}$

g) $\sqrt{729}$

h) $-\sqrt{784}$

i) $\sqrt{\frac{16}{25}}$

Challenge Problems

4. If $\frac{x}{y} = -2$. Find $\sqrt{\frac{x^2}{y^2} + \frac{y^2}{x^2}}$

5. Simplify: $\sqrt{(81)^2} + \sqrt{(49)^2}$

6. If x = 16 and y = 625. Find $\left(2\sqrt{x+y}\right)^2$.

Definition 1.4 : If a number y ≥ 0 is the square of a positive number x (x ≥ 0), then the number x is called the square root of y. This can be written as $x = \sqrt{y}$.

Example13: Find

a) $\sqrt{0.01}$ c) $\sqrt{0.81}$ e) $\sqrt{0.7921}$ g) $\sqrt{48.8601}$

b) $\sqrt{0.25}$ d) $\sqrt{0.6889}$ f) $\sqrt{0.9025}$

Solution

a) $\sqrt{0.01} = \sqrt{0.1 \times 0.1} = 0.1$

b) $\sqrt{0.25} = \sqrt{0.5 \times 0.5} = 0.5$

c) $\sqrt{0.81} = \sqrt{0.9 \times 0.9} = 0.9$

d) $\sqrt{0.6889} = \sqrt{0.83 \times 0.83} = 0.83$

e) $\sqrt{0.7921} = \sqrt{0.89 \times 0.89} = 0.89$

f) $\sqrt{0.9025} = \sqrt{0.95 \times 0.95} = 0.95$

g) $\sqrt{48.8601} = \sqrt{6.99 \times 6.99} = 6.99$

Exercise 1E

Simplify the square roots.

a) $\sqrt{35.88}$ c) $\sqrt{89.87}$ e) $\sqrt{62.25}$

b) $\sqrt{36.46}$ d) $\sqrt{99.80}$ f) $\sqrt{97.81}$

1.2.1 Square Roots of Perfect Squares

Group work 1.4

Discuss with your group.

1. **Find the prime factorization of the following numbers by using the factor trees.**

a) 64 c) 121 e) 324 g) 625 i) 700

b) 81 d) 289 f) 400 h) 676

Note: The following properties of squares are important:

$(ab)^2 = a^2 \times b^2$ and $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$ (where $b \neq 0$).

Thus $(2 \times 3)^2 = 2^2 \times 3^2 = 36$ and $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$.

Remember a number is called a perfect square, if it is the square of a rational number.

The following properties are useful to simplify square roots of numbers.

Properties of Square roots, for $a \geq 0$, $b \geq 0$.

If $\sqrt{a}$ and $\sqrt{b}$ represent rational numbers, then

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } b \neq 0.$$

Example: 14 Determine whether each of the following numbers is a perfect square or not.

a) 36 c) 81 e) $\frac{16}{625}$ g) 11

b) 49 d) $\frac{49}{25}$ f) 7

Solution: a) 36 is a perfect square, because $36 = 6^2$.

b) 49 is a perfect square, because $49 = 7^2$.

c) 81 is a perfect square, because $81 = 9^2$.

d) $\frac{49}{25}$ is a perfect square, becaus $\frac{49}{25} = \left(\frac{7}{5}\right)^2$.

e) $\frac{16}{625}$ is a perfect square, because $\frac{16}{625} = \left(\frac{4}{25}\right)^2$.

f) 7 is not a perfect square since there is no rational number whose square is equal to 7. In other words there is no rational number n such that $n^2 = 7$.

g) 11 is not a perfect square since there is no rational number whose square is equal to 11. In short there is no rational number n such that $n^2 = 11$.

Example 15: Use prime factorization and find the square root of each of the following numbers.

a) $\sqrt{324}$ b) $\sqrt{400}$ c) $\sqrt{484}$

Solution: a) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

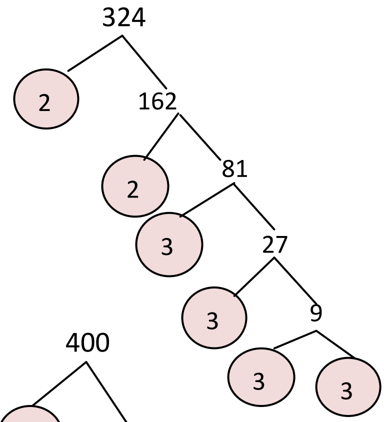
Now arrange the factors so that 324 is a product of two identical sets of prime factors.

i.e $324 = (2 \times 2 \times 3\times3 \times 3\times3)$

$= (2 \times 3 \times 3) \times (2 \times 3 \times 3)$

$= 18 \times 18 = 18^2$

So, $\sqrt{324} = \sqrt{18\times18} = 18$



b) $400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

Now arrange the factors so that 400 is a product of two identical sets of prime factors.

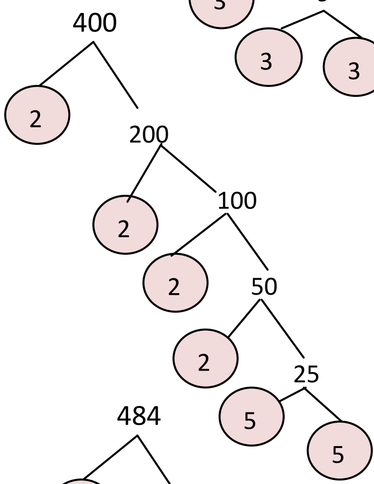
i.e $400 = (2\times 2 \times 5)\times(2 \times 2\times5)$

$= 20 \times 20$

$= 20^2$

So $\sqrt{400} = \sqrt{20\times20}$

$= 20$



c) $484 = 2\times2\times 11\times11$, now arrange the factors so that 484 is a product of two identical sets of prime factors.

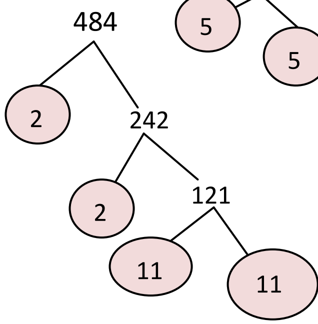
i.e $484 = 2\times2 \times 11\times11$

$= (2 \times 11) \times (2 \times 11)$

$= 22 \times 22 = 22^2$

So $\sqrt{484} = \sqrt{22\times22}$

$= 22$



Exercise 1F

1. Determine whether each of the following statements is true or false.

a) $\sqrt{64\times25} = \sqrt{64}\times\sqrt{25}$

b) $\sqrt{\frac{64}{4}} = 4$

c) $\sqrt{\frac{32}{64}} = \frac{\sqrt{32}}{\sqrt{64}}$

d) $\sqrt{\frac{0}{1296}} = 0$

e) $\sqrt{\frac{1296}{0}} = 1$

f) $\sqrt{\frac{729}{1444}} = \frac{27}{38}$

2. Evaluate each of the following.

a) $\sqrt{0.25}$ c) $\sqrt{\frac{1296}{1024}}$ e) $\sqrt{\frac{81}{324}}$

b) $\sqrt{0.0625}$ d) $\sqrt{\frac{625}{1024}}$ f) $\sqrt{\frac{144}{400}}$

Challenge Problem

3. Simplify a) $\sqrt{625-0}-\sqrt{172-3}$

b) $\sqrt{81\times 625}$

c) $\sqrt{\left(\frac{1}{64}\right)^2}$

4. Does every number have two square roots? Explain.
5. Which of the following are perfect squares?
{0, 1, 4, 7, 12, 16, 25, 30, 36, 42, 49}
6. Which of the following are perfect squares?
{50, 64, 72, 81, 95, 100, 121, 140, 144, 169}
7. Copy and complete.

a) $3^2 + 4^2 + 12^2 = 13^2$ c) $6^2 + 7^2 +$ ______ = ______

b) $5^2 + 6^2 +$ ____ = ______ d) $x^2 + (x + 1)^2 +$ ____ = ____

Using the square root table

The same table which you can use to determine squares of numbers can be used to find the approximate square roots, of numbers.

Example 16: Find $\sqrt{17.89}$ from the numerical table.

Solution:

Step i. Find the number 17.89 in the body of the table for the function $y = x^2$.

Step ii. On the row containing this number move to the left and read 4.2 under x.

These are the first two digits of the square root of 17. 89

Step iii. To get the third digit start from 17.89 move vertically up ward and read 3.

Therefore $\sqrt{17.89} \approx 4.23$

X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
				2nd						
3.0										
4.0										
4.2		1st		17.89						
5.0										
6.0										
7.0										
8.0										
9.9										

Figure 1.7 Table of square roots

If the radicand is not found in the body of the table, you can consider the number which is closer to it.

Example 17: Find $\sqrt{10.59}$

Solution: i) It is not possible to find the number 10. 59 directly in the table of squares. But in this case find two numbers in the table which are closer to it, one from left (i.e. 10.56) and one form right (10.63) that means 10.56 < 10. 59 < 10.63.

ii) Find the nearest number to (10.59) form those two numbers. So the nearest number is 10.56 thus $\sqrt{10.59} \approx \sqrt{10.56} = 3.25$.

Example 18: Find $\sqrt{83.60}$

Solution:

i. It is not possible to find the number 83. 60 directly in the table of squares. But find two numbers which are closer to it, one from left (i.e. 83.54) and one from right (i.e 83.72) that means $83.54 < 83.60 < 83.72$.

ii. Find the nearest number from these two numbers. Therefore the nearest number is 83. 54, so $\sqrt{83.60} \approx \sqrt{83.54} = 9.14$.

> **Note:** To find the square root of a number greater than 100 you can use the method illustrated by the following example.

Example 19: Find the square root of each of the following.

a) $\sqrt{6496}$ b) $\sqrt{9801}$ c) $\sqrt{9880}$ d) $\sqrt{9506}$

Solution:

a) $\sqrt{6496} = \sqrt{64.96 \times 100}$

$= \sqrt{64.96} \times \sqrt{100}$

$= 8.06 \times 10$

$= 80.6$

c) $\sqrt{9880} = \sqrt{98.80 \times 100}$

$= \sqrt{98.80} \times \sqrt{100}$

$= 9.94 \times 10$

$= 99.4$

b) $\sqrt{9801} = \sqrt{98.01 \times 100}$

$= \sqrt{98.01} \times \sqrt{100}$

$= 9.90 \times 10$

$= 99$

d) $\sqrt{9506} = \sqrt{95.06 \times 100}$

$= \sqrt{95.06} \times \sqrt{100}$

$= 9.75 \times 10$

$= 97.5$

Example 20: Find the square root of each of the following numbers by using the table

a) $\sqrt{98.41}$ c) $\sqrt{984100}$ e) $\sqrt{0.009841}$

b) $\sqrt{9841}$ d) $\sqrt{0.9841}$ f) $\sqrt{0.00009841}$

Solution:

a) $\sqrt{98.41} = 9.92$

b) $\sqrt{9841} = \sqrt{98.41 \times 100}$

$= \sqrt{98.41} \times \sqrt{100}$

$= 9.92 \times 10$

$= 99.2$

c) $\sqrt{984100} = \sqrt{98.41 \times 10000}$

$= \sqrt{98.41} \times \sqrt{10000}$

$= 9.92 \times 100$

$= 992$

d) $\sqrt{0.9841} = \sqrt{98.41 \times \frac{1}{100}}$

$= \sqrt{98.41} \times \sqrt{\frac{1}{100}}$

$= 9.92 \times \frac{1}{10}$

$= 0.992$

e) $\sqrt{0.009841} = \sqrt{98.41 \times \frac{1}{10,000}}$

$= \sqrt{98.41} \times \sqrt{\frac{1}{10,000}}$

$= 9.92 \times \frac{1}{100}$

$= 0.0992$

f) $\sqrt{0.00009841} = \sqrt{98.41} \times \sqrt{\frac{1}{1,000,000}}$

$= 9.92 \times \frac{1}{1,000}$

$= 0.00992$

Exercise 1G

1. Find the square root of each of the following numbers from the table.

a) 15.37	d) 153.1	g) 997	j) 5494
b) 40.70	e) 162.8	h) 6034	k) 5295
c) 121.3	f) 163.7	i) 6076	l) 3874

2. Use the table of squares to find approximate value of each of the following.

a) $\sqrt{6.553}$ c) $\sqrt{24.56}$

b) $\sqrt{8.761}$ d) $\sqrt{29.78}$

1.3 Cubes and Cube Roots

1.3.1 Cube of a Number

If the number to be cubed is 'a', then the product $a \times a \times a$ which is usually written as a^3 and is read as 'a' cubed. For example 3 cubed gives 27 because $3 \times 3 \times 3 = 27$.

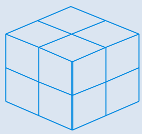
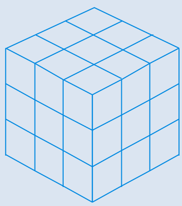
The product $3\times3\times3$ can be written as 3^3 and which is read as 3 cubed.

Activity 1.2

Discuss with your friends

1. Copy and complete this Table 1.3

   Number of small cubes

	Standard form	Factor form	Power form
a) 	1	$1 \times 1 \times 1$	1^3
b) 	8	$2 \times$ ___ $\times$ ___	2^3
c)  Figure 1.8	27	___ $\times$ ___ $\times$ ___	$\square^{\square}$

2. a) Which of these numbers are cubic numbers?

64	100	125	216	500	1000
1728	3150	4096	8000	8820	15625

   b) Write the cubic numbers from part (a) in power form.

3. Find a^3 in each of the following.

   a) $a = 2$ c) $a = 10$ e) $a = 0.5$

   b) $a = -2$ d) $a = \frac{1}{4}$ f) $a = 0.25$

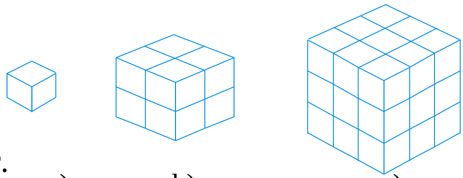
Definition 1.5: A cube number is the result of multiplying a rational number by itself, then multiplying by the number again.

For example, some few cube numbers are:

a) $1 \times 1 \times 1 = 1$ is the 1^{st} cube number.

b) $2 \times 2 \times 2 = 8$ is the 2^{nd} cube number.

c) $3 \times 3 \times 3 = 27$ is the 3^{rd} cube number.



a) b) c)

Figure 1.9 A cube number can be shown as a pattern of cubes

Example 21: Find the numbers whose cube are the following.

a) 4,913 b) 6,859 c) 9,261 d) 29,791

Solution:

a) $4{,}913 = 17 \times 17 \times 17 = 17^3$

b) $6{,}859 = 19 \times 19 \times 19 = 19^3$

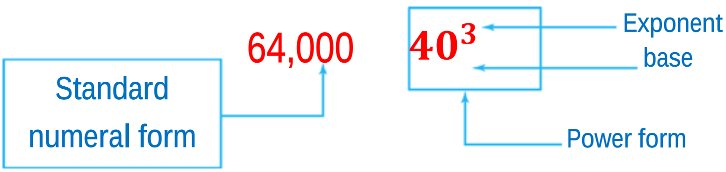
c) $9{,}261 = 21 \times 21 \times 21 = 21^3$

d) $29{,}791 = 31 \times 31 \times 31 = 31^3$

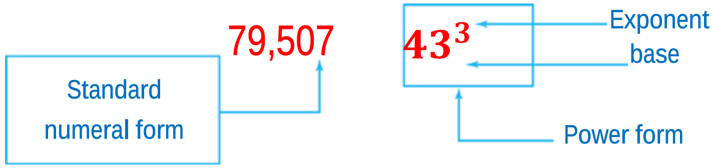
Example 22: Identify the base, exponent, power form and standard numeral form:

a) 40^3 b) 43^3

Solution: a)



b)



Example 23: In Table1.4 below integers are as values of x, find x^3 and complete the table 1.4.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3											

Solution:

When $x = -4$, $x^3 = (-4)^3 = -4 \times -4 \times -4 = -64$
When $x = -3$, $x^3 = (-3)^3 = -3 \times -3 \times -3 = -27$
When $x = -2$, $x^3 = (-2)^3 = -2 \times -2 \times -2 = -8$
When $x = -1$, $x^3 = (-1)^3 = -1 \times -1 \times -1 = -1$
When $x = 0$, $x^3 = 0^3 = 0 \times 0 \times 0 = 0$
When $x = 1$, $x^3 = 1^3 = 1 \times 1 \times 1 = 1$
When $x = 2$, $x^3 = 2^3 = 2 \times 2 \times 2 = 8$
When $x = 3$, $x^3 = 3^3 = 3 \times 3 \times 3 = 27$
When $x = 4$, $x^3 = 4^3 = 4 \times 4 \times 4 = 64$
When $x = 5$, $x^3 = 5^3 = 5 \times 5 \times 5 = 125$
When $x = 6$, $x^3 = 6^3 = 6 \times 6 \times 6 = 216$

Lastly you have:

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3	-64	-27	-8	-1	0	1	8	27	64	125	216

The examples above illustrate the following theorem. This theorem is called **existence theorem.**

Theorem 1.2: Existence theorem

For each rational number x, there is a rational number y such that $y = x^3$.

Rough calculations could be used for approximating and checking the results in cubing rational numbers. The following examples illustrate the situation.

Example 24: Find the approximate values of x^3 in each of the following.

a) $x = 2.2$ b) $x = 0.065$ c) $x = 9.54$

Solution:

a. $2.2 \approx 2$ thus $(2.2)^3 \approx 2^3 = 8$

b. $0.065 \approx 0.07$ thus $(0.065)^3 \approx \left(\frac{7}{100}\right)^3 = \frac{343}{1,000,000}$
$= 0.000343$

c. $9.54 \approx 10$ thus $(9.54)^3 \approx 10^3 = 1,000$

Exercise 1H

1. Determine whether each of the following statements is true or false.

a) $4^3 = 16 \times 4$ c) $(-3)^3 = 27$ e) $\left(\frac{4}{3}\right)^3 = \frac{64}{125}$

b) $4^3 = 64$ d) $\left(\frac{3}{4}\right)^3 = \frac{27}{16}$ f) $\sqrt[3]{64} = 4$

2. Find x^3 in each of the following.

a) $x = 8$ c) $x = -4$ e) $x = \frac{-1}{5}$

b) $x = 0.4$ d) $x = -\frac{1}{4}$ f) $x = -0.2$

3. Find the approximate values of x^3 in each of the following.

a) $x = -2.49$ c) $x = 2.98$

b) $x = 2.29$ d) $x = 0.025$

Challenge Problem

4. The dimensions of a cuboid are 4xcm, 6xcm and 10xcm. Find

a) The total surface area

b) The volume

Figure 1.10 Cuboid

Table of Cubes

Activity 1.3

Discuss with your friends

Use the table of cubes to find the cubes of each of the following.

a) 2.26	c) 5.99	e) 8.86	g) 9.58	i) 9.99
b) 5.12	d) 8.48	f) 9.48	h) 9.89	j) 9.10

To find the cubes of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work, a table of cubes is prepared and presented in the "Numerical Tables" at the end of this textbook.

In this table the first column headed by ‘x’ lists numbers starting from 1.0. The remaining columns are headed respectively by the digit 0 to 9.0. Now if we want to determine the cube of a number, for example 1.95 Proceed as follows.

Step i. Find the row which starts with 1.9 (or under the column headed by x).

Step ii. Move to the right until you get the number under column 5 (or find the column headed by 5).

Step iii. Then read the number at the intersection of the row in step (i) and the column step (ii) therefore we find that $(1.95)^3 = 7.415$. See the illustration below.

X	0	1	2	3	4	5	6	7	8	9
1.0										
1.9						7.415				
2.0										
3.0										
4.0										
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.11 Tables of cubes

Note that the steps (i) to (iii) are often shortened saying “1.9 under 5”

Mostly the values obtained from the table of cubes are only a approximate values which of course serves almost for all practical purposes.

Group work 1.5

Find the cube of the number 7.89.

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.

d) compare your answer from “a” to “c”.

e) write your generalization.

Example 25: Find the cube of the number 6.95.

Solution:

Do rough calculation and compare your answer with the value obtained from the table.

i. Rough Calculation

$6.95 \approx 7$ and $7^3 = 343$

$(6.95)^3 \approx 343$

ii. Value Obtained from the Table

Step i. Find the row which starts with 6.9

Step ii. Find the column head by 5

Step iii. Read the number, that is the intersection of the row in (i) and the column (ii), therefore $(6.95)^3 = 335.75$

iii. Exact Value

Multiply $6.95 \times 6.95 \times 6.95 = 335.702375$

so $(6.95)^3 = 335.702375$

This examples shows that the result obtained from the numerical tables is an approximation and more closer to the exact value.

Exercise 11

1. Use the table of cubes to find the cube of each of the following.

a) 3.55	c) 6.58	e) 7.02	g) 9.86	i) 9.90	k) 9.97
b) 4.86	d) 6.95	f) 8.86	h) 9.88	j) 9.94	l) 9.99

1.3.2 Cube Root of a Number

Group work 1.6

Discuss with your group.

1. **In Figure 1.12 to the right, the volume of a cube is 64 m^3. What is the length of each edge?**
2. **Can you define a "cube root" of a number precisely by your own word?**
3. **Find the cube root of $12{,}167 \times 42{,}875$.**

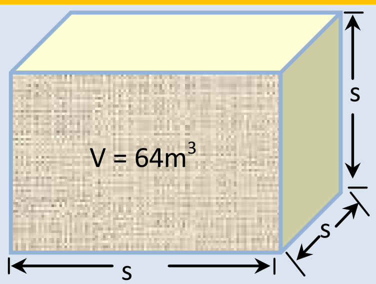


Figure 1.12 Cube

Definition 1.6: The cube root of a given number is one of the three identical factors whose product is the given number.

Example 26:

a) $0\times 0\times 0 = 0$, so 0 is the cube root of 0.

b) $5\times5\times5 = 125$, so 5 is the cube root of 125.

c) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$, so $\frac{1}{5}$ is the cube root of $\frac{1}{125}$.

Note: i) $4^3 = 64$, 64 is the cube of 4 and 4 is the cube root of 64

i.e $\sqrt[3]{64} = 4$.

ii) $\sqrt{\ }$ is a radical sign.

Or symbolically: $\sqrt[n]{x}$ — Index: n; Radicand: x; Radical sign: $\sqrt{\ }$

iii) The relation of cubing and extracting cube root can be expressed as follows:

a → **Cubing** → $a^3 = b$

a ← **Cube root** ← $a^3 = b$

iv) a is the cube root of b and written as $a = \sqrt[3]{b}$.

When no index is written, the radical sign indicates a square root.

For example $\sqrt[3]{512}$ is read as "the cube root of 512".

The number 3 is called **the index** and 512 is called **the radicand**.

Cube Roots of Perfect Cubes

Group work 1.7

Discuss with your group

1. Find the cube root of the perfect cubes.

a) $\sqrt[3]{27}$ b) $\sqrt[3]{\frac{1}{27}}$ c) $\sqrt[3]{125}$ d) $\sqrt[3]{-64}$

2. Which of the following are perfect cubes?

{42,60,64,90,111,125,133,150,216}

3. Which of the following are perfect cubes?

$\{3,6,8,9,12,27,y^3, y^8, y^9, y^{12}, y^{27}\}$

Note: The following properties of cubes are important: $(ab)^3 = a^3 \times b^3$ and $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ (where $b \neq 0$).

Thus $(2 \times 2)^3 = 2^3 \times 2^3 = 8 \times 8 = 64$ and $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$.

A number is called a perfect cube, if it is the cube of a rational number.

Definition 1.7: A rational number x is called a perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$.

Example 27:

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$, $64 = 4^3$ and $125 = 5^3$.

Thus 1, 8, 27 , 64 and 125 are **perfect cubes.**

Note: A perfect cube is a number that is a product of three identical factors of a rational number and its cube root is also a rational number.

Example 28: Find the cube root of each of the following.

a) 216 b) $\frac{1}{8}$ c) -64 d) -27

Solution:

a) $\sqrt[3]{216} = \sqrt[3]{6\times6\times6} = 6$

b) $\sqrt[3]{\frac{1}{8}} = \sqrt[3]{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}} = \frac{1}{2}$

c) $\sqrt[3]{-64} = \sqrt[3]{-4\times-4\times-4} = -4$

d) $\sqrt[3]{-27} = \sqrt[3]{-3\times-3\times-3} = -3$ -3

Exercise 1J

1. Determine whether each of the following statements is true or false.

a) $\sqrt[3]{17281} = 26$ b) $\sqrt[3]{\frac{1}{729}} = \frac{1}{90}$ c) $\sqrt[3]{-64} = \pm4$ d) $\sqrt[3]{\frac{-1}{625}} = \frac{1}{20}$

2. Find the cube root of each of the following.

a) 0 b) 343 c) 1000 d) 0. 001 e) $\frac{1}{729}$ f) $\frac{-1}{9261}$ g) $\sqrt[3]{x^3}$ h) $\frac{0}{27}$ i) $\frac{-1}{1331}$

3. Evaluate each of the following.

a) $\sqrt[3]{-27}$ b) $\sqrt[3]{\frac{1}{8}}$ c) $\sqrt[3]{27}$ d) $-\sqrt[3]{\frac{1}{64}}$ e) $\sqrt[3]{\frac{-8}{27}}$ f) $\sqrt[3]{-1000}$ g) $\sqrt[3]{\frac{-27}{64}}$ h) $-\sqrt[3]{-64}$ i) $\sqrt[3]{\frac{-1}{125}}$

Challenge Problem

4. Simplify: a) $5\sqrt{18} - 3\sqrt{72} + 4\sqrt{50}$ b) $\frac{2\sqrt{5}\times7\sqrt{2}}{\sqrt{14}\times\sqrt{45}}$

5. Simplify the expressions. Assume all variables represent positive rational number.

a) $\sqrt[3]{\frac{y^5}{27y^3}}$ b) $\sqrt[3]{16z^3}$ c) $\sqrt[3]{16a^3}$ d) $\sqrt[3]{\frac{b^4}{27b}}$ e) $\sqrt[3]{\frac{x^5}{x^2}}$ f) $\sqrt[3]{15m^4n^{22}}$ g) $\sqrt[3]{\frac{y^{11}}{y^2}}$ h) $\sqrt[3]{20s^{15}t^{11}}$

Table of Cube Roots

The same table which you can used to determine cubes of numbers can be used to find the approximate cube roots, of numbers.

Example 29: Find $\sqrt[3]{64.48}$ from the numerical table.

Solution: Find the value using rough calculations.

$$64.48 \approx 64; \sqrt[3]{64.48} \approx \sqrt[3]{64}$$

$$\approx \sqrt[3]{4 \times 4 \times 4} = 4$$

***Step i*:** Find the number 64. 48 in the body of the table for the relation $y = x^3$.

***Step ii*:** Move to the left on the row containing this number to get 4.0 under x. These are the first two digits of the required cube root of 64. 48.

***Step iii*:** To get the third digits start from 64.48 and move vertically upward and read 1 at the top.

There fore $\sqrt[3]{64.48} \approx 4.01$.

X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
3.0										
4.0		64.48								
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.13 Tables of cube roots

Example 30:

In Figure 1.14 below, find the exact volume of the boxes.

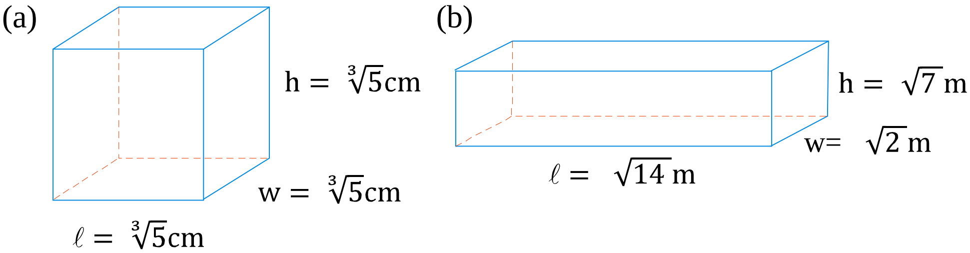


Figure 1.14

Solution

a) $V = \ell \times w \times h$

But the box is a cube, all the side of a cube are equal.

i.e $\ell = w = h = s$

$V = s \times s \times s = s^3$

$V = \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm}$

$V = \left(\sqrt[3]{5\text{cm}}\right)^3$

$V = \left(5^{\frac{1}{3}}\right)^3$

$V = 5\ \text{cm}^3$

Therefore, the volume of the box is 5cm^3.

b) $V = \ell \times w \times h$

$V = \sqrt{14}\text{m} \times \sqrt{2}\text{m} \times \sqrt{7}\text{m}$

$V = \sqrt{14}\text{m} \times \sqrt{14}\text{m}^2$

$V = \left(\sqrt{14 \times 14}\right)\text{m}^3$

$V = 14\text{m}^3$

Therefore, the volume of the box is 14m^3.

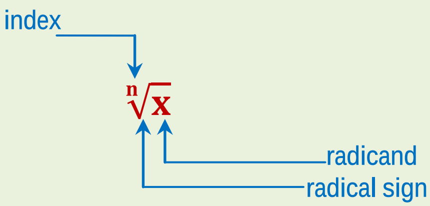
Exercise 1k

1. Use the table of cube to find the cube root of each of the following.

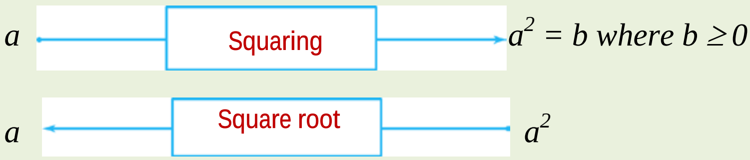
a) 32.77 c) 302.5 e) 3114

b) 42.6 d) 329.5 f) 3238

Summary for unit 1

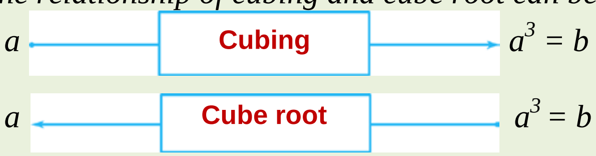
1. *The process of multiplying a number by itself is called* ***squaring*** *the number.*
2. *For each rational number x there is a rational number y ($y \geq 0$) such that $x^2 = y$.*
3. *A square root of a number is one of its two equal factors.*
4. *A rational number x is called a* ***perfect square,*** *if and only if $x = n^2$ for some $n \in \mathbb{Q}$.*
5. *The process of multiplying a number by itself three times is called* ***cubing*** *the number.*
6. *The cube root of a given number is one of the three identical factors whose product is the given number.*
7. *A rational number x is called a* ***perfect cube,*** *if and only if $x = n^3$ for some $n \in \mathbb{Q}$.*
8. 

9. *The relationship of squaring and square root can be expressed as follows:*



- *a is the square root of b and written as $a = \sqrt{b}$*

10. *The relationship of cubing and cube root can be expressed as follows:*



- *a is the cube root of b and written as $a = \sqrt[3]{b}$*

Miscellaneous Exercise 1

1. Determine whether each of the following statements is true or false.

a) $\frac{3\sqrt{8}}{2\sqrt{32}} = \frac{-3}{4}$

b) $\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}}$

c) $\sqrt{\frac{2}{5}}\sqrt{\frac{125}{8}} = 2.5$

d) $\sqrt{\frac{1}{7}} \times \sqrt{63} = \pm 3$

e) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

f) $\sqrt{0.25} = \frac{-1}{2}$

g) $\sqrt{0.0036} = 0.06$

2. Simplify each expression.

a) $\sqrt{\frac{36}{324}}$

b) $\frac{\sqrt{50}}{\sqrt{2}}$

c) $8\sqrt{\frac{25}{4}}$

d) $\sqrt{\frac{16}{4}}$

e) $2\sqrt{2}\left[\frac{3}{\sqrt{2}} + \sqrt{2}\right]$

3. Simplify each expression.

a) $\sqrt{600}$

b) $\sqrt{50} + \sqrt{18}$

c) $(5\sqrt{6})^2$

d) $\sqrt{3}(\sqrt{3} + \sqrt{6})$

e) $\sqrt{19^2}$

f) $\sqrt{64 + 36}$

g) $\sqrt{2}(\sqrt{2} + \sqrt{6})$

h) $\sqrt{2}(\sqrt{3} + \sqrt{8})$

4. Simplifying radical expressions (where $x \neq 0$).

a) $\frac{\sqrt[3]{32}}{\sqrt[3]{-4}}$

b) $\frac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

c) $\frac{\sqrt{12x^4}}{\sqrt{3x}}$

d) $\sqrt[3]{80n^5}$

e) $\sqrt[3]{p^{17}q^{18}}$

5. Study the pattern and find a and b

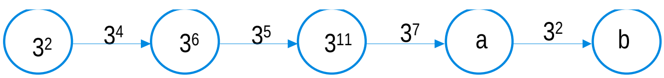


Figure 1.15

6. Study the pattern and find a, b, c and d.

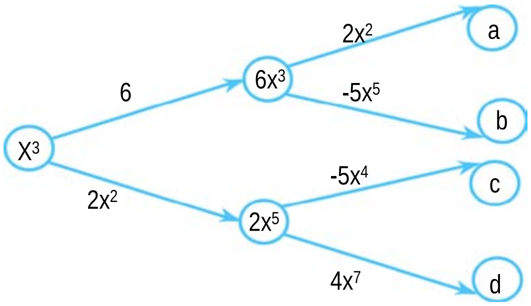


Figure 1.16

7. An amoeba is a single cell animal. When the cell splits by a process called "fission" there are then two animals. In a few hours a single amoeba can become a large colony of amoebas as shown to the right.

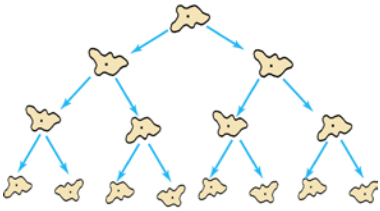


Figure 1.17

Number of splits	Number of amoeba cells
0	1
1	2
2	$4 = 2 \times 2 = 2^2$
3	$8 = 2 \times 2 \times 2 = 2^3$

How many amoebas would there be

a) After 4 splits?

b) After 5 splits?

c) after 6 splits?

d) after 10 splits?

8. Using only the numbers in the circular table, write down, all that are:

a) square numbers

b) cube numbers

Figure 1.18

9. Find the exact perimeter of a square whose side length is $5\sqrt[3]{16}$ cm.

10. The length of the sides of a cubes is related to the volume of the cube according to the formula: $x = \sqrt[3]{V}$.

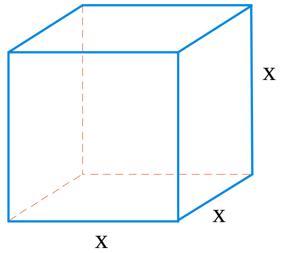


Figure 1.19 Cube

a) What is the volume of the cube if the side length is 25cm.

b) What is the volume of the cube if the side length is 40 cm.

11. In Figure 1.20 to the right find:

a) the surface area of a cube.

b) the volume of a cube.

c) compare the surface area and the volume of a given a cube

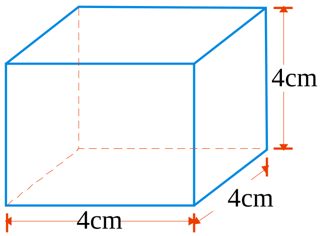


Figure 1.20 Cube

12. Prove that the difference of the square of an even number is multiple of 4.

13. Show that 64 can be written as either 2^6 or 4^3.

14. Look at this number pattern.

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

This pattern continues.

a) Write down the next line of the pattern.

b) Use the pattern to work out 6666667^2.

15. Find three consecutive square numbers whose sum is 149.

16. Find the square root of $25x^2 - 40xy + 16y^2$.

17. Find the square root of $\frac{64a^2}{9b^2} + 4 + \frac{32a}{3b}$.

18. Find the cube root of $27a^3 + 54a^2b + 36ab^2 + 8b^3$.

UNIT 2

FURTHER ON WORKING WITH VARIABLES

Unit outcomes

After Completing this unit, you should be able to:

- solve life related problems using variables.
- multiply binomial by monomial and determine the product of binomials.
- determine highest common factor of algebraic expressions.

Introduction

By now you are well aware of the importance of variables in mathematics. In this unit you will learn more about variables, specially you will learn about mathematical expression, its component parts and uses of variables in formulas and solving problems. In addition to these you will study special expressions known as binomials and how to perform addition and multiplication on them.

2.1. Further on Algebraic Terms and Expressions

2.1.1 Use of variables in formula

Group Work 2.1

Discuss with your friends

1. What a variable is?
2. Find what number I am left with if
   a. I start with x, double it and then subtract 6.
   b. I start with x, add 4 and then square the result.
   c. I start with x, take away 5, double the result and then divide by 3.
   d. I start with w, subtract x and then square the result.
   e. I start with n add p, cube the result and then divide by a.
3. Translate the following word problems in to mathematical expression.
   a. Eighteen subtracted from 3.
   b. The difference of -5 and 11.
   c. Negative thirteen subtracted from – 10.
   d. Twenty less than 32.
4. Describe each of the following sets using variables.
   a. The set of odd natural numbers.
   b. The solution set of $3x - 1 \geq 4$.
   c. The solution set of $x + 6 = 24$.
   d. The set of all natural number less than 10.

Definition 2.1: ***A variable*** **is a symbol or letter such as x, y and z used to represent an unknown number (value).**

Example 1: Describe each of the following sets using variables.

a. The solution set of $3x - 5 > 6$.

b. The solution set of $2x + 1 = 10$.

Solution

a) $3x - 5 > 6$ ……………. Given inequality

$3x - 5 + 5 > 6 + 5$ ……... Adding 5 from both sides

$3x > 11$ ………………… Simplifying

$\frac{3x}{3} > \frac{11}{3}$ ………………….. Dividing both sides by 3

$x > \frac{11}{3}$

The solution set of $3x - 5 > 6$ is $\left\{x : x > \frac{11}{3}\right\}$.

b) $2x + 1 = 10$ ………….. Given equation.

$2x + 1 - 1 = 10 - 1$ ……Subtracting 1 from both sides.

$2x = 9$ …….. ... Simplifying.

$\frac{2x}{2} = \frac{9}{2}$ ……….. Dividing both sides by 2.

$x = \frac{9}{2}$

The solution set of $2x + 1 = 10$ is $x = \frac{9}{2}$ or S.S $= \left\{\frac{9}{2}\right\}$

Example 2: Find the perimeter of a rectangle in terms of its length ℓ and width w.

Solution Let P represent the perimeter of the rectangle.

Then P = AB + BC + CD + DA

$= \ell + w + \ell + w$

$= 2\ell + 2w$

$= 2(\ell + w)$

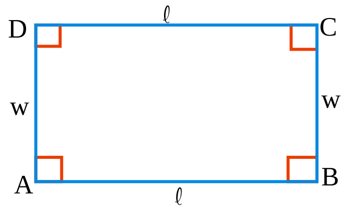


Figure 2.1

Example 3: The volume of a rectangular prism equals the product of the numbers which measures of the length, the width and the height. Formulate the statement using variables.

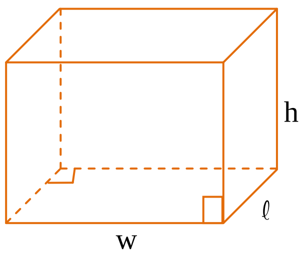


Figure 2.2 A rectangular prism

Solution let ℓ represent the length, w the width and h the height of the prism. If V represents the volume of the prism, then

$V = \ell \times w \times h$

$V = \ell wh$

Example 4: Express the area of a triangle in terms of its base 'b' and altitude 'h'.

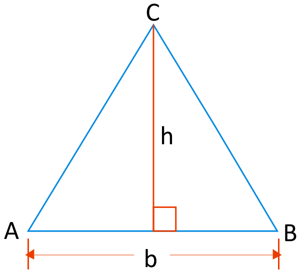


Figure 2.3 triangle

Solution Let "b" represent the base and "h" the height of the triangle.

$A = \frac{1}{2} bh$

Example 5: The area of a trapezium (see Figure 2.4) below can be given by the formula $A = \frac{1}{2}(b_1 + b_2)$ where A = area, h = height, b_1 = upper base and b_2 = lower base. If the area is 170 cm^2, height 17cm and b_2 = 12cm then:

a) Express b_1 in terms of the other variables in the formula for A.

b) Use the equation you obtained to find b_1.

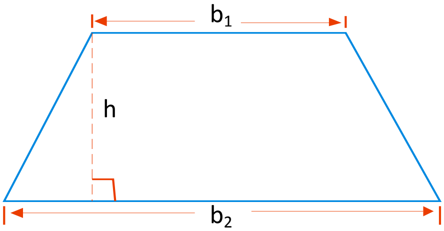


Figure 2.4 Trapezium

Solution

a) $\frac{1}{2} h (b_1 + b_2) = A$ …. Given equation

$h (b_1 + b_2) = 2A$ ….. Multiplying A by 2

$b_1 + b_2 = \frac{2A}{h}$ ….. Dividing both sides by h

$b_1 = \frac{2A}{h} - b_2$ ….. Subtracting b_2 from both sides

$b_1 = \frac{2A - b_2 h}{h}$ …… Simplifying

b) For (a) above we have

$b_1 = \frac{2A - b_2 h}{h}$

$b_1 = \frac{2(170) - 17(12)}{17}$

$b_1 = \frac{17(20-12)}{17}$

$b_1 = 8cm$

Therefore the upper base (b_1) is 8cm.

✓ **Check:** $A = \frac{1}{2}h(b_1 + b_2)$ when $b_1 = 8cm$

$170cm^2 \stackrel{?}{=} \frac{1}{2}(17cm)\ (8cm + 12cm)$

$170\ cm^2 \stackrel{?}{=} \frac{17}{2}\ cm\ (20cm)$

$170\ cm^2 = 170\ cm^2$ (True)

Exercise 2A

Solve each of the following word problems.

1. The perimeter of a rectangular field is 1000m. If the length is given as x, find the width in terms of x.
2. Find
   i) The perimeter of a square in terms of its side of length ″s″ unit.
   ii) The area of a square interms of its side of length ″s″ units.
3. Express the volume of the cube in Figure 2.5.

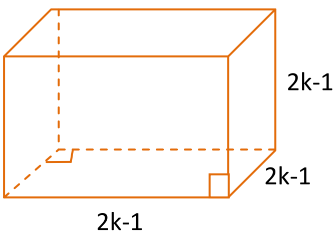


Figure 2.5 Cube

4. The area of a trapezoid is given by the formula $A = \left(\frac{b_1 + b_2}{2}\right)h$ then give the height h interms of its bases b_1 & b_2.
5. A man is 8x years old now. How old he will be in:
   a. 10 years time? b. 6x years time? c. 5y years time?
6. How many days (d) are there in the given number of weeks (w) below?
   a. 6 weeks c. y weeks
   b. 104 weeks d. 14 weeks

2.1.2 Variables, Terms and Expressions

Activity 2.1

Discuss with your teacher before starting the lesson.

1. What do we mean by like terms? Given an example.
2. Are $7a^3$, $5a^2$ and 12a like terms? Explain.
3. What is an algebraic expression?
4. What is a monomial?
5. What is a binomial?
6. What is a trinomial?
7. What are unlike terms? Give an example.

Definition 2.2: **Algebraic expressions are formed by using numbers, letters and the operations of addition, subtraction, multiplication, division, raising to power and taking roots.**

Some examples of algebraic expressions are:

$x + 10$, $y – 16$, $2x^2 + 5x – 8$, $x – 92$, $2x + 10$, etc.

Note:

i. An algebraic expression that contains variables is called an expression in certain variables. For examples the expression $7xy + 6z$ is an algebraic expression with variables x, y and z.

ii. An algebraic expression that contains no variable at all is called constant. For example, the algebraic expression $72 - 16\pi$ is constant.

iii. The terms of an algebraic expression are parts of the expression that are connected by plus or minus signs.

Examples 6: List the terms of the expression $5x^2 – 13x + 20$.

Solution: The terms of the expression $5x^2 – 13x + 20$ are $5x^2$, $- 13x$, and 20.

Definition 2.3: **An algebraic expression in algebra which contains one term is called a monomial.**

Example 7: $8x$, $13a^2b^2$, $\frac{-2}{3}$, $18xy$, $0.2a^3b^3$ are all monomial.

> Definition 2.4: **An algebraic expression in algebra which contains two terms is called a binomial.**

Examples 8: $2x + 2y$, $2a - 3b$, $5p^2 + 8$, $3x^2 + 6$, $n^3 - 3$ are all binomial.

> Definition 2.5: **An algebraic expression in algebra which contains three terms is called a trinomial.**

Examples 9: $4x^2 + 3x + 10$, $3x^2 - 5x + 2$, $ax^2 + bx + c$ are all trinomial.

> Definition 2.6: Terms which have the same variables, with the corresponding variables are raised to the same powers are called like terms; other wise called unlike terms.

For example:

Like terms	Unlike terms
$34xy$ and $-8xy$	$12xy$ and $6x$ …… Different variables.
$18p^2q^3$ and p^2q^3	$8p^2q^3$ and $16p^3q^2$ …Different power
$5w$ and $6w$	$10w$ and 20 …… Different variables.
7 and 20	14 and $10a$…… Different variables.

Example 10: Which of the pairs are like terms: $80ab$ and $70b$ or $4c^2d^2$, and $-6c^2 d^2$.

Solution $4c^2d^2$ and $-6c^2d^2$ are like terms but $80\ ab$ and $70\ b$ are unlike terms.

Note:

i. Constant terms with out variables, (or all constant terms) are like terms.
ii. Only like terms can be added or subtracted to form a more simplified expression.
iii. Adding or subtracting like terms is called combining like terms.
iv. If an algebraic expression contains two or more like terms, these terms can be combined into a single term by using distributive property.

Example 11: Simplify by collecting like terms.

a. $18x + 27 - 6x - 2$
b. $18k - 10k - 12k + 16 + 7$

Solution

a. $18x + 27 - 6x - 2$

$= 18x - 6x + 27 - 2$ ……. Collecting like terms

$= 12x + 25$ ……………… Simplifying

b. $18k - 10k - 12k + 16 + 7$

$= 18k - 22k + 16 + 7$ ….. Collecting like terms

$= -4k + 23$ …….. ……… Simplifying

Example 12: Simplify the following expressions

a. $(6a + 9x) + (24a - 27x)$
b. $(10x + 15a) - (5x + 10y)$
c. $-(4x - 6y) - (3y + 5x) - 2x)$

Solution

a. $(6a + 9x) + (24a - 27x)$

$= 6a + 9x + 24a - 27x$ ……. Removing brackets

$= 6a + 24a + 9x - 27x$ ……. Collecting like terms

$= 30a - 18x$ ….. Simplifying

b. $(10x + 15a) - (5x + 10y)$

= 10x + 15a – 5x – 10y ……. Removing brackets
= 10x – 5x + 15a – 10y ……. Collecting like terms
= 5x + 15a – 10y …… . . . . . .Simplifying

c. –(4x – 6y) – (3y + 5x) – 2x
= -4x + 6y – 3y – 5x – 2x ……Removing brackets
= -4x – 5x – 2x + 6y – 3y ……Collecting like terms
= -11x + 3y ………………… Simplifying

Group work 2.2

1. Three rods A, B and C have lengths of (x + 1) cm; (x + 2) cm and (x – 3)cm respectively, as shown below.

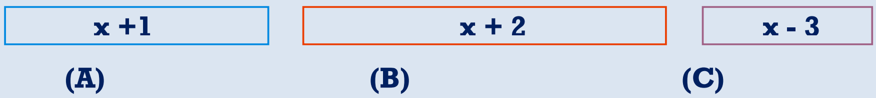


Figure 2.6

In the figures below express the length ℓ interms of x.

Give your answers in their simplest form.

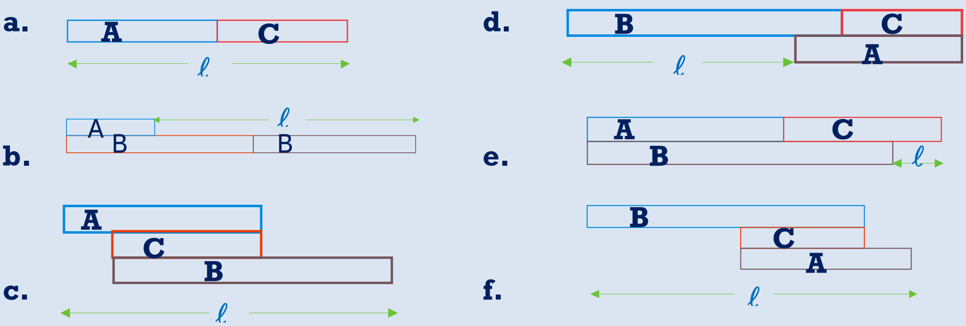


Figure 2.7

2. When an algebraic expression was simplified it became 2a + b.

a. Write down as many different expressions as you can which simplify to 2a + b.

b. What is the most complex expression you can think of that simplifies to 2a + b?

c. What is the simplest expression you can think of that simplifies to 2a + b?

Note: An algebraic formula uses letters to represent a relationship between quantities.

Exercise 2B

1. Explain why the terms $4x$ and $4x^2$ are not like terms.
2. Explain why the terms $14w^3$ and $14z^3$ are not like terms.
3. Categorize the following expressions as a monomial, a binomial or a trinomial.

a. 26	d. $16x^2$	g. $20w^4 - 10w^2$
b. $50\ bc^2$	e. $10a^2 + 5a$	h. $2t - 10t^4 - 10a$
c. $90 + x$	f. $27x + \frac{3}{2}$	i. $70z + 13z^2 - 16$

4. Work out the value of these algebraic expressions using the values given.
   a. $2(a + 3)$ if $a = 5$
   b. $4(x + y)$ if $x = 5$ and $y = -3$
   c. $\frac{7-x}{y}$ if $x = -3$ and $y = -2$
   d. $\frac{2a + b}{c}$ if $a = 3$, $b = 4$ and $c = 2$
   e. $2(b + c)^2 - 3(b - c)^2$ if $b = 8$ and $c = -4$
   f. $(a + b)^2 + (a + c)^2$ if $a = 2$, $b = 8$ and $c = -4$
   g. $c(a + b)^3$ if $a = 3$, $b = 5$ and $c = 40$

Challenge Problems

5. Solve for d if $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ if $x_1 = 3$, $y_1 = 4$ and $x_2 = 12$, $y_2 = 37$.
6. $y\frac{[3x+6y(x-20)]}{2x+12}$ if $x = 5$ and $y = \frac{1}{2}$
7. Collect like terms together.

a. $xy + ab - cd + 2xy - ab + dc$	d. $5 + 2y + 3y^2 - 8y - 6 + 2y^2 + 3$
b. $3x^2 + 4x + 6 - x^2 - 3x - 3$	e. $6x^2 - 7x + 8 - 3x^2 + 5x - 10$
c. $3y^2 - 6x + y^2 + x^2 + 7x + 4x^2$	f. $2x^2 - 3x + 8 + x^2 + 4x + 4$

2.1.3 Use of Variables to Solve Problems

Activity 2.2

Discuss with your teacher

1. Prove that the sum of two even numbers is an even numbers.
2. If the perimeter of a rectangle is 120cm and the length is 8cm more than the width, find the area.
3. The sum of three consecutive integers is 159. What are the integers?
4. The height of a ballon from the ground increases at a steady rate of x metres in t hours. How far will the ballon rise in n hours?

In this topic you are going to use variables to solve problems involving some unknown values and to prove a given statement.

? What is a proof?

A **proof** is an argument to show that a given statements is true. The argument depends on known facts, such as definitions, postulates and proved theorems.

Example13: **(Application involving consecutive integers)**

The sum of two consecutive odd integers is -188. Find the integers.

Solution: Let x represent the first odd integer, hence

x + 2 represents the second odd integer.

(First integer) + (Second integer) = total ……. Write an equation in words.

x + (x + 2) = -188 …… Write a mathematical equation

$x + x + 2 = -188$

$2x + 2 = -188$

$2x = -190$

$\frac{2x}{2} = \frac{-190}{2}$

$x = -95$

Therefore the integer are -95 and -93.

Example14: **(Application involving Ages)**

The ratio of present ages of a mother and her son is 12 : 5. The mother's age, at the time of birth of the son was 21 years. Find their present ages.

$\left(\text{Hint: } \frac{x}{y} = \frac{12}{5}\right)$

Solution:

Let x be the present age of the mother and y be that of her son.

Thus x:y = 12:5 or $\frac{x}{y} = \frac{12}{5}$

$5x - 12y = 0$ …... Equation 1

$x - y = 21$ …... Equation 2

From equation 2, we get $x = 21 + y$ …… Equation 3

Substituting equation 3 in to equation 1, we will get:

$5(21 + y) - 12y = 0$

$105 + 5y - 12y = 0$

$105 - 7y = 0$

$105 = 7y$

$y = 15$

Thus $x = 21 + y$

$x = 21 + 15 \Rightarrow x = 36$

Therefore, the present ages of a mother and her son are 36 years and 15 years respectively.

Exercise 2C

Solve each of the following word problems.

1. A 10 meter piece of wire is cut in to two pieces. One piece is 2 meters longer than the other. How long are the pieces?
2. The perimeter of a college basket ball court is 96 m and the length is 14m more than the width. What are the dimensions?
3. Ten times the smallest of three consecutive integers is twenty two more than three times the sum of the integers. Find the integers.
4. The surface area "S" of a sphere of radius r is given by the formula: $S = 4\pi r^2$.
   Find (i) the surface area of a sphere whose radius is 5 cm.
   (ii) the radius of a sphere whose surface area is $17\frac{1}{9}$ cm^2.
5. By what number must be 566 be divided so as to give a quotient 15 and remainder 11?
6. I thought of a number, doubled it, then added 3. The result multiplied by 4 came to 52. What was the number I thought of ?

7. One number is three times another, and four times the smaller added to five times the greater amounts to 133; find them.

Challenge Problems

8. If a certain number is increased by 5, one – half of the result is three – fifths of the excess of 61 over the number. Find the number.
9. Divide 54 in to two parts so that four times the greater equals five times the less.
10. Prove that the sum of any 5 consecutive natural numbers is divisible by 5.

2.2 Multiplication of Binomials

In grade seven you have studied about certain properties of multiplication and addition such as the commutative and associative properties of addition and multiplication and the distributive of multiplication over addition. In this sub–unit you will learn how to perform multiplication of monomial by binomial and multiplication of binomial by binomial.

2.2.1 Multiplication of Monomial by Binomial

Activity 2.3

Discuss with your friends /partners/	
1. Multiply 4a by 2ab	3. Multiply 6b by (3a + 15b)
2. Multiply 4b by (2ab + 6b)	4. Multiply 7ab by (3ab – 6a)

You begin this topic, let us look at some examples:

Example 15: Multiply 2x by 4yz

Solution:

$$2x \times 4yz = 2 \times x \times 4 \times y \times z$$
$$= (2 \times 4)(x \times y \times z)$$
$$= 8\,xyz$$

Example 16: Multiply $4c^2$ by $(16\,abc - 5a^2\,bc)$

Solution:

$$4c^2 \times (16abc - 5a^2bc)$$
$$= (4c^2 \times 16abc) - (4c^2 \times 5a^2bc)$$
$$= (4 \times 16 \times c^2 \times c \times a \times b) - (4 \times 5 \times c^2 \times c \times a^2 \times b)$$
$$= 64\,c^3\,ab - 20\,c^3\,a^2b$$

Example 17: Multiply 4rt (5pq – 3pq)

Solution:

$4rt \times (5pq - 3pq)$
$= (4rt \times 5pq) - (4rt \times 3pq)$
$= (4 \times 5 \times r \times t \times p \times q) - (4 \times 3 \times r \times t \times p \times q)$
$= (20rt\,pq - 12\,rt\,pq)$
$= (20-12)\,rt\,pq$
$= 8pqrt$

? Do you recall the properties used in examples 15, 16, and 17 above?

1. Distributive properties

Group Work 2.3

1. In Figure 2.8 below, find the area of the shaded region.

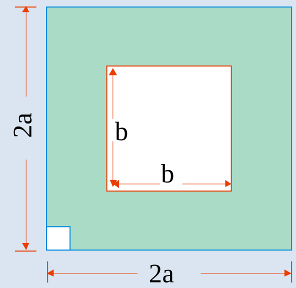


Figure 2.8

2. If the area of a rectangle is found by multiplying the length times the width, express the area of the rectangle in Figure 2.9 in two ways to illustrate the distributive property for a(b + c).

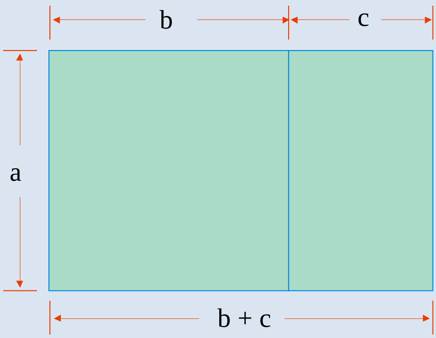


Figure 2.9

3. Express the shaded area of the rectangle in Figure 2.10 in two ways to illustrate the distributive property for a (b – c).

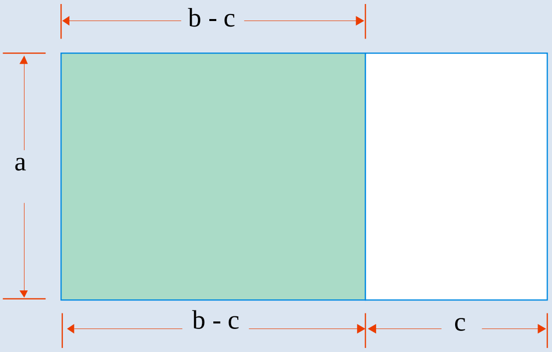


Figure 2.10

4. In Figure 2.11, find the area of each rectangle.

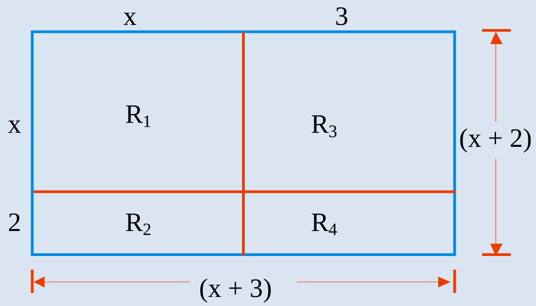


Figure 2.11

Consider the rectangle in Figure 2.12 which has been divided in to two smaller ones:

The area A of the bigger rectangle is given by: $A = a(b + c)$.

The area of the smaller rectangles are given by $A_1 = ab$ and $A_2 = ac$; but the sum of the areas of the two smaller rectangles are given by,

$A_1 + A_2$ gives the area A of the bigger rectangle:

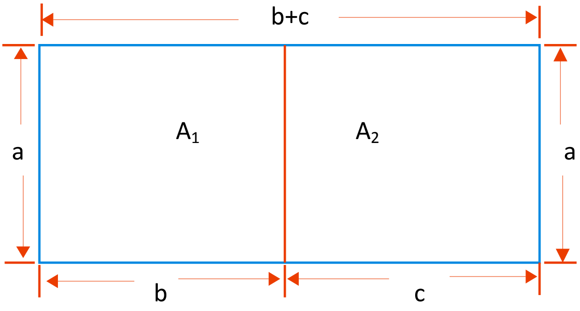


Figure 2.12 Rectangle

That means: $A = A_1 + A_2$

$a(b + c) = ab + ac$

This suggests that a(b + c) = a × b + a × c the factor out side the bracket multiply each number in the bracket, this process of removing the bracket in a product is known as **expansion**.

Similarly consider another rectangle as in Figure 2.13 below.

Area of the shaded region = area of the bigger rectangle–area of the un shaded region.

Therefore, a (b – c) = ab – ac.

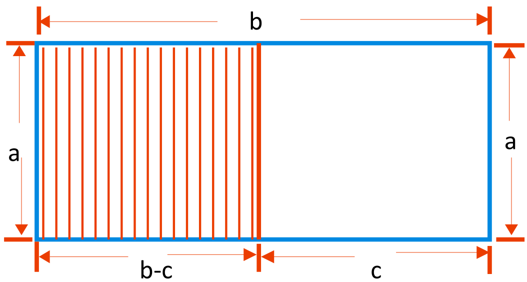


Figure 2.13 Rectangle

You have seen that the above two examples on area of rectangle, this could be generalized as in the following way:

Note: For any rational numbers a, b, and c

a. a(b + c) = ab + ac b. a (b – c) = ab – ac

These two properties are called the distributive properties of multiplication over Addition (subtraction).

Exercise 2D

1. Expand these expressions by using the distributive properties to remove the brackets in and then simplify.
   1. 2(a + b) + 3 (a + b)
   2. 5(2a – b) + 49(a + b)
   3. 4(5a+c) + 2(3a – c)
   4. 5(4t – 3s) + 8(3t + 2s)
   5. 5(3z + b) + 4(b – 2z)
   6. 7(2d + 3e) + 6 (2e – 2d)
   7. 3(p + 2q) + 3(5p – 2q)
   8. 5(5q + 4h) + 4(h – 5q)
   9. 6(p + 2q + 3r) + 2(3p – 4q + 9r)
   10. 2(a + 2b – 3c) + 3(5a – b + 4c) + 4(a + b + c)

Challenge Problems

11. Remove the brackets and simplify.

a) $(x + 1)^2 + (x + 2)^2$

b) $(y - 3)^2 + (y - 4)^2$

c) $(x - 2)^2 + (x + 4)^2$

d) $(x + 2)^2 - (x - 4)^2$

e) $(2x + 1)^2 + (3x + 2)^2$

f) $(2x - 3)^2 + (5x + 4)^2$

2.2.2 Multiplication of Binomial by Binomial

Activity 2.4

Discuss with your friends / partners

Find the following products

1. $(2x + 8)(3x - 6)$
2. $(5a + 4)(4a + 6)$
3. $(2x - 8)(2x + 8)$
4. $(2x - 10)(x - 8)$
5. $(2a^2 - ab)(20 + x)$
6. $(3x^2 + 2x - 5)(x - 1)$

Sometimes you will need to multiply brackets expressions. For example $(a + b)(c + d)$.

This means $(a + b)$ multiplied by $(c + d)$ or $(a + b) \times (c + d)$.

Look at the rectangles 2.14 below.

The area 'A' of the whole rectangle is $(a + b)(c + d)$. It is the same as the sum of the areas of the four rectangle so: $A = A_1 + A_2 + A_3 + A_4$

$$(a + b)(c + d) = ac + ad + bc + bd$$

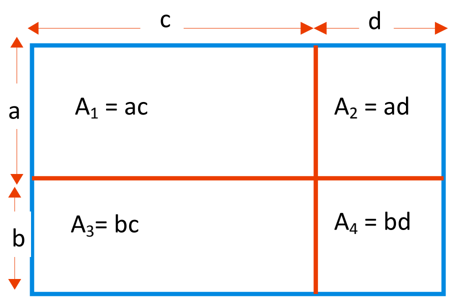
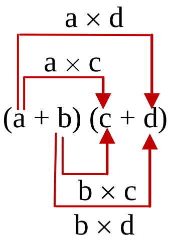


Figure 2.14 Rectangle

Notice that each term in the first brackets is multiplied by each term in the second brackets:



You can also think of the area of the rectangle as the sum the areas of two separate part (the upper two rectangles plus the lower two rectangle) see Figure 2.15:

Thus, (a + b) (c + d) = a (c + d) + b(c + d)

Think of multiplying each term in the first bracket by the whole of the second bracket. These are two ways of thinking about the same process. The end result is the same. This is called **multiplying out** the brackets.

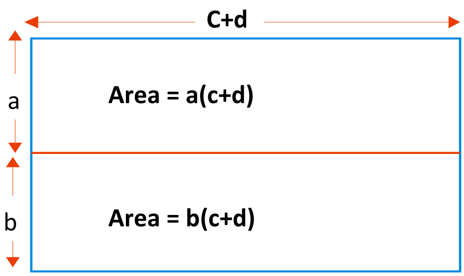


Figure 2.15 Rectangle

This process could be described as follows.

Note: If (a + b) and (c +d) are any two binomials their product

(a + b) × (c + d) is defined as:

$(a + b) \times (c + d) = ac + ad + bc + bd$

Example18: Multiply (4x + 4) by (3x + 8)

Solution

$$(4x + 4)(3x + 8) = (4x \times 3x) + (4x \times 8) + (4 \times 3x) + (4 \times 8)$$
$$= 12x^2 + 32x + 12x + 32$$
$$= 12x^2 + 44x + 32$$

Example 19: Multiply (2x + 10) by (3x - 6)

Solution

$$(2x + 10)(3x - 6) = (2x \times 3x) - (6 \times 2x) + (10 \times 3x) - (6 \times 10)$$
$$= 6x^2 - 12x + 30x - 60$$
$$= 6x^2 + 18x - 60$$

Example20: Multiply (2x - 3) by (4x - 12)

Solution

$$(2x - 3)(4x - 12) = (2x \times 4x) - (12 \times 2x) - (3 \times 4x) + (3 \times 12)$$
$$= 8x^2 - 24x - 12x + 36$$
$$= 8x^2 - 36x + 36$$

In the multiplication of two binomials such as those shown in example 20 above, the product $2x \times 4x = 8x^2$ and $-3 \times -12 = 36$ are called **end products**. Similarly, the product $-12 \times 2x = -24x$ and $-3 \times 4x = -12x$ are called **cross product**. Thus the product of any two binomials could be defined as the sum of the **end products** and the **cross products**. The sum of the cross products is written in the **middle**.

Example21: Multiply (4x - 6) by (4x + 10)

Solution

end product

$$(4x - 6)(4x + 10) = (4x \times 4x) + (4x \times 10) - (6 \times 4x) - (6 \times 10)$$

Cross product

$$= 16x^2 + 40x - 24x - 60$$
$$= 16x^2 + 16x - 60$$

Example22: Multiply (4x-10) by (6x-2)

Solution:

end product

$(4x - 10)(6x - 2) = (4x \times 6x) - (2 \times 4x) - (10 \times 6x) + (10 \times 2)$

cross product

$= 24x^2 - 8x - 60x + 20$

$= 24x^2 - 68x + 20$

Exercise 2E

Find the products of the following binomials.

1. $(2x + 2y)(2x - 2y)$
2. $(3x + 16)(2x - 18)$
3. $(-4x - 6)(-20x + 10)$
4. $-5[(4x+y)(3x+2b)]$
5. $\left(\frac{3}{2} - \frac{2}{3}x\right)(2x + 1)$
6. $\left(\frac{x}{8} + \frac{x}{8}\right)\left(\frac{x}{8} - \frac{x}{4}\right)$
7. $\left(\frac{3}{2}x + \frac{4}{3}x\right)\left(\frac{2}{3}x + \frac{3}{5}x\right)$
8. $\left(\frac{4}{5}ab - \frac{3}{5}ab\right)\left(-4ab - \frac{3}{2}ab\right)$
9. $\left(\frac{2}{5}ab + \frac{3}{5}a^2b^2\right)\left(\frac{3}{7}a^2b^2 + \frac{3}{7}a^2b^2\right)$

Challenge Problems

10. $(2x^2 + 4x - 6)(x^2 + 4)$
11. $(2x^2 - 4x - 6)\left(\frac{3}{2}x^2 - 6\right)$
12. $(4x^2 + 4x - 10)(5x - 5)$

2.3 Highest Common Factors

Activity 2.5

Discuss with your teacher before starting the lesson.

1. Define and explain the following key terms:
   a. Factorizing a number
   b. Prime factorization
2. Find the HCF of the following.
   a. 72 and 220
   b. 36, 48 and 72
   c. 120, 150 and 200

3. Find the HCF of the following:
   a. $20xyz$ and $18x^2z^2$
   b. $5x^3y$ and $10xy^2$
   c. $3a^2b^2$, $6a^3b$ and $9a^3b^3$
4. Factorize the following expressions.
   a. $\frac{3}{19}ac - \frac{1}{19}ad$
   b. $x(2b + 3) + y(2b + 3)$
   c. $\frac{5a^2b^2}{4} + \frac{15}{6}a^4b^2$
   d. $a^2(c+2d)-b^2(c+2d)$

Factorizing

This unit is devoted to the method of describing an expression is called **Factorizing**. To factorize an integer means to write the integer as a product of two or more integers. To factorize a monomial or a Binomial means to express the monomial or Binomial as a product of two or more monomial or Binomials. In the product $2\times5 = 10$, for example, 2 and 5 are factors of 10. In the product $(3x + 4)(2x) = 6x^2 + 8x$, the expressions $(3x + 4)$ and $2x$ are factors of $6x^2 + 8x$.

Example 23: Factorize each monomial in to its linear factors with coefficient of prime numbers.

a. $15x^3$ b. $25x^3$

Solution:

a. $15x^3 = (3\times 5) \times (x \times x \times x)$
$= (3x) \times (5x) \times (x).$

b. $25x^3 = (5 \times 5) \times (x \times x \times x)$
$= (5x) \times (5x)(x).$

Example 24: Factorize each of the following expressions.

a. $6x^2 + 12$ b. $5x^4 + 20x^3$

Solution:

a. $6x^2 + 12 = 6x^2 + 6 \times 2$
$= 6(x^2 + 2)$

b. $5x^4 + 20x^3 = 5x^3 \times x + 5x^3 \times 4$
$= 5x^3(x + 4)$

Exercise 2F

Factorize each of the following expressions.

1. $2x^2 + 6x$
2. $18xy^2 – 12xy^3$
3. $5x^3y + 10xy^2$
4. $16a^2b + 24ab^2$
5. $12ab^2c^3 + 16ac^4$
6. $3a^4b – 5bc^3$
7. $6x^4yz + 15x^3y^2z$
8. $8a^2b^3c^4 – 12a^3b^2c^3$
9. $8xy^2 + 28xyz – 4xy$
10. $-10mn^3 + 4m^2n – 6mn^2$

Challenge Problems

11. $7a^2b^3 + 5ab^2 + 3a^2b$
12. $2a^3b^3 + 3a^3b^2 + 4a^2b$
13. $-30abc + 24abc – 18a^2b$
14. $16x^4 – 24x^3 + 32x^2$
15. $10x^3 + 25x^2 + 15x$

Highest common factor of two integers

Group Work 2.4

Discuss with your group

For Exercise 1 – 4 factor out the HCF.

1. $15x^2 + 5x$

2. $5q^4 – 10q^5$

3. $y(5y + 1) – 9(5y + 1)$

4. $5x(x – 4) – 2(x – 4)$

You begin the study of factorization by factoring integers. The number 20 for example can be factored as 1×20, 2×10, 4×5 or 2×2×5. The product 2×2×5 (or equivalently $2^2×5$) consists only of prime numbers and is called the **prime factorization**.

The **highest common factor** (denoted by HCF) of two or more integers is the highest factor common to each integer. To find the highest common factor of two integers, it is often helpful to express the numbers as a product of prime factors as shown in the next example.

Example 25: Find the highest common factor of each pair of integers.

a. 24 and 36
b. 105 and 40

Solution:

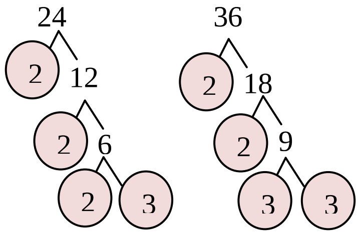
First find the prime factorization of each number by multiplication or by factor tree method.

a. i. by multiplication

Factors of 24 = $2 \times 2 \times 2 \times 3$

Factors of 36 = $2 \times 2 \times 3 \times 3$

ii. By using factor trees



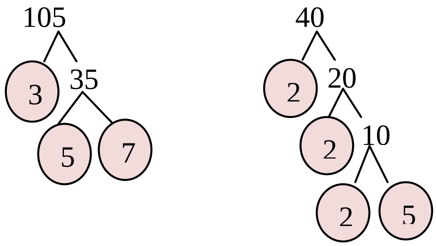
The numbers 24 and 36 share two factors of 2 and one factor of 3. Therefore, the highest common factor is $2 \times 2 \times 3 = 12$

b. i. By multiplication

Factors of 105 = $3 \times 7 \times 5$

Factors of 40 = $2 \times 2 \times 2 \times 5$

ii. By using factor trees



Therefore, the highest common factors is 5.

Highest common factor (HCF) of two or more monomials

Example 26: Find the HCF among each group of terms.

a. $7x^3$, $14x^2$, $21x^4$

b. $8c^2d^7e$, $6c^3d^4$

Solution:

List the factors of each term.

a) $7x^3 = 7 \times x \times x \times x$

$14x^2 = 2 \times 7 \times x \times x$

$21x^4 = 3 \times 7 \times x \times x \times x \times x$

Therefore, the HCF is $7x^2$.

b) $\left.\begin{array}{l} 8c^2d^7e = 2^3c^2d^7e \\ 6c^3d^4 = 2 \times 3c^3d^4 \end{array}\right\}$ The common factors are the common powers of 2, c and d appearing in both factorization to determine the HCF we will take the common least powers. Thus

The lowest power of 2 is : 2^1

The lowest power of c is : c^2

The lowest power of d is : d^4

Therefore, the HCF is $2c^2d^4$.

Example 27: Find the highest common factor between the terms:

$3x(a + b)$ and $2y(a + b)$

Solution

$\left.\begin{array}{l} 3x\,(a + b) \\ 2y(a + b) \end{array}\right\}$ The only common factor is the binomial $(a + b)$.

Therefore, the HCF is $(a + b)$.

Factorizing out the highest common factor

Factorization process is the reverse of multiplication process. Both processes use the distributive property: $ab + ac = a\,(b + c)$

Example 28: Multiply: $5y(y^2 + 3y + 1)$

$= 5y\,(y^2) + 5y(3y) + 5y(1)$

$= 5y^3 + 15y^2 + 5y$

Factor: $5y^3 + 15y^2 + 5y$

$= 5y(y^2) + 5y(3y) + 5y(1)$

$= 5y\,(y^2 + 3y + 1)$

Example 29: Find the highest common factors

a. $6x^2 + 3x$ b. $15y^3 + 12y^4$ c. $9a^4b - 18a^5b + 27a^6b$

Solution a. The HCF of $6x^2 + 3x$ is $3x$ … Observe that $3x$ is a common factor.

$6x^2 + 3x = (3x \times 2x) + (3x \times 1)$ … Write each term as the product of $3x$ and another factor.

$= 3x\,(2x + 1)$ …… Use the distributive property to factor out the HCF.

Therefore, the HCF of $6x^2 + 3x$ is $3x$.

✓ **Check:** $3x\,(2x + 1) = 6x^2 + 3x$

b. The HCF of $15y^3 + 12y^4$ is $3y^3$ …Observe that $3y^3$ is a common factor.

$15y^3 + 12y^4 = (3y^3 \times 5) + (3y^3 \times 4y)$ …. Write each term as the product of $3y^3$ and another factor.

$= 3y^3(5 + 4y)$ ……Use the distributive property to factor out the HCF.

Therefore, the HCF of $15y^3 + 12y^4$ is $3y^3$.

c. $9a^4b - 18a^5b + 27a^6b$ is $9a^4b$ …Observe that $9a^4b$ is a common factor.

$= (9a^4b \times 1) - (9a^4b \times 2a) + (9a^4b \times 3a^2)$ …… Write each term as the product of $9a^4b$ and another factor.

$= 9a^4b(1 - 2a + 3a^2)$ …… Use the distributive property to factor out the HCF.

Therefore the HCF of $9a^4b - 18a^5b + 27a^6b$ is $9a^4b$.

Factorizing, out a binomial factor

The distributive property may also be used to factor out a common factor that consists of more than one term such as a binomial as shown in the next example.

Example 30: Factor out the highest common factor:
$2x(5x + 3) - 5(5x + 3)$

Solution

$2x(5x + 3) - 5(5x + 3)$ …….. The Highest common factor is the binomial $5x + 3$

$= (5x + 3) \times (2x) - (5x + 3) \times (5)$ ……… Write each term as the product of $(5x + 3)$ and another factor.

$= (5x + 3)(2x - 5)$ ….. Use the distributive property to factor out the HCF.

✓Check: $(5x + 3)(2x - 5) = (5x + 3)(2x) + (5x + 3)(-5)$

$= 2x(5x + 3) - 5(5x + 3)$

Exercise 2G

1. Find the highest common factor among each group of terms.
   - a. $-8xy$ and $20y$
   - b. $20xyz$ and $15yz^2$
   - c. $6x$ and $3x^2$
   - d. $2ab$, $6abc$ and $4a^2c$
   - e. $3x^2y$, $6xy^2$ and $9xyz$
   - f. $15a^3b^2$ and $20ab^3c$
   - g. $6ab^4c^2$ and $12a^2b^3cd$
2. Find the highest common factor of the pairs of the terms given below.
   - a. $(2a - b)$ and $3(2a - b)$
   - b. $7(x - y)$ and $9(x - y)$
   - c. $14(3x + 1)^2$ and $7(3x + 1)$
   - d. $a^2(x + y)$ and $a^3(x + y)^2$
   - e. $21x(x + 3)$ and $7x^2(x + 3)$
   - f. $5y^3(y - 2)$ and $-20y(y - 2)$
3. Factor out the highest common factor.
   - a. $13(a + 6) - 4b(a + 6)$
   - b. $7(x^2 + 2) - y(x^2 + 2)$
   - c. $8x(y^2 - 2) + (y^2 - 2)$
   - d. $4(x + 5)^2 + 5x(x + 5) - (x + 5)$
   - e. $6(z - 1)^3 + 7z(z - 1)^2 - (z - 1)$
   - f. $x^4 - 4x$

Challenge Problems

4. Factor by grouping: $3ax + 12a + 2bx + 8b$

Summary For Unit 2

1. *A variable is a symbol or letter used to represent an unspecified value in expression.*
2. *An algebraic expression is a collection of variables and constant under algebraic operations of addition or subtraction. For example, $y + 10$ and $2t - 2 \times 8$ are algebraic expressions.*

The symbols used to show the four basic operations of addition, subtraction, multiplication and division are summarized in Table 2.1

Table 2.1		
Operation	Symbols	Translation
Addition	x + y	✓ Sum of x and y ✓ x plus y ✓ y added to x ✓ y more than x ✓ x increased by y ✓ the total of x and y
Subtraction	x – y	✓ difference of x and y ✓ x minus y ✓ y subtracted from x ✓ x decreased by y ✓ y less than x
Multiplication	$x \times y$, x(y), xy	✓ product of x and y ✓ x times y ✓ x multiplied by y
Division	$x \div y$, $\frac{x}{y}$, $^{x}/_{y}$	✓ Quotient of x and y ✓ x divided by y ✓ y divided into x ✓ ratio of x and y ✓ x over y

3. *For any rational numbers a, b and c*

   a. $a(b + c) = ab + ac$

   b. $a(b - c) = ab - ac$

   These two properties are called the ***distributive properties****.*

4. If (a + b) and (c + d) are any two binomials whose product (a + b) (c + d) is defined as (a + b) (c + d) = ac + ad + bc + bd.

5. The highest common factor (HCF) of numbers is the greatest number which is a common factor of the numbers.

The procedures of one of the ways to find the HCF is given below:

1) List the factors of the numbers.

2) Find the common factors of the numbers.

3) Determine the highest common factors of these common factors.

Miscellaneous Exercise 2

I. State whether each statement is true or false for all positive integers x, y, z and w.

1. If a number y has z positive integer factors, then y and 2z integer factors.
2. If 2 is a factor of y and 3 is a factor of y, then 6 is a factor of y.
3. If y has exactly 2 positive integer factors, then y is a prime numbers.
4. If y has exactly 3 positive integer factors, then y is a square.
5. If y has exactly 4 positive integer factors, then y is a cube.
6. If x is a factor of y and y is a factor z, then x is a factor of z.

II. Choose the correct answer from the given four alternatives.

7. A triangle with sides 6, 8 and 10 has the same perimeter as a square with sides of length _______?

   a. 6 b. 4 c. 8 d. 12

8. If x + y = 10 and x – y = 6, what is the value of $x^3 - y^3$?

   a. 604 b. 504 c. 520 d. -520

9. If ab + 5a + 3b + 15 = 24 and a + 3 = 6, then b + 5 = _____ ?

   a. 5 b. 50 c.4 d. 12

10. If ab = 5 and $a^2 + b^2 = 25$, then $(a + b)^2$ = ________ ?

    a. 35 b. 20 c. 15 d. 30

11. If n is an integer, what is the sum of the next three consecutive even integers greater than 2n?

    a. 6n + 12 b. 6n + 10 c. 6n + 4 d. 6n + 8

12. One of the following equation is false.

a. $A = \frac{1}{2}bh$ for $h = \frac{2A}{b}$

c. $P = 2(\ell + w)$ for $\ell = \frac{p}{2} - w$

b. $A = 2s^2 + 4sh$ for $h = \frac{A - 2s^2}{4s}$

d. $A = \frac{1}{2}bh$ for $h = \sqrt{\frac{2A}{h}}$

13. If x = 6 and y = 2, then what is the value of $3x^2 - 4\left(2y - \frac{4}{12}\right) + 8$.

a. $\frac{304}{3}$ b. $\frac{-304}{3}$ c. $\frac{-348}{3}$ d. $\frac{108}{3}$

14. Find the value of y, if $y = x^2 - 6$ and x = 7.

a. 49 b. 7 c. 43 d. 45

15. If x = 2 and y = 3, then what is the value of $y^x + xy \times y + x$?

a. 9 b. -29 c. 29 d. 18

16. If a = 4 and b = 7, then what is the value of $\frac{a + \frac{a}{b}}{a - \frac{a}{b}}$?

a. 8 b. 1 c. $\frac{4}{3}$ d. $\frac{8}{3}$

III. Work out Problems

17. Simplify each of the following expressions.

a. $(x^3 + 2x - 3) - (x^2 - 2x + 4)$

b. $2x(3x + 4) - 3(x + 5)$

c. $x(y^2 + 5xy) + 2xy(3x - 2y)$

d. $2(a^2b^2 - 4a^3b^3) - 8(ab^2 - 3a^2b^2)$

18. Express the volume of this cube.

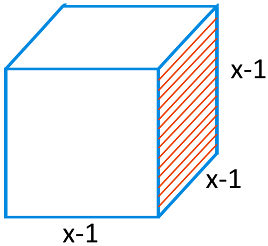


Figure 2.16 cube

19. Find the surface area of this cube.

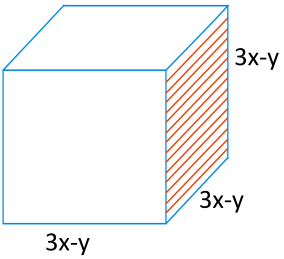


Figure 2.17 cube

20. Prove that the sum of five consecutive natural number is even.
21. Prove that 6(n + 6) – (2n + 3) is odd numbers for all $n \in \mathbb{N}$
22. Multiply the expressions.
    a. $(7x + y)(7x - y)$
    b. $(5k + 3t)(5k + 3t)$
    c. $(7x - 3y)(3x - 8y)$
    d. $(5z + 3)(z^2 + 4z - 1)$
    e. $\left(\frac{1}{3}m\text{-}n\right)^2$
    f. $(5a - 4b)(2a - b)$
    g. $\left(\frac{1}{5}x + 6\right)(5x - 3)$
    h. $(2h + 2{\cdot}7)(2h - 2{\cdot}7)$
    i. $(k - 3)^3$
    j. $(k + 3)^3$
23. Find the highest common factor for each expression.
    a. $12x^2 - 6x$
    b. $8x(x - 2) - 2(x - 2)$
    c. $8(y + 5) + 9y(y + 5)$
    d. $y(5y + 1) - 8(5y + 1)$
    e. $4x(3x - y) + 5(3x - y)$
    f. $2(5x + 9) + 8x(5x + 9)$
    g. $8q^9 + 24q^3$
24. Find three consecutive numbers whose sum shall equal 45.
25. Find three consecutive numbers such that twice the greatest added to three times the least amount to 34.
26. Find two numbers whose sum is 36 and whose difference is 10.
27. If a is one factor of x, what is the other factor?
28. Find the value of (x+5) (x+2) + (x–3) (x –4) in its simplest form. What is the numerical value when x = -6?
29. Simplify (x + 2) (x + 10) – (x – 5) (x – 4). Find the numerical value of this expression when x = -3.

UNIT 3

LINEAR EQUATIONs AND INEQUALITIES

Unit outcomes

After Completing this unit, you Should be able to:

- understand the concept equations and inequalities.
- develop your skills on rearranging and solving linear equations and inequalities.
- apply the rule of transformation of equations and inequalities for solving problems.
- draw a line through the origin whose equation is given.

Introduction

In this unit you will expand the knowledge you already have on solving linear equations and inequalities by employing the very important properties known as the associative property and distributive property of multiplication over addition and apply these to solve problems from real life. More over you will learn how to set up a coordinate plane and drawing straight lines using their equation.

3.1 Further on Solutions of Linear Equations

Group Work 3.1

Discuss with your friends/partners.

1. Solve the following linear equations using equivalent transformation.

   a. $6x - 8 = 26$
   b. $14x + 6x = 64$
   c. $5x - 17 - 2x = 6x - 1 - x$
   d. $5x - 8 = -8 + 3x - x$

2. Solve the following linear equations.

   a. $7(x-1) - x = 3 - 5x + 3(4x - 3)$
   b. $0.60x + 3.6 = 0.40(x + 12)$
   c. $8(x + 2) + 4x + 3 = 5x + 4 + 5(x + 1)$
   d. $8y - (5y - 9) = -160$
   e. $3(x + 7) + (x - 8)6 = 600$

3. Do you recall the four basic transformation rules of linear equations. Explain.

3.1.1 Solution of Linear Equations Involving Brackets

Activity 3.1

Discuss with your friends/partners.

Solve the following linear equations.

a. $4(2x + 3) = 3(x + 8)$
b. $6(5x - 7) = 4(3x + 7)$
c. $4(8y + 3) = 6(7y + 5)$
d. $3(6t + 7) = 5(4t + 7)$
e. $7(9d - 5) = 12(5d - 6)$
f. $10x - (2x + 3) = 21$

To solve an equation containing brackets such as $5(4x + 6) = 50 - (2x + 10)$, you transform it into an equaivalent equation that does not have brackets. To do this it is necessary to remember the following rules.

Note: For rational numbers a, b and c,

a) $a + (b + c) = a + b + c$
b) $a - (b + c) = a - b - c$
c) $a(b + c) = ab + ac$
d) $a(b - c) = ab - ac$

Example 1: Solve: x – 2(x -1) = 1 – 4(x + 1) Using the above rules.

Solution

x – 2 (x – 1) = 1 -4 (x +1) …Given equation

x – 2x + 2 = 1 – 4x – 4 ……. Removing brackets

x – 2x +4 x = 1 - 4 – 2………Collecting like terms

3x = -5 …………….Simplifying

$\frac{3x}{3} = \frac{-5}{3}$………..……Dividing both sides by 3

x = $\frac{-5}{3}$………..…… x is solved.

✓ Check: For x = $\frac{-5}{3}$

$$\frac{-5}{3} - 2\left(\frac{-5}{3} - 1\right) \stackrel{?}{=} 1 - 4\left(\frac{-5}{3} + 1\right)$$

$$\frac{-5}{3} + \frac{10}{3} + 2 \stackrel{?}{=} 1 + \frac{20}{3} - 4$$

$$\frac{-5}{3} + \frac{10}{3} + \frac{6}{3} \stackrel{?}{=} \frac{3}{3} + \frac{20}{3} - \frac{12}{3}$$

$$\frac{11}{3} = \frac{11}{3} \text{ (True)}$$

Example 2: Solve 4(x – 1) +3 (x + 2) = 5 (x – 4) using the above rules.

Solution

4(x -1) + 3(x + 2) = 5(x – 4) ……Given equation

4x- 4 + 3x + 6 = 5x - 20 ………Removing brackets

4x + 3x – 5x = -20 + 4 – 6…...…..Collecting like terms

2x = -22 ………………Simplifying

$\frac{2x}{2} = \frac{-22}{2}$ ……………...Dividing both sides by 2

x = - 11 ………..……X is solved

✓ Check: For x = -11

$$4(-11 - 1) + 3(-11 + 2) \stackrel{?}{=} 5(-11 - 4)$$

$$4(-12) + 3(-9) \stackrel{?}{=} 5(-15)$$

$$-48 - 27 \stackrel{?}{=} -75$$

- 75 = - 75 (True)

Exercise 3A

1. Solve each of the following equations, and check your answer in the original equations.
   a. $7x - 2x + 6 = 9x - 32$
   b. $21 - 6x = 10 - 4x$
   c. $2x - 16 = 16 - 2x$
   d. $8 - 4y = 10 - 10y$
   e. $8x + 4 = 3x - 4$
   f. $2x + 3 = 7x + 9$
   g. $5x - 17 = 2x + 4$
   h. $4x + 9 = 3x + 17$
2. Solve each of the following equations, and check your answer in the original equations.
   a. $7 - (x + 1) = 9 - (2x - 1)$
   b. $3y + 70 + 3(y - 1) = 2(2y + 6)$
   c. $5(1 - 2x) - 3(4 + 4x) = 0$
   d. $3 - 2(2x + 1) = x + 17$
   e. $4(8y + 3) = 6(7y + 5)$
   f. $8(2k - 6) = 5(3k - 7)$
   g. $5(2a + 1) + 3(3a - 4) = 4(3a - 6)$

Challenge problems

3. Solve the equation $8x + 10 - 2x = 12 + 6x - 2$.
4. solve the equation $-16(2x - 8) - (18x - 6) = -12 + 2(6x - 6)$.
5. Solve the equation $(8x - 4)(6x + 4) = (4x + 3)(12x - 1)$.
6. Solve for x in each of the following equations:
   a. $m(x + n) = n$
   b. $x(a + b) = b(c - x)$
   c. $mx = n(m + x)$

3.1.2 Solution of Linear Equations Involving Fractions

Group work 3.2

Discuss with your friends/partners.

1. Work out

a. $\frac{2}{7} + \frac{3}{50}$

b. $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$

c. $2\frac{9}{10} + 1\frac{5}{8}$

d. $3\frac{2}{5} + 2\frac{7}{15}$

e. $1\frac{3}{4} + 2\frac{5}{16}$

2. Work out.

a. $\frac{21}{4} - \frac{1}{15}$ b. $4\frac{7}{8} - 1\frac{2}{5}$ c. $6\frac{1}{5} - 5\frac{1}{7}$ d. $7\frac{4}{7} - 4\frac{2}{5}$

3. Work out.

a. $\frac{2}{35} \times 2\frac{5}{6}$ b. $2\frac{1}{3} \times \frac{7}{10}$ c. $21\frac{1}{7} \times 1\frac{3}{5}$ d. $3\frac{5}{6} \times 2\frac{5}{7}$

4. Work out

a. $3\frac{5}{9} \div \frac{20}{9}$ b. $36\frac{7}{3} \div 2\frac{2}{5}$ c. $4\frac{3}{5} \div \frac{2}{3}$ d. $2\frac{3}{2} \div \frac{15}{2}$

5. In a school, $\frac{7}{16}$ of the students are girls. What fraction of the students are boys?

6. A box containing tomatoes has a total weight of $5\frac{7}{8}$ kg.
The empty box has a weight of $1\frac{1}{4}$kg. what is the weight of the tomatoes?

7. A machine takes $5\frac{1}{2}$ minutes to produce a special type of container. How long would the machine take to produce 15 container?

From grade 5 and 6 mathematics lesson you have learnt about addition, subtraction, multiplication and division of fractions. All of these are shown on the following discussion.

Adding fractions

It is easy to add fractions when the denominators (bottom) are the same:

Easy to add:

$$\frac{35}{29} + \frac{39}{29} = \frac{74}{29}$$

Denominators are the same

what about this?

$$\frac{38}{9} + \frac{37}{11} = ?$$

Denominators are different

Adding fractions with the same denominator

$$\frac{7}{20} + \frac{2}{20} = \frac{9}{20}$$

Add the numerators (top)

Add them over the same denominators, (bottom)

Example 3:

Adding fractions with different denominators

$$\frac{38}{9}+\frac{37}{11}=?$$

First find equivalent fractions to these ones which have the same denominator (bottom):

Fractions equivalent to $\frac{38}{9}$

$$\frac{38\times 11}{9\times 11}=\frac{418}{99}$$

Fractions equivalent to $\frac{37}{11}$

$$\frac{37\times 9}{11\times 9}=\frac{333}{99}$$

$$\frac{418}{99}+\frac{333}{99}=\frac{751}{99}$$

Equivalent to $\frac{38}{9}$ Equivalent to $\frac{37}{11}$

So $\frac{38}{9}+\frac{37}{11}=\frac{751}{99}$

Note: To add fractions, find equivalent fractions that have the same denominator or (bottom).

Subtracting fractions

It is easy to subtract fractions when the denominators (bottom) are the same:

Easy to subtract:

$$\frac{7}{12}-\frac{26}{12}=\frac{-19}{12}$$

Denomintaors are the same

What about this?

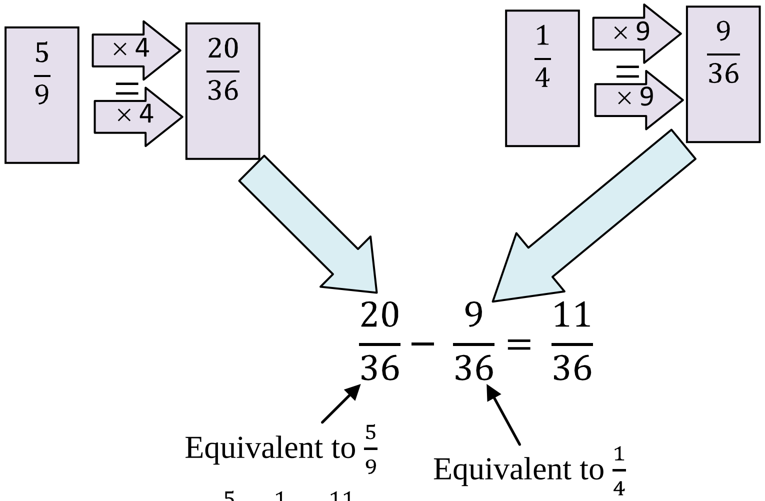
$$\frac{5}{9}-\frac{1}{4}=?$$

Denominators are different.

Example 4: (Subtracting fraction with different denominators)

work out $\frac{5}{9}-\frac{1}{4}$

Solution: Find equivalent fractions to these ones which have the same denominator (bottom). An easy way is to change both denominator to 36 because $9\times4 = 36$ is LCM of the denominators.



So $\frac{5}{9} - \frac{1}{4} = \frac{11}{36}$

Note: To subtract fractions, find equivalent fractions that have the same denominator (bottom).

Multiplying fractions

To multiply two fractions, multiply the numerators together and multiply the denominators together.

For example,

$$\frac{50}{18} \times \frac{7}{10} = \frac{350}{180}$$

← **Multiply the numerators (top)**
← **Multiply the denominators (bottom)**

You can simplify this to $\frac{35}{18}$ (by dividing the top and bottom of $\frac{350}{180}$ by 10).

Therefore, $\frac{50}{18} \times \frac{7}{10} = \frac{35}{18}$

Dividing fractions

To divide fractions, invert or take the reciprocal of the dividing fraction (turn it upside down) and multiply by the divisor.

For example

Chang the "÷ " $\frac{1}{21} \div \frac{3}{7} = ?$

Sign in to change the fraction you are dividing by up side down.

a "×" sign This is called **inverting** the fraction.

$\frac{1}{21} \times \frac{7}{3} = \frac{1}{9}$

Now let us consider linear equations having fractional coefficients.

Example 5: Solve $\frac{x+1}{3} + \frac{x-1}{10} = 12.$

Solution: $\frac{x+1}{3} + \frac{x-1}{10} = 12$ …..Given equation

The LCM of the denominators is 3 × 10 = 30 since 3 and 10 do not have any common factors.

Therefore, multiplying both sides by 30.

$30\left(\frac{x+1}{3} + \frac{x-1}{10}\right) = 30 \times 12$

$30\left(\frac{x+1}{3}\right) + 30\left(\frac{x-1}{10}\right) = 30 \times 12$……..By the distributive property

$10(x + 1) + 3(x - 1) = 360$ ………Simplifying

$10x + 10 + 3x - 3 = 360$ ………Removing brackets

$13x + 7 = 360$ ………Collecting like terms

$13x + 7 - 7 = 360 - 7$……Subtracting 7 from both sides

$13x = 353$………Simplifying

$\frac{13x}{13} = \frac{353}{13}$ … … ….Dividing both sides by 13

$x = \frac{353}{13}$

The solution set is $\left\{\frac{353}{13}\right\}$.

✓ Check: $\frac{x+1}{3} + \frac{x-1}{10} = 12$

$\frac{\frac{353}{13}+1}{3} + \frac{\frac{353}{13}-1}{10} \stackrel{?}{=} 12$

$\frac{353+13}{39} + \frac{353-13}{130} \stackrel{?}{=} 12$

$\frac{366}{39} + \frac{340}{130} \stackrel{?}{=} 12$

$$\frac{47580+13260}{5070} \stackrel{?}{=} 12$$

$$\frac{60840}{5070} \stackrel{?}{=} 12$$

$$12 = 12 \text{ (True)}$$

Example 6: Solve $\frac{7}{24} = \frac{x}{8} + \frac{1}{6}$.

Solution: The LCM of the denominators is 24.

$24\left(\frac{7}{24}\right) = 24\left(\frac{x}{8} + \frac{1}{6}\right)$ ….Multiplying both sides by 24.

$24\left(\frac{7}{24}\right) = 24\left(\left(\frac{x}{8}\right) + 24\left(\frac{1}{6}\right)\right)$…Distributive property

$7 = 3x + 4$ …………Removing brackets

$7 - 4 = 3x + 4 - 4$………Subtracting 4 from both sides

$3 = 3x$ …………Simplifying

Or $\frac{3x}{3} = \frac{3}{3}$… … ..Dividing both sides by 3

$x = \frac{3}{3} = 1$

The solution set is {1}.

Example 7: Solve $\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4} = 16$.

Solution: The LCM of the denominators is 12.

$12\left(\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4}\right) = (12 \times 16)$…Multiplying both sides by 12.

$12\left(\frac{x+1}{2}\right) + 12\left(\frac{x+2}{3}\right) + 12\left(\frac{x+3}{4}\right) = 12\times16$……Distributive property

$6(x + 1) + 4(x + 2) + 3(x + 3) = 12\times16$……Simplifying

$6x + 6 + 4x + 8 + 3x + 9 = 192$………Removing brackets

$13x + 23 - 23 = 192 - 23$...Subtracting 23 from both sides

$13x = 169$ …………Simplifying

$\frac{13x}{13} = \frac{169}{13}$……….Dividing both sides by 13

$x = 13$

The solution set is {13}.

✓ **Check:** $\frac{13+1}{2} + \frac{13+2}{3} + \frac{13+3}{4} \stackrel{?}{=} 16$

$$\frac{14}{2} + \frac{15}{3} + \frac{16}{4} \stackrel{?}{=} 16$$

$$7 + 5 + 4 \stackrel{?}{=} 16$$

$$16 = 16 \text{ (True)}$$

Example 8: Solve $\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$.

Solution: The LCM of the denominators 3 and 2 is 6.

$\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$ ………Given equation

$6\left[\frac{1}{3}(x+7) - \frac{1}{2}(x+1)\right] = 6\times4$ ……Multiply both sides by 6.

$6\left[\frac{1}{3}(x+7)\right] - 6\left[\frac{1}{2}(x+1)\right] = 6\times4$ …Distributive property

$2(x + 7) - 3(x + 1) = 24$ …Simplifying

$2x + 14 - 3x - 3 = 24$ …Removing brackets

$2x - 3x + 14 - 3 = 24$

$-x + 11 = 24$ ….Collecting like terms

$-x + 11 - 11 = 24 - 11$...Subtracting 11 from both sides

$-x = 13$ ……………….Simplifying

$\frac{-x}{-1} = \frac{13}{-1}$………..Dividing both sides by -1.

$x = -13$

The solution set is $\{-13\}$.

✓ Check:

$\frac{1}{3}(x+7) - \frac{1}{2}(x+1) \stackrel{?}{=} 4$

$\frac{1}{3}(-13+7) - \frac{1}{2}(-13+1) \stackrel{?}{=} 4$

$\frac{1}{3}(-6) - \frac{1}{2}(-12) \stackrel{?}{=} 4$

$-2 + 6 \stackrel{?}{=} 4$

$4 = 4$ (True)

Exercise 3B

1. Solve each of the following equations.

a. $\frac{x}{10} = \frac{2}{3}$

b. $\frac{6n}{2} - \frac{3n}{2} = 3\frac{1}{2}$

c. $\frac{-x}{2} + 6 = -3\frac{2}{8}$

d. $\frac{-3}{5} + \frac{x}{10} = \frac{-1}{5} - \frac{x}{5}$

e. $\frac{2x}{5} - \frac{2}{3} = \frac{x}{2} + 6$

f. $\frac{3x}{7} + \frac{35x}{8} = 10$

g. $\frac{5x}{13} + \frac{5x}{26} = 1$

h. $\frac{12}{23} - x = 4$

2. Solve each of the following equations and check your answer in each case by inserting the solution in original equation.

   a. $\frac{5x}{6} + \frac{2}{3} = \frac{-1x}{6} - \frac{5}{3}$

   b. $\frac{3}{7}x - \frac{1}{4} = \frac{-4x}{7} - \frac{5}{4}$

   c. $\frac{24}{5}w + 14 = 62 - \frac{6}{10}w$

   d. $\frac{9x}{7} - 10 = \frac{48x}{7} + 14$

   e. $\frac{2x+2}{2} + \frac{3x+6}{3} + \frac{4x+16}{4} = -6$

   f. $\frac{4+2x}{6x} = \frac{12}{5x} + \frac{2}{15}$

   g. $\frac{2x+7}{3} - \frac{x-9}{2} = \frac{5}{2}$

   h. $\frac{2x+3}{6} - \frac{x-5}{4} = \frac{3}{8}$

Challenge Problems

3. Solve the following equations.

   a. $12 - \frac{x-2}{2} = \frac{6-x}{4} + \frac{x-4}{4}$

   b. $\frac{2x-10}{11} - \frac{2x-4}{7} = 10x - 17\frac{1}{2}$

   c. $\frac{x+9}{4} - \frac{x-12}{5} = 6^{2}$

   d. $0.78 - \frac{1}{25}h = \frac{3}{5}h - 0.5$

3.1.3 Solve Word Problems Using Linear Equations

Group work 3.3

1. **Two complementary angles are drawn such that one angle is 10° more than seven times the other angle. Find the measure of each angle.**
2. **A certain number of two digits is three times the sum of its digits, and if 45 be added to it the digits will be reversed; find the number.**

Mathematical problems can be expressed in different ways. Common ways of expressing mathematical problems are verbal or words and formulas or open statements. In this sub-unit you will learn how to translate verbal problems to formulas or mathematical expressions so that you can solve it easily. It is important to translate world problems to open statements because it will be clear and concise.

Although there is no one definite procedure which will insure success to translate word problems to open statements to solve it, the following steps will help to develop the skill.

Table 3.1 Problem – solving Flow chart for word problems

Step	Flow chart	Explanation
Step 1	Read the problem carefully.	• Familiarize yourself with the problem. Identify the unknown, and if possible estimate the answer.
Step 2	Assign lables to unknown quantities	• Identify the unknown quantity or quantities. Let x represent one of the unknowns. Draw a picture and write down relevant formulas.
Step 3	Develop a verbal models	• Write an equation in words.
Step 4	Write a mathematical equation	• Replace the verbal model with a mathematical equation using x or other variable.
Step 5	Solve the equation	• Solve for the variable using steps for solving linear equations.
Step 6	Interpet the results and write the final answer in words	• Once you have obtained a numerical value for the variable, recall what it represents in the context of the problems.
Step 7	Check all answers back into the original statements of the problems	

Example 9: The sum of a number and negative ten is negative fifteen. Find the number.

Solution:

Let x represent the unknown number — Step 1: Read the problem; Step 2: Label the unknown

(a number) + (-10) = -15 — Step 3: Develop a verbal model

$x + (-10) = -15$ — Step 4: Write the equation

$x + (-10) + 10 = -15 + 10$ — Step 5: Solve for x

$x = -5$

Therefore, the number is -5. — Step 6: Write the final answer in words.

Example 10: **(Applications involving sales Tax)**

A video game is purchased for a total of Birr 48.15 including sales tax. If the tax rate is 7%. Find the original price of the video game before sales tax is added.

Solution: Step 1: Read the problem

Let x represent the price of the video game.

0.07x represents the amount of sales tax. Step 2: Label variables

$\begin{pmatrix}\text{orignal}\\ \text{price}\end{pmatrix} + \begin{pmatrix}\text{sales}\\ \text{tax}\end{pmatrix} = \begin{pmatrix}\text{total}\\ \text{cost}\end{pmatrix}$ Step 3: Write a verbal equation

x + 0.07x = Birr 48.15 Step 4: Write a mathematical equation

1.07x = 48 .15 Step 5: Solve for x multiply by 100 to clear decimals

100(1.07x) = 100(48.15)

107x = 4815

$\frac{107x}{107} = \frac{4815}{107}$ Step 6: Divide both sides by 107

$x = \frac{4815}{107}$

x = 45 Step 7: Interpret the results and write the answer in words.

Therefore, the original price was Birr 45.

Example 11: **(Applications involving consecutive integers)**

Find three consecutive even numbers which add 792.

Solution: Step 1: Read the problem

Let the smallest even number be x. Step 2: label the unknow

Then the other even numbers are (x+2) and (x+4) Step 3: Develop a verbal model

Because they are consecutive even numbers.

x+(x+2) +(x+4) = 792 Step 4: Write the equation

3x + 6 = 792

3x = 786

x = 262

The three even numbers are 262, 264 and 266. Step 5: Write the final answer in Word

✓ **Check**

262 + 264 +266 = 792 Step 6: Check

Example 12: (Applications involving ages)

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution: let m = the man's age and w = the wife's age.

$\Rightarrow$ m + w= 96 ……………Translated equation (1)

$\Rightarrow$ m = 6 + w ……………...Translated equation (2)

$\Rightarrow$ (6 + w) + w = 96 ……... Substituting equation (2) into 1

$\Rightarrow$ 2w + 6 = 96 ……………Collecting like terms

$\Rightarrow$ 2w = 96 – 6 ……………Subtracting 6

$\Rightarrow$ 2w = 90 ………………..Collecting like terms

$\Rightarrow$ w = 45 …………………Divided both sides by 2

Therefore, the age of his wife is 45 years old.

Exercise 3C

Solve each problem by forming an equation.

1. The sum of three consecutive numbers is 276. Find the numbers.
2. The sum of three consecutive odd number is 177. Find the numbers.
3. Find three consecutive even numbers which add 1524.
4. When a number is doubled and then added to 13, the result is 38. Find the number.
5. Two angles of an isosceles triangle are x and (x+10). Find two possible values of x.
6. A man is 32 years older than his son. Ten years ago he was three times as old as his son. Find the present age of each.

Challenge Problems

7. A shop –keeper buys 20kg of sugar at Birr y per kg. He sells 16kgs at Birr $\left(y+\frac{3}{4}\right)$ per kg and the rest at Birr (y + 1) per kg. what is his profit.
8. A grocer buys x kg of potatoes at Birr 1.50 per kg and y kg of onions at Birr 2.25 per kg. how much money does he pay in Birr?

9. If P is the smallest of four consecutive even integers, what is their sum interms of P?
10. The sum of a certain number and a second number is -42. The first number minus the second number is 52. Find the numbers.

3.2 Further on Linear Inequalities

Activity 3.2

Discuss with your friends/partners.

1. Can you recall the definition of linear inequality?
2. Discuss the four rules of transformation of linear inequalities using examples and discuss the result with your teacher.
3. Solve the following linear inequalities.

a. $4x - 16 < 12, x \in \mathbb{W}$

b. $\frac{2}{3}x < -4(x-5), x \in \mathbb{Z}^+$

c. $20 - \frac{3}{2}x \geq \frac{3}{2}x - 18, x \in \mathbb{Q}^+$

d. $0.5(x-8) \leq 10 + \frac{3}{2}x, x \in \mathbb{Q}$

From grade 6 and 7 mathematics lesson you have learnt about to solve linear inequalities in one variable based on the given domain.

Example 13: (Solving an inequality)

Solve the inequality $-3x + 8 \leq 22$.

Solution:

$-3x + 8 \leq 22$............Given inequalities

$-3x + 8 - 8 \leq 22 - 8$ ....Subtracting 8 from both sides

$-3x \leq 14$ ......Simplifying

$\frac{-3x}{-3} \geq \frac{14}{-3}$...Dividing both sides by -3; reverse the inequality sign

$x \geq \frac{-14}{3}$.......Simplifying

Therefore, the solution set is $\left\{x: x \geq \frac{-14}{3}\right\}$.

Example 14: (Solving an inequality)

Solve the inequality $24x - 3 < 4x + 10, x \in \mathbb{Q}$.

Solution:

$24x - 3 < 4x + 10 \; x \in \mathbb{Q}$ ....Given inequalities

$24x - 3 + 3 < (4x + 10) + 3$ ....Adding 3 from both sides

$24x < 4x + 13$....................Simplifying

$24x - 4x < 4x - 4x + 13$……..Subtracting 4x from both sides

$20x < 13$……………..Simplifying

$\frac{20x}{20} < \frac{13}{20}$……..Dividing both sides by 20

$x < \frac{13}{20}$……….Simplify

Therefore, the solution set is $\left\{x: x < \frac{13}{20}\right\}$.

Example 15: **(Solving an inequality)**

Solve the inequality $4x - 6 > 10$, $x \in \mathbb{N}$.

Solution: $4x - 6 > 10$ …………Given inequalities

$(4x - 6) + 6 > 10 + 6$ ……..Adding 6 from both sides

$4x > 16$ ………..Simplifying

$\frac{4x}{4} > \frac{16}{4}$…………..Dividing both sides by 4

$x > 4$

The solution of the inequality is $x > 4$.

Therefore, the solution set is $\{x: x > 4\} = \{5, 6, 7, 8, 9, \ldots\}$.

Example 16: **(Solving an inequality)**

Solve the inequality $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x, x \in \mathbb{Z}$.

Solution: $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x$………Given inequalities

$12\left(\frac{-1}{4}x + \frac{1}{6}\right) \leq 12\left(2 + \frac{2x}{3}\right)$……Multiply both sides by 12 to clear fractions.

$12\left(\frac{-1}{4}x\right) + 12\left(\frac{1}{6}\right) \leq 12(2) + 12\left(\frac{2}{3}x\right)$…….Apply the distributive property

$-3x + 2 \leq 24 + 8x$ …………Simplifying

$-3x - 8x + 2 \leq 24 + 8x - 8x$ ……..Subtracting 8x from both sides

$-11x + 2 \leq 24$ ………..Collecting like terms

$-11x + 2 - 2 \leq 24 - 2$ ………Subtracting 2 from both sides

$-11x \leq 22$ …………..Simplifying

$\frac{-11x}{-11} \geq \frac{22}{-11}$ …………Dividing both sides by -11. Reverse the inequality sign.

$x \geq -2$

There fore, the solution set is {-2, -1, 0, 1, 2, 3, …}.

Example 17: **(Solving an inequality)**

Solve the inequality $3x - 2(2x - 7) \leq 2(3 + x) - 4$, $x \in \mathbb{N}$.

Solution:

$3x - 2(2x - 7) \leq 2(3 + x) - 4$ …Given inequalities

$3x - 4x + 14 \leq 6 + 2x - 4$ ………Removing brackets

$-x + 14 \leq 2x + 2$ ………………...Simplifying

$-x - 2x + 14 \leq 2x - 2x + 2$ ……….Subtracting 2x from both sides

$-3x + 14 \leq 2$ ……………………..Simplifying.

$-3x + 14 - 14 \leq 2 - 14$…………...Subtracting 14 from both sides

$-3x \leq -12$ ……………………….Simplifying

$\frac{-3x}{-3} \geq \frac{-12}{-3}$ …………….Dividing both sides by -3. Reverse the inequality sign

$x \geq 4$

The solution of the inequality is $x \geq 4$.

Therefore, the solution set is $\{x: x \geq 4\} = \{4, 5, 6, 7, 8, 9, ...\}$.

Exercise 3D

1. Solve the following inequalities:
   a. $\frac{1}{2}(x + 4) \geq \frac{3}{4}(x - 2)$
   b. $\frac{x}{4} + 5 \leq x + 4$
   c. $8x - 5 > 13 - x$
   d. $4x + 6 > 3x + 3$
   e. $\frac{1}{4}x + 7 \leq \frac{1}{3}x - 2$
   f. $9 + \frac{1}{3}x \geq 4 - \frac{1}{2}x$
   g. $\frac{1}{2}(2x + 3) > 0$
2. Solve each of the following linear inequality in the given domain.
   a. $4 - \frac{5}{6}x > \frac{3}{2}x - 8$, $x \in \mathbb{Q}$
   b. $4y - 6 < \frac{1}{2}(28 - 2y)$, $y \in \mathbb{W}$
   c. $\frac{5}{3}x < -8(x - 6)$, $x \in \mathbb{Z}^+$
   d. $-2(12 - 2x) \geq 3x - 24$, $x \in \mathbb{Q}^+$
   e. $5x + 6 \leq 3x + 20$, $x \in \mathbb{N}$
   f. $\frac{3y}{4} + \frac{1}{6} > \frac{17}{10}$, $y \in \mathbb{Z}$
   g. $6x \geq 16 + 2x - 4$, $x \in \mathbb{Z}$
   h. $10x + 12 \leq 6x + 40$, $x \in \mathbb{N}$
3. Eight times a number increased by 4 times the number is less than 36. What is the number?
4. If five times a whole number increased by 3 is less than 13, then find the solution set.

Challenge Problems

5. Solve each of the following linear inequalities:
   a. 3 (x + 2) – (2x – 7)≤ (5x -1) -2(x + 6)
   b. 6 – 8(y + 3) +5y > 5y –(2y -5) +13
   c. $-2 - \frac{W}{4} \leq \frac{1+w}{3}$
   d. -0. 703 < 0.122 x -2 .472
   e. 3.88 – 1.335t ≥5.66

3.3 Cartesian Coordinate System

3.3.1 The Four Quadrants of the Cartesian Coordinate Plane

Group work 3.4

1. Write down the coordinates of all the points marked red in Figure 3.1 to the right .

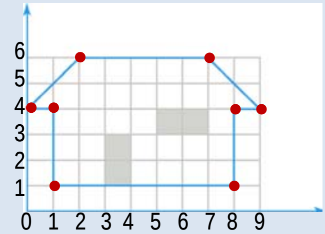


Figure 3.1

2. Write the coordinates of the points A, B, C, D, E, F, G and H shown in Figures 3.2 to the right.

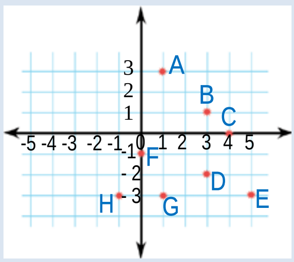


Figure 3.2

3. Name the quadrant in which the point of p(x, y) lines when:
   a. x > 0 , y > 0
   b. x < 0, y>0
   c. x > 0, y < 0
   d. x <0, y <0

For determing the position of a point on a plane you have to draw two mutually perpendicular number lines. The horizontal line is called the **X –axis**, while the vertical line is called the **Y-axis**. These two axes together set up a plane called the **Cartesian coordinate planes**. The point of intersection of these two axis is called the **origin**. On a suitably chosen scale, points representing numbers on the X-axis are called **X-coordinates or abscissa**, while chose on the y-axis are called **Y-coordinates or ordinat**. The x-coordinate to the right of the y-axis are positive, while those to the left are negative. The y- coordinates above and below the X-axis are positive and negative respectively. Let XOX' and YOY' be the **X-axis and the Y-axis** respectively and let P be any point in the given plane. For determing the coordinates of the point P, you draw lines through P parallel to the coordinate axis, meeting the X-axis in M and the y-axis in N.

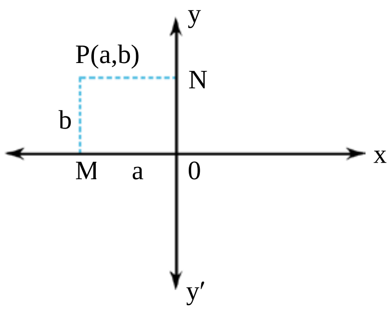


Figure 3.3

The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise (counter clockwise) direction, the quadrants which you come across are called **the first**, **the second**, **the third** and **the fourth quadrants** respectively.

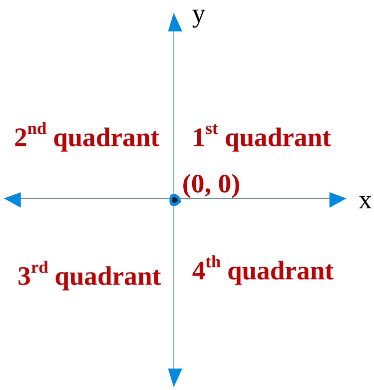


Figure 3.4

Note: i. In the first quadrant all points have a positive abscissa and a positive ordinate.

ii. In the second quadrant all points have a negative abscissa and a positive ordinate.

iii. In the third quadrant all points have a negative abscissa and a negative ordinate.

iv. In the fourth quadrant all points have a positive abscissa and a negative ordinate.

EXERCISE 3E

1. Draw a pair of coordinate axes, and plot the point associated with each of the following ordered pair of numbers.

A(-3, 4)	D (0, -3)	G (0, 6)
B(4, 6)	E (-3, -2)	H (2.5, 3)
C(4, -3)	F (-5, 6)	I (-2, 4.5)

2. Based on the given Figure 3.5 to the right answer the following questions.

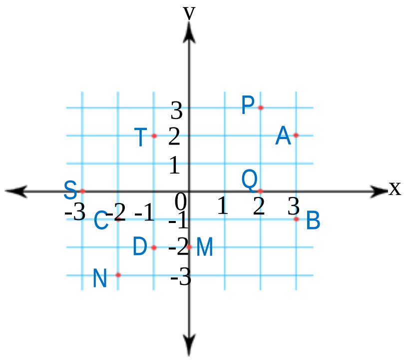


Figure 3.5

a. Write the coordinates of the point A, B, P, S, N and T.
b. Which point has the coordinates (-1, -2)?
c. Which coordinate of the points Q is zero?
d. Which coordinate of the points D and M is the same?
e. To which axis is the line DM parallel?
f. To which axis is the line AT parallel?
g. If F is any point on the line AT, state its y-coordinate.
h. To which axis is the line PQ parallel?

Challenge Problems

3. Answer the following:

    a. On which axis does the point A(0,6) lie?

    b. In which quadrant does the point B(-3, -6) lie?

    c. Write the coordinates of the point of intersection of the x-axis and y- axis.

3.3.2 Coordinates and Straight Lines

Group Work 3.5

1. **Write down the equations of the lines marked (a) to (d) in the given Figure 3.6 to the right.**

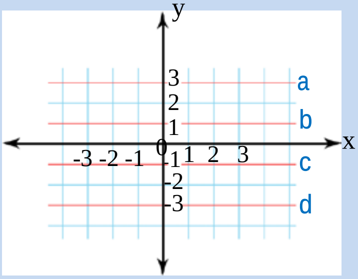


Figure 3.6

2. **Write down the equations of the lines marked (a) to (d) in the given Figure 3.7 to the right.**
3. **Draw the graphs of the following equations on the same coordinate system:**

    a. y = x **c. y = 4x**

    b. y = -x **d. y = -4x**

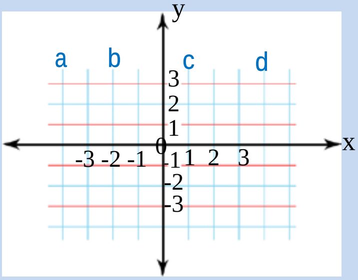


Figure 3.7

4. True or false. If the statement is false, rewrite it to be true.

a. The line x = 30 is horizontal. **b. The line y = -24 is horizontal.**

5. True or false. If the statement is false, rewrite it to be true.

a. A line parallel to the y – axis is vertical.

b. A line perpendicular to the x – axis is vertical.

For exercise 6 – 9, identify the equation as representing a vertical line or a horizontal line.

6. 2x + 7 =10 **7. 9 = 3 + 4y** **8. -3y + 2 = 9** **9. 7 = -2x – 5**

10. Write an equation representing the x – axis.

11. write an equation representing the y – axis.

Graph of an equation of the form x = a (a ∈ ℚ)

The graph of the equation x= a (a∈ ℚ, a≠0) is a line parallel to the y-axis and at a distance of **a unit** from it.

> **Note:** i. If a > 0, then the line lies to the right of the y-axis.
> ii. If a <0, then the line lies to the left of the y-axis.
> iii. The graph of the equation x= 0 is the y-axis.

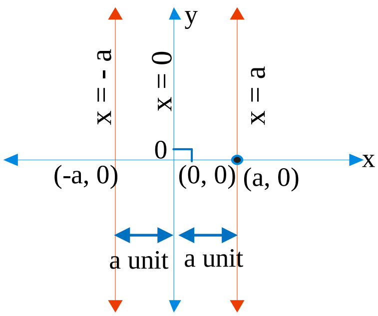


Figure 3.8

Example 18: Draw the graphs of the following straight lines.

a. x = 6 b. x = -6

Solution: First by drawing tables of values for x, and y in which x-is constant and following this you plot these points and realize that the points lie vertical line.

a..

x	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
y	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8

b.

x	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	-5	-4	-3	-2	-1	0	1	2	3	4	5

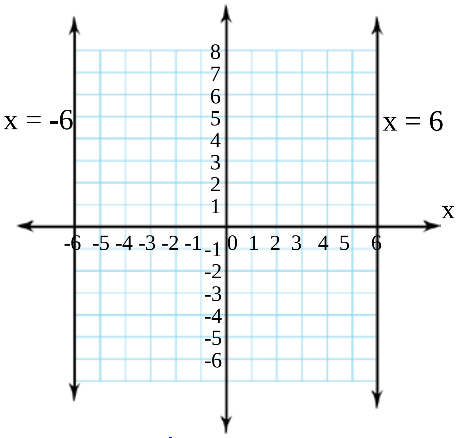


Figure 3.9

Graph of an equation of the form y = b (b ∈ ℚ)

The graph of the equation y= b (b∈ ℚ, b≠0) is the line parallel to the x-axis and at a distance of b from it.

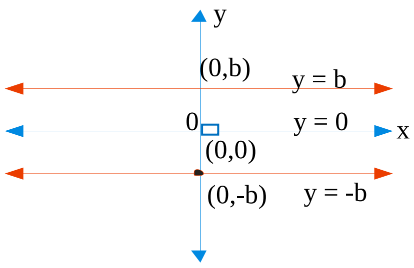


Figure 3.10

Note: i. If b >0, then the line lies above the x-axis.
ii. If b < 0, then the line lies below the x-axis.
iii. The graph of the equation y= 0 is the x-axis.

Example 19: Draw the graphs of y = 4.

Solution: First by drawing tables of values for x, and y in which y is constant and following this you plot these points and realize that the points lie horizontal line.

x	-4	-3	-2	-1	0	1	2	3	4
y	4	4	4	4	4	4	4	4	4

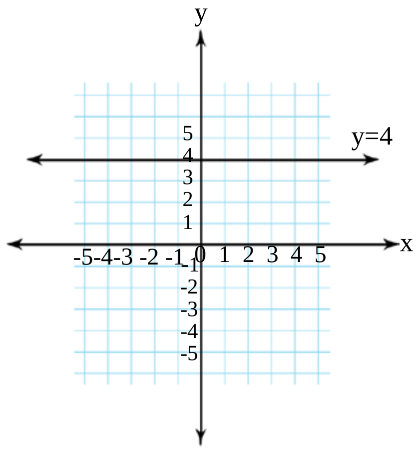
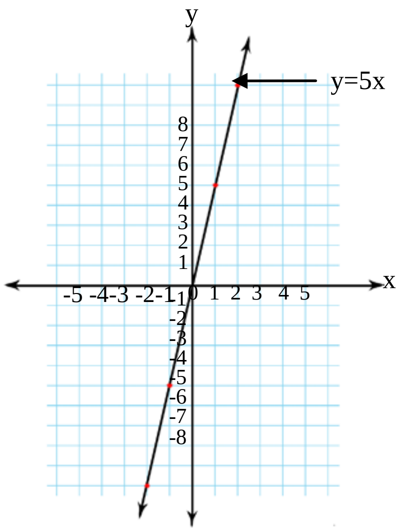


Figure 3.11

Graph of an equation of the form y = mx (m∈ ℚ and m ≠0)

In grade 6 and 7 mathematics lesson we discussed about y = kx, where y is directly proportional to x, with constant of proportionality k. For example y = 4x where y is directly proportional to x with constant of poroportionality 4. Similarly how to draw the graph of y= mx, (m∈ℚ), look at the following examples.

Example 20: Draw the graphs of y = 5x.



Solution:

Step i: Choose some values for x, for example let x= -2,-1, 0, 1 and 2.

Step ii: Put these values of x into the equation y = 5x:

When x= -2: y =5(-2) = -10

When x = -1: y = 5(-1) = -5

When x = 0: y = 5(0) = 0

When x = 1: y = 5(1) = 5

When x = 2: y = 5(2) = 10

Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	-10	-5	0	5	10

Step iv: Plot the points (-2, -10), (-1, -5), (0, 0) (1, 5) and (2, 10) and join them to get a straight line.

Figure 3.12

Step v: Lable the line y = 5x.

Example 21: Draw the graphs of y = -5x.

Solution:

Step i: Choose some values for x, for example let x= -2, -1, 0, 1 and 2.

Step ii: Put these values of x into the equation y= -5x

When x= -2: y = -5(-2) =10

When x = -1: y= -5(-1) = 5

When x = 0: y = -5(0)=0

When x= 2: y = -5(2) = -10

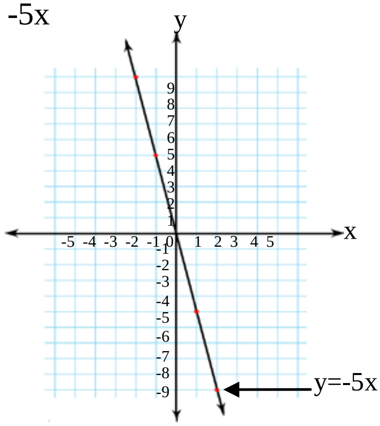


Figure 3.13

Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	10	5	0	-5	-10

Step iv: Plot the points (-2, 10), (-1, 5), (0, 0) (1, -5) and (2, -10) and join them to get a straight line.

Step v: Label the line y=-5x

3.3.3. The Slope "m" Of Straight Line

Activity 3.3

Discuss with your friends.

1. What is a slope?

2. What is the slope of a line parallel to the y-axis?

3. What is the slope of a horizontal line?

4. What is the slope of a line parallel to the x – axis?

5. What is the slope of a line that rises from left to right?

6. What is the slope of a line that falls from left to right?

7. a. Draw a line with a negative slope.

b. Draw a line with a positive slope.

c. Draw a line with an undefined slope.

d. Draw a line with a slope of zero.

From your every day experience, you might be familiar with the idea of **slope**. In this sub – topic you learnt how to calculate the slope of a line by dividing the change in the y – value by change in the x – value, where the y – value is the vertical height gained or lost and the x – value is the horizontal distance travelled.

$$\textbf{Slope} = \frac{\text{change in } y - \text{value}}{\text{change in } x - \text{value}}$$

In Figure 3.14 to the right, consider a line drawn through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$. From P to Q the change in the x coordinate is $(x_2 - x_1)$ and the change in the y coordinate is $(y_2 - y_1)$. By definition, the slope of the line AB is given by:

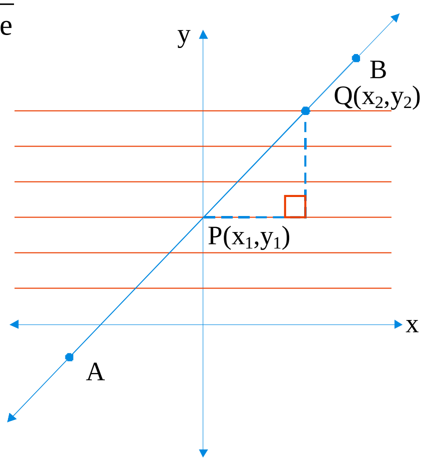


Figure 3.14

$$\frac{y_2 - y_1}{x_2 - x_1}; x_2 \neq x_2$$

Note: If we denote the slope of a line by the letter "m".

Definition 3.1: If $x_1 \neq x_2$ the slope of the line through the points (x_1, y_1) and (x_2, y_2) is the ratio:

$$\text{Slope} = m = \frac{\text{Change in } y-\text{value}}{\text{change in } x-\text{value}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Group work 3.6

Discuss with your friends (partners).

1. In Figure 3.15 below, determine the slope of the roof.

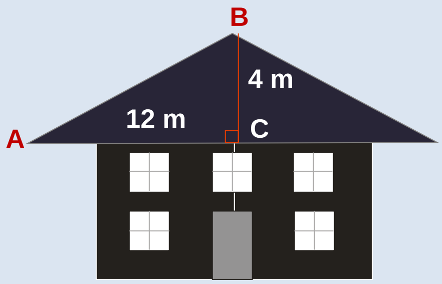


Figure 3.15

2. State the slope of the straight line that contains the points p(1, -1) and Q(8, 10).
3. Find the slope of a line segment through points (-7, 2) and (8,6).
4. Find the slope of each line.

   a) $y = 4$ b) $x = 7$

Example 22: Find the slope of the line passing through the point P(-4, 2) and Q(8, -4).

Solution: $\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{8-(-4)} = \frac{-6}{12} = \frac{-1}{2}$

Therefore, $-\frac{1}{2}$ is the coefficient of x in the line equation $y = -\frac{1}{2}x$.

Example 23: Find the slope of the line passing through each of the following pairs of points.

a) P(4, -6) and Q(10, -6) b) P $\left(\frac{-1}{4},-4\right)$ and Q$\left(\frac{-1}{4},4\right)$

Solution:

a. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-6)}{10-4} = \frac{-6+6}{6} = \frac{0}{6} = 0$

b. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-4)}{\frac{-1}{4}-\left(\frac{-1}{4}\right)} = \frac{8}{0}$ undefined

Note: i. The horizontal line has a slope of 0.
ii. The vertical line has no slope (not defined).

Example 24: Draw the graphs of the following equations on the same Cartesian coordinate plane.

a. $y = \frac{7}{6}x$ b. y = -3x c. y = 4x d. $y = \frac{2}{3}x$

Solution: First to draw the graph of the equation to calculated some ordered pairs that belongs to each equation shown in the table below.

x	-3	-2	-1	0	1	2	3
$y=\frac{7}{6}x$	$\frac{-7}{2}$	$\frac{-7}{3}$	$\frac{-7}{6}$	0	$\frac{7}{6}$	$\frac{7}{3}$	$\frac{7}{2}$
y = -3x	9	6	3	0	-3	-6	-9
y = 4x	-12	-8	-4	0	4	8	12
$y=\frac{2}{3}x$	-2	$\frac{-4}{3}$	$\frac{-2}{3}$	0	$\frac{2}{3}$	$\frac{4}{3}$	2

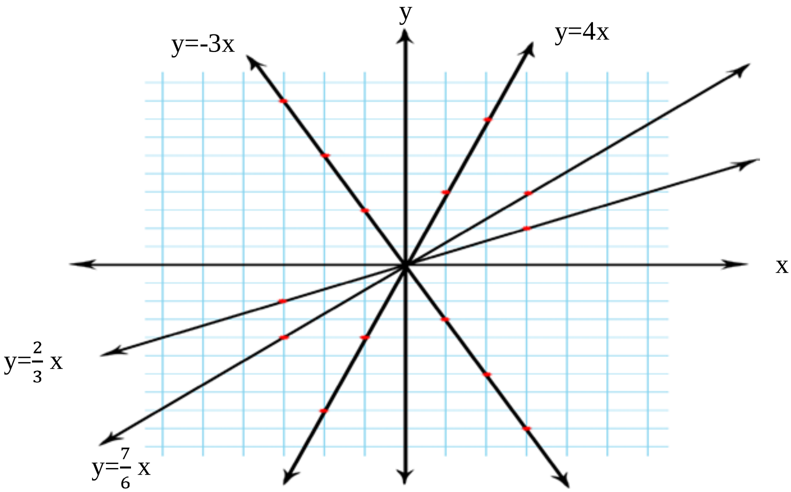


Figure 3.16

From the above graphs, you can generalize that:

i. All orderd pairs, that satisfy each linear equation of the form $y = mx$ $(m \in \mathbb{Q}, m \neq 0)$ lies on a straight lines that pass through the origin.

ii. The equation of the line $y = mx$, m is called the **slope** of the line, and the graph passes the **1st and 3rd quadrants if m > 0**, and the graph passes through the **2nd and 4th quadrants if m< 0.**

EXERCISE 3F

1. Draw the graphs of the following equations on the same coordinate system:

   a. $y= -6x$ b. $y = 6x$ c. $y = \frac{5}{2}x$ d. $y = \frac{-5}{2}x$

2. Draw the graphs of the following equations on the same coordinate system:

   a. $y + 4x = 0$ c. $x = 3$ e. $2x - y = 0$

   b. $2y = 5x$ d. $x + 4 = 0$ f. $\frac{3}{2}x - \frac{y}{2} = 0$

3. Complete the following tables for drawing the graph of $y = \frac{2x}{3}$

x	1	6	3
y			
(x, y)			

Challenge Problems

4. Point (3, 2) lies on the line $ax+2y = 10$. Find a .
5. Point (m, 5) lies on the line given by the equation $5x - y = 20$. Find m.
6. Draw and complete a table of values for the graphs $y=2x -1$ and $y= x- 2$
7. a. Show that the choice of an ordered pair to use as (x_1, y_1) does not affect the slope of the line through (2, 3) and (-3, 5).

   b. Show that $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

   For Exercise 8 – 11 fined the slope of the line that passes through the two points.

8. $P\left(\frac{-2}{7}, \frac{1}{3}\right)$ and $Q\left(\frac{8}{7}, \frac{-5}{6}\right)$
9. $A\left(\frac{1}{2}, \frac{3}{5}\right)$ and $B\left(\frac{1}{4}, \frac{-4}{5}\right)$
10. C (0, 24) and D (30, 0)

11. $E\left(0, \frac{5}{7}\right)$ and $F\left(0, \frac{9}{26}\right)$
12. Find the slope between the points
    A (a + b, 4m – n) and B (a – b, m + 2n)
13. Find the slope between the points
    C(3c – d, s + t) and D (c – 2d, s – t)
14. Write the equation of the line which has the given slope "m" and which passes through the given point.
    a. (2, 10) and m = - 4 b. (4, -4) and m = $\frac{3}{2}$ c. (0, 0) and m = $\frac{3}{5}$
15. State the slope and y-intercept of the line 2x + y + 1 = 0.
16. Find the slope and y-intercept of $y - y_o = m(x - x_o)$ where x_o and y_o are constants.
17. Find the slope and y-intercept of each line:
    a. (x + 2) (x + 3) = (x – 2) (x – 3) + y
    b. x = mu + b
18. State the slope and y-intercept of each linear equations.
    a. 6(x + y) = 3(x – y)
    b. 2(x + y) = 5(y + 1)
    c. 5x + 10y – 20 = 0
19. Write the slope-intercept equation of the line that passes through (2,5) and (-1,3).

Summary For unit 3

1. *You can transform an equation into an equivalent equation that does not have brackets. To do this it is necessary to remember the following rules.*

   a. $a + (b + c) = a + b + c$ **c.** $a(b + c) = ab + ac$
   b. $a - (b + c) = a - b - c$ **d.** $a(b - c) = ab - ac$

2. *The following rules are used to transform a given equation to an equivalent equation.*
   a. *For all rational numbers a, b and c:*
   If $a = b$ *then* $a + c = b + c$ *and* $a - c = b - c$. *that is, the same number may be added to both sides and the same number maybe subtracted from both sides without affecting the equality.*
   b. *For all rational numbers a, b and c where* $c \neq 0$:
   If $a = b$ *then* $ac = bc$ *and* $\frac{a}{c} = \frac{b}{c}$. *That is both sides may be multiplied by the same non-zero number and both sides may be divided by the same non-zero number without affecting the equality.*
3. *To solve word problems, the following steps will help you to develop the skill. The steps are:*
   a. *Read the problem carefully, and make certain that you understand the meanings of all words.*
   b. *Read the problem a second time to get an overview of the situation being described and to determine the known facts as well as what is to be found.*
   c. *Sketch any figure, diagram or chart (if any) that might be helpful in analyzing the problem.*
   d. *Choose a variable to represent an unknown quantity in the problem.*
   e. *Form an equation containing the variable which translates the conditions of the problem.*
   f. *Solve the equation.*
   g. *Check all answer back into the original statements of the problem.*
4. *The following rules are used to transform a given inequality to an equivalent inequality.*
   a. *For all rational numbers a ,b and c, if* $a < b$ *then* $a + c < b + c$ *or* $a - c < b - c$. *That is, if the same number is added to or subtracted from both sides of an inequality, the direction of the inequality remains unchanged.*

b. For all rational numbers a, b and c

i. If **$a < b$ and $c > 0$,** then **$ac < bc$ or $\frac{a}{c} < \frac{b}{c}$**. That is, if both sides of an inequality are multiplied or divided by the same positive number, the direction of the inequality is unchanged.

ii. If **$a < b$ and $c < 0$,** then **$ac > bc$ or $\frac{a}{c} > \frac{b}{c}$**. That is, if both sides are multipled or divided by the same negative number, the direction of the inequality is **reversed**.

5. The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise direction, the quadrants which you come across are called **the first**, **the second**, **the third** and the **fourth** quadrants respectively.

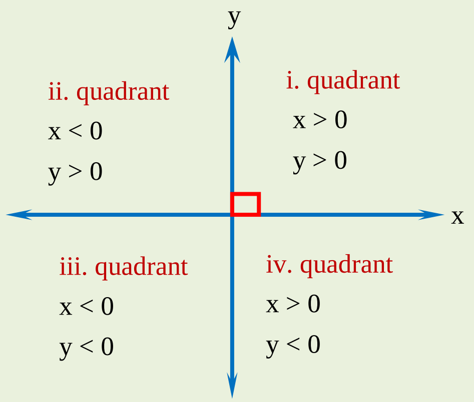


Figure 3.17

6. If $x_1 \neq x_2$ the slope of the line through the points (x_1, y_1) and (x_2, y_2) is the ratio:

$$\textbf{Slope} = m = \frac{\text{Change in } y\text{-value}}{\text{Change in } x\text{-value}}$$

$$= \frac{Y_2 - Y_1}{X_2 - X_1}$$

7. All orderd pairs, that satisfy each linear equation of the form $y = mx$ ($m \in \mathbb{Q}$, $m \neq 0$) lies on a straight lines that pass through the origin.

8. The equation of the line $y = mx$, m is called the slope of the line, and the graph passes the 1st and 3rd quadrants if $m > 0$, and the graph passes through the 2^{nd} and 4^{th} quadrants if $m < 0$.

Miscellaneous Exercise 3

I. Write true for the correct statements and false for the incorrect ones.

1. For any rational numbers a, b and c, then a(b + c) = ab + ac.
2. If any rational number a > 0, ax +b > 0, then the solution set is $\left\{x: x \geq \frac{-b}{a}\right\}$.
3. If any rational number a < 0, ax +b > 0, then the solution set is $\left\{x: x < \frac{-b}{a}\right\}$.
4. The equation of the line y = 4x, 4 is the slope of the line and the graph passes the 2nd and 3rd quadrants, since 4 > 0.
5. The equation of the line y= 4x + 6 that pass through the origin of coordinates.
6. The graphs of the equation y= b (b ∊ ℚ, b ≠ 0), if b > 0 then the equation of the line lies above the x-axis.
7. The graph of the equation x= a (a ∊ ℚ, a ≠ 0), if a < 0 then the equation of the lines to right of the y-axis.

II. Choose the correct answer from the given alternatives

8. In one of the following linear equations does pass through the origin?

   a. $y = \frac{3}{7}x + 10$ c. $y = \frac{5}{8}x$

   b. $y = -3x + \frac{3}{5}$ d. y = 2x - 6

9. The solution set of the equation $\frac{3x+2}{5} - \frac{2x-5}{3} = 2$ is:

   a. {1} b. {−1} c. $\left\{\frac{1}{2}\right\}$ d. $\left\{\frac{3}{2}\right\}$

10. The solution set of the equation 2x +3 (5 -3x) =7 (5- 3) is:

    a. {5} b. $\frac{1}{7}$ c. {3} d. $\left\{\frac{5}{3}\right\}$

11. If $\frac{2}{5x} = 2 + \frac{1}{x}$, (x ≠0), then which of the following is the correct value of x?

    a. $\frac{1}{8}$ b. $\frac{3}{10}$ c. $\frac{-7}{10}$ d. $\frac{-3}{10}$

12. If x is a natural number, then what is the solution set of the inequality $0.2x - \frac{1}{5} \leq 0.1x$?

    a. {x: x≤ 0 or x≥ 1} b. ϕ c. {1, 2} d. {0, 1, 2}

13. Which one of the following equations has no solution in the set of integers $\mathbb{Z}$?
    a. $6x + 4 = 10$ c. $9 – 12x = 3$
    b. $8x + 2 = 4x – 6$ d. $\frac{3}{2}x - 3 = 3x$
14. What is the solution set of the inequality $20(4x - 6) \leq 80$ in the set of positive integers?
    a. $\{1, 2, 3, 4\}$ b. $\{1, 2\}$ c. ϕ d. $\{0, 1, 2, 3, 4\}$
15. The sum of the ages of a boy and his sister is 32 years. The boy is 6 years older than his sister. How old is his sister?
    a. 15 b. 19 c. 14 d. 13

III. Work out problems

16. Solve each of the following linear equation by the rules of transformation.
    a. $4x + 36 = 86 – 8x$
    b. $12x – 8 + 2x - 17 = 3x – 4 – 8 + 74$
    c. $4(2x - 10) = 70 + 6x$
    d. $20 – 2x = 62 (x – 3)$
    e. $2(6y – 18) - 102 = 78 – 18(y+2)$
    f. $7(x + 26 + 2x) = 5(x + 7)$
17. Sove each of the following equations.
    a. $\frac{2x+7}{3} - \frac{x-9}{2} = \frac{5}{2}$ b. $\frac{x+3}{6} - \frac{x-5}{4} = \frac{3}{8}$ c. $\frac{x+2}{3} + \frac{x+3}{8} = \frac{5}{6}$
18. Solve each of the following linear inequalities by the rules of transformation.
    a. $6x – 2 < 22$ c. $8x – 44 < 12(x - 7)$ e. $\frac{x}{5} - 8 > \frac{-x}{3}$
    b. $-9 \leq 3x + 12$ d. $8(x - 3) \geq 15x - 10$ f. $6(2+6x) \geq 10x - 12$
19. **(word problems)**
    a. The sum of three consecutive odd integers is 129. Find the integers.
    b. Two of the angles in a triangle are complementary. The third angle is twice the measure of one of the complementany angles. What is the measure of each of the angles?
    c. Abebe is 12 years old and his sister Aster is 2 years old. In how many years will Abebe be exactly twice as old as Aster?
20. Draw the graphs of the equations $y = \frac{8}{3}x$ and $y = -\frac{8}{3}x$ on the same coordinate plane. Name their point of intersection as p. State the coordinate of the point p.

21. Find the equation of the line with y-intercept (0,8) and slope $\frac{3}{5}$.
22. Find the slope and y-intercept of $y = 10x - \frac{1}{3}$.
23. Find the slopes of the lines containing these points.
    a) (4,-3) and (6, -4)
    b) $\left(\frac{1}{8}, \frac{1}{4}\right)$ and $\left(\frac{3}{4}, \frac{1}{2}\right)$
    c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $\left(\frac{3}{2}, \frac{3}{4}\right)$
24. Find the slope of the line x = -24.
25. Find a and b, if the points P(6,0) and Q (3,2) lie on the graph of ax + by = 12.
26. Points P(3,0) and Q(-3,4) are on the line ax + by = 6. Find the values of a and b.
27. Point (a,a) lies on the graph of the equation 3y = 2x – 4. Find the value of a.
28. Find anequation of the line containing (3,-4) and having slope -2. If this line contains the points (a,8) and (5,b), find a and b.