MATHEMATICS

Grade 6

Student Textbook

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Acknowledgements

The redesign, printing and distribution of this student textbook has been funded through the General Education Quality Improvement Project (GEQIP), which aims to improve the quality of education for Grades 1–12 students in government schools throughout Ethiopia.

The Federal Democratic Republic of Ethiopia received funding for GEQIP through credit/financing from the International Development Associations (IDA), the Fast Track Initiative Catalytic Fund (FTI CF) and other development partners – Finland, Italian Development Cooperation, the Netherlands and UK aid from the Department for International Development (DFID).

The Ministry of Education wishes to thank the many individuals, groups and other bodies involved – directly and indirectly – in publishing the textbook and accompanying teacher guide.

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First edition, 2003(E.C.)

ISBN: 978-99944-2-215-9

Developed, printed and distributed for the Federal Democratic Republic of Ethiopia, Ministry of Education by:

Al Ghurair Printing and Publishing House CO. (LLC)
PO Box 5613
Dubai
U.A.E.

In collaboration with
Kuraz International Publisher P.L.C
P.O. Box 100767
Addis Ababa
Ethiopia

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UNIT 1

BASIC CONCEPTS OF SETS

Unit outcomes: **After completing this unit you should be able to:**

- understand the concept of set.
- describe the relation between two sets.
- perform two operations (intersection and union) on sets.

Introduction

The idea of a set is familiar in everyday life. Do you have a set of books, a set of tools, or a set of pens? Each of these sets is regarded as a unit. Sets, however need not consists of physical objects; they may well consist of abstract ideas. For instance, the 'Ten commandments' is a set of moral laws. The constitution is the basic set of laws of Ethiopia.

You will study sets in this unit not only because much of elementary mathematics can be based on this concept, but also because many mathematical ideas can be stated most simply.

1.1 Introduction to sets

The idea of collection of objects is familiar in everyday life. In our daily life we talk of a collection of things such as a class of students, a herd of cattle, a flock of sheep, a swarm of bees, etc. Can you think of more names for collections of things? In mathematics, a collection of things is called a **set**.

Definition 1.1: A **set** is a collection of well defined objects.

Group work 1.1

You can talk of the set of students in your class. How many students are there in your class? Which one is greater? The number of male students or the number of female students? Can you give other examples of sets?

Definition 1.2: Each object of a set is called an **element** of the set or member of the set.

Can you list some of the elements of the set of students in your class? What are the elements in the set of all vowels in English alphabet?

Note: A set can contain any variety of objects. For example, we may have a set that consists of the following things: a book, a pen, an orange and a bottle.

Group work 1.2

Study the picture

1. Write different sets you see on the picture. You may use classifications such as children, birds, hens and cats etc.
2. How many elements are there in each set?

Figure 1.1

1 BASIC CONCEPTS OF SETS

Let us take a set whose elements are 1, 2, 3, 4 and 5. To describe this set, you may use the notation $\{1, 2, 3, 4, 5\}$ which means" the set of natural numbers less than 6". The symbol $\{\ldots\}$ is used to group the members of a set. Usually we use a capital letter to designate a set. For example, $A = \{1, 2, 3, 4, 5\}$ denote the above set.

How would you use the braces notation to describe the set of students in your class? How about the set of countries you have visited? Choose the more example of a set that affects you personally, and state how you would describe it with the braces notation.

The notation $\{1, 2, 3, 4, 5\}$ is one way of describing a set which is called **tabulation or complete listing** method. In this method, all of the members of the set are listed. We read $A = \{1, 2, 3, 4, 5\}$ as "A is the set whose members are 1,2,3,4, and 5 " or "A is the set of natural numbers less than 6".

The way we describe a set should tell us what items belong to the set and what items are not members of the set.

Activity 1.1

Use braces to write the members of each of the following sets, or state that the set has no members.

a) The months of the year.

b) The whole numbers less than 99.

c) Students in grade 6 that are 3 years of age.

Example 1

Let set M be the set of multiples of 2 between 1 and 9. Here M = {2, 4, 6, 8}. You can see that 2, 4, 6 and 8 are members of the set M.

Example 2

The circled numbers form a set of even counting numbers up to 100. We can call it set E. Can you list the elements of set E?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

E = { 2, 4, 6, 8, ---, 96, 98, 100 }

Figure 1.2

The even numbers in set E are members or elements of set E. Braces are written before the first member and after the last member. Set E has many members, and so only the first three or four members are written and the last three or four members. In between, dots show the missing members.

$4 \in$ E means 4 is a member of set E.

$3 \notin$ E means 3 is not a member of set E.

You may represent the set of odd numbers up to 99 as D = {1, 3 , 5, 7, ---, 95, 97 , 99}. This is a second way of representing a set which is called **partial listing** method. Observe that $5 \in D$ but $8 \notin D$.

You may also represent factors (divisors) of 12 as F= {1 , 2 , 3 , 4 , 6, 12} and multiples of 3 less than 100 as M= {3, 6 , 9, 12, ---, 93, 96, 99} . Can you list elements of the set of factors of 20? Multiples of 7 less than 100?

Activity 1.2

Identify whether each of the following statements is true or false.

a) $3 \in \{1, 2, 3, 4\}$.

b) $3 \notin \{1, 2, \{3\}, 4\}$.

c) $\{3\} \in \{3\}$.

d) $3 \in \{33, 44, 55\}$.

Notice that a set with no element is called **empty or null** set and is denoted by { } or ϕ

Example 3

The set of students in your class who are 100 years old may represent an empty set. Can you give other examples of empty sets?

Activity 1.3

Which of the following sets are empty?

1. The set of months whose names have less than four letters.
2. The set of multiples of 3 which are less than 7.
3. The set of months whose names begin with letter 'y'.

Another way to specify a set consists in giving a rule or condition that enables us to decide whether or not any given objects belong to the set.

For example, P, the set of all females who are living in Addis Ababa can be described as

P= {x|x is a female living in Addis Ababa} which is read as "P is the set of x such that x is a female living in Addis Ababa".
This method of describing a set is called the **set builder notation.**

Example 4

The set of whole numbers can be described as
W={x|x is a whole number}

Exercise1 A

1. Identify whether each of the following statements is true or false.
   a) $1 \in \{1, 2, 3, 4, 5\}$.
   b) $0 \in \{2, 4, 6, 8, 10\}$.
   c) $3 \notin \{2, 10, 18, 26\}$.
   d) $\frac{1}{2} \notin \left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$.
   e) 2 is an element of the set of factors of 20.
   f) 72 is an element of the set of multiples of 6.
2. List the elements of the following sets.
   a) The set of factors of 24.
   b) The set of whole numbers between 3 and 11.
   c) The set of whole numbers less than 12.
   d) The set of multiples of 8 which are greater than 20 but less than 40.
3. Name the following sets.
   a) $S = \{0, 4, 8, 12, 16\}$.
   b) $R = \{a, e, i, o, u\}$.
   c) $Q = \{0, 1, 2, 3, 4, 5\}$.
4. Which of the following sets are empty?
   a) The set of factors of 6 which are greater than 10.
   b) The set of common factors of 16 and 24.
   c) The set of all human beings who are 3 meters tall.
   d) The set of consonants in the English alphabet.
   e) The set of countries in East Africa whose name start with the letter V.

f) The set of teachers in your school who are ten years old.

g) The set of cats that can fly.

1.2 Relations Among Sets

Let A = {1, 2, 3} and B = {1, 2, 3, 4}. Observe that every element of A is also an element of the set B. In such a case we say that Set A is a subset of set B.

Definition 1.3: A set A is a subset of set B if every element of set A is also an element of set B.

It is symbolically denoted by $A \subseteq B$ which is read as "set A is subset of set B".

If set A is not a subset of set B, we denote this by $A \nsubseteq B$.

Example 5

Let P = {a, b}, Q = {a, b, c }, then $P \subseteq Q$ but $Q \nsubseteq P$. It is also true that $P \subseteq P$, and $Q \subseteq Q$.

Activity 1.4

1. Is every set a subset of itself?
2. Is empty set a subset of every set?

Out of the set {2, 3, 4} we can form a set with no element, 1 element, 2 elements or 3 elements as follows

A = { }, B = { 2}, C = {3} , D = {4} , E = {2, 3} , F = {2, 4}, G = {3, 4}
H = {2, 3, 4}. These sets are subsets of the original set {2, 3, 4}.

✓ Consider two sets M = {2, 3} and P = { 2, 3, 4}. Observe that $M \subseteq P$ and there exists one element in P which is not an element of M (i.e $4 \in P$ but $4 \notin M$). In such a case we call set M is a proper subset of set P. It is denoted by $M \subset P$ which is read as ' M is a proper subset of P'.

Definition 1.4: Set A is a **proper subset** of set B if every element of set A is an element of the set B but there exists at least one element in B which is not an element of the set A.

Example 6

Given set p = {a, b, c}, the sets ϕ, {a}, {b}, {c}, {a, b}, {a, c} and {b, c} are proper subsets of set P.

Note

1. Empty set is a proper subset of every other set.
2. A set is not a proper subset of itself.

Definition 1.5: If two sets, A and B, have equal number of elements, then the two sets are called **equivalent sets**. We denote equivalent sets as $A \leftrightarrow B$.

Example 7

Let A = {1, 2, 3}, B = {a, b, c}. Then A and B are equivalent sets ($A \leftrightarrow B$) because both sets have three elements. Can you give other examples of equivalent sets?

Definition 1.6: If two sets have identical elements, then they are called **equal sets**.

Example 8

Let A = {1, 2, 3, 6} and
B = The set of divisors of 6. Then A = B, i.e
A and B are equal sets. Can you give your own examples of equal sets?

Exercise 1. B

1. Identify whether each of the following statements is true or false?

   a) $\{c, d, e\} \subseteq \{c, d, e\}$.

   b) $\{c, d, e\} \subset \{c, d, e\}$.

   c) The set of divisors of 6 is a subset of the set of divisors of 12.

   d) The set of multiples of 2 is a proper subset of the set of multiples of 4.

   e) If $A = \{6, 8, 9\}$ and $B = \{a, b, c\}$, then $A \leftrightarrow B$.

   f) The set of whole number less than 10 and the set of multiples of 2 which are less than 10 are equal sets.

2. Find all the pairs of equal sets from those below.

   $A = \{0, 2, 4, 6, 8\}$

   $B = \{1, 3, 5, 7\}$

   C = The set of even numbers less than 9.

   $D = \{1, 3, 5, 7, 9\}$

   $E = \{2, 3, 5, 7, 9, 11\}$

   $F = \{21, 23, 25, 27\}$

   $G = \{11, 9, 7, 5, 3\}$

   H = The set of odd numbers less than 10.

   $I = \{2, 4, 6\}$

   J = The set of odd numbers between 20 and 28.

3. Name any sets that are equivalent in question 2.

4. Which of the following are empty sets?

   a) The set of all human beings born with wings.

   b) The set of all even numbers which are greater than 2 and less than 4.

   c) The set of all odd numbers between 4 and 8.

   d) The set of all the distances which are greater than a meter and also less than a centimeter.

5. Let $A = \{a, c, e\}$.

   a) Form all subsets of A. How many are they?

   b) Form all proper subsets of A. How many are they?

1.3 Operations on Sets

You will study about intersection of sets, union of sets and a simple visual way of describing relationships between sets.

1.3.1 The Intersection of Sets

Activity 1.5

1. List the elements which belong to both sets
   $P = \{2, 4, 6, 8\}$ and $Q = \{4, 8, 12\}$?
2. Let A be the set of all multiples of 7 between 3 and 20 and B be the set of divisors of 8, Then list the elements which are common to both sets A and B.

Look at the following example carefully.

Example 9

Let $A = \{1, 2, 3, 7, 9\}$ and $B = \{7, 9, 11, 13\}$
The set of elements which belong to both sets A and B is called the intersection of set A and set B. It is denoted by $A \cap B$. We may write $A \cap B = \{7, 9\}$.

Definition 1.7: The set of elements that is common to the sets A and B is called the intersection of A and B and is denoted by $A \cap B$.

Example 10

Let $R = \{a, b, c, d\}$ and $T = \{a, d, e\}$, then $R \cap T = \{a, d\}$.

Exercise 1.C

1. Find $A \cap B$, if
   a) $A = \{2, 4, 6, 8\}$ and $B = \{4, 8, 12, 16\}$.
   b) A = The set of divisors of 10.
      B = The set of divisors of 12.
   c) A = The set of multiples of 3 which are less than 20.
      B = The set of multiples of 6 which are less than 20.
   d) A = The set of even numbers which are less than 8.
      B = The set of odd numbers which are less than 8.
2. In which of the above cases (question 1) is that $A \cap B = \phi$?
3. If A = B, then what can you say about $A \cap B$?

1.3.2 The Union of Sets

Activity 1.6

List the elements which belong to either set

$A = \{1, 2, 3, 4\}$ or $B = \{3, 4, 5\}$?

Definition 1.8: The set of elements which belong to either set P or set Q or both P and Q is called the **union** of set P and set Q. It is denoted by PUQ.

Group work 1.3

Form all subsets of set P where

$P = A \cup B$, and

$A = \{2, 4\}$, $B = \{4, 6\}$

Example 11

Let $P = \{1, 3, 5\}$, and $Q = \{2, 3, 4, 6\}$. The union of these sets is written as $P \cup Q = \{1, 2, 3, 4, 5, 6\}$.

Exercise 1.D

1. Find P ⋃ Q if

   a) P = {2, 3, 4, 5} and Q = {3, 4, 5, 6,7}.

   b) P = The set of odd numbers between 10 and 20.

      Q = The set of even numbers between 11 and 19.

   c) P = {a, b, c, 2, 3, 5} and Q = { d, e, 1, 4}.

   d) P = {Ayal, Alemu, Bekele, Chala} and Q = {Derartu, Habtamu, Hagos, Mohammed}.

   e) P = { 2^2, 3^2, 4^2, 5^2} and Q = { 4, 9, 13, 16, 25}.

2. If A = ϕ, then what can you say about A ⋃ B?

3. If A = B, then what can you say about A ⋃ B for any set B?

4. If A = {3, 4, 5}, B = { 1, 3, 6, 7} and C = { 8, 10 , 12} . Then find

a) A ⋃ B	d) A ⋂ B
b) A ⋃ C	e) A ⋂ C
c) B ⋃ C	f) B ⋂ C

1.3.3 Venn Diagram

The English Mathematician John Venn (1834 - 1923) invented a simple visual way of describing relationships between sets. His diagrams, now called **Venn diagrams** use circles to represent sets. Venn diagrams are learned best through examples.

Figure 1.3

Example 12

Consider the set of cows and the set of mammals. Because every member of the set of cows is also a member of the set of mammals, we say that the set of cows is a subset of the set of mammals. As shown in Figure 1.4, we represent this relationship in a Venn diagram by drawing the circle for cows inside the circle for mammals.

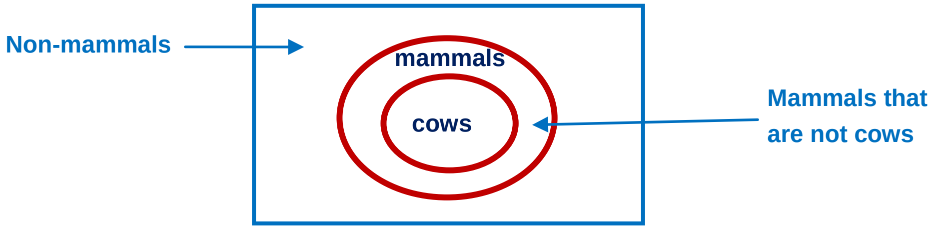


Figure 1.4

The circles are enclosed by a rectangle, so this diagram has three regions:

- The inside of the cows circle represents all **cows**.
- The region outside the cows circle but inside the **mammals** circle represents mammals that are not **cows** (such as bears, whales and people).
- The region outside the **mammals** circle represents non-mammals; from the context, we can interpret this region to represent animals (or living things) that are not mammals, such as birds, fish and insects.

Notice that (i) The diagram illustrates only the relationship between the sets; the sizes of circles do not matter.

(ii) The set of **cows** is a subset of the set of **mammals**.

Activity 1.7

We may represent sets $A = \{1, 3, 5, 6\}$ and $B = \{1, 2, 3, 4\}$ using Venn – diagram shown (Figure 1.5). list elements that belong to both sets A and B or the set $A \cap B$.

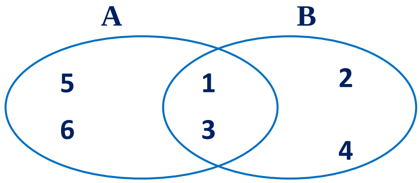


Figure 1.5

Definition 1.9: Two sets are said to be **disjoint** if their intersection is empty set. That is, $A \cap B = \phi$ implies sets A and B are disjoint.

Example 13

Consider the set of dogs and the set of cats. A domestic animal can be either a dog or a cat, but not both. We draw the Venn diagram with separated circles that do not touch , and we say that the set of dogs and the set of cats are disjoint sets (Figure 1.6). Again, we enclose the circles in a rectangle. This time, the context suggests that the region outside both circles represents domestic animals that are neither dogs nor cats, such as cows, sheep and goats.

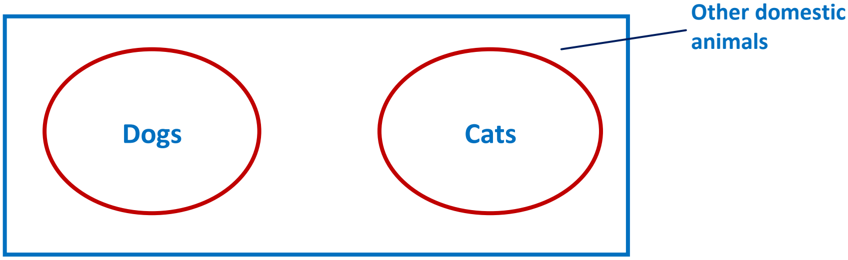


Figure 1.6

The set of dogs is disjoint from the set of cats.

For our last general case, consider the set of nurses and the set of women.

Example 14

As shown in figure 1.7, these sets are overlapping because it is possible for a person to be both a woman and a nurse. Because the two sets have intersection, this diagram has four regions.

- **The overlapping region represents people who are both women and nurses. (i.e. female nurses)**
- **The non-overlapping region of the nurses circle represents nurses who are not women-that is, male nurses.**

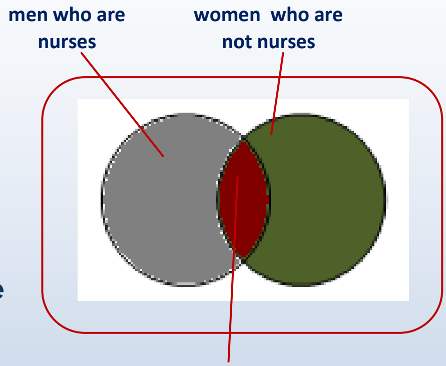


Figure 1.7

- **The non-overlapping region of the women circle represents women who are not nurses.**
- **The region outside both circles to represent people who are neither nurses nor women-that is, men who are not nurses.**

Note: The sizes of the regions are not important; for example, the small size of the overlapping region does not imply that female nurses are less common than male nurses. Speaking more generally, we use overlapping circles whenever two sets might have members in common.

Example 15

Let A = The set of even whole numbers less than 11. The Venn diagram for A looks like this. Notice that zero (0) is included in the set of even numbers.

B = the set of odd whole numbers less than 10. The Venn diagram for B looks like this.

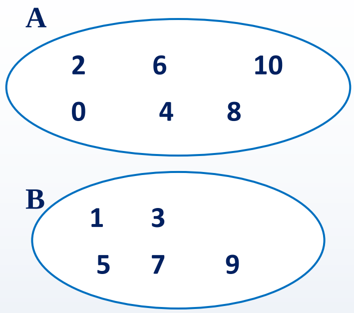


Figure 1.8

What would the Venn diagram for the empty set look like? How many members are there in A? How many members are there in B?

Are there any members that are common to both sets?

Solution: You may represent the two sets by a Venn diagram as shown in figure 1.9. The two sets are disjoint as there are no members in common.

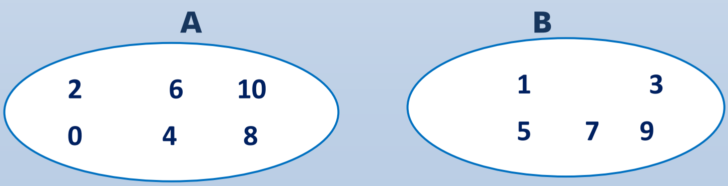


Figure 1.9

Group work 1.4

Which of the following describes the shaded region in the Venn-diagram (Figure 1.10)

a) P ⋂ Q ⋂ R c) P ⋃ Q ⋃ R

b) P ⋂ Q d) Q ⋂ R

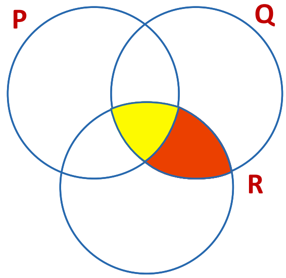


Figure 1.10

Example 16

Use a Venn diagram to show common factors of 24 and 30.

Solution: First make a list of the factors of 24 and 30

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 or

A = {1, 2, 3, 4, 6, 8, 12, 24}

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 or

B = {1, 2, 3, 5, 6, 10, 15, 30}

Then, use a Venn diagram to summarize the information. Draw a rectangle and then draw two overlapping circles inside the rectangle.

Label one circle A to represent factors of 24 and the other circle B to represent factors of 30. Write the elements common to both sets where the circle over lap.

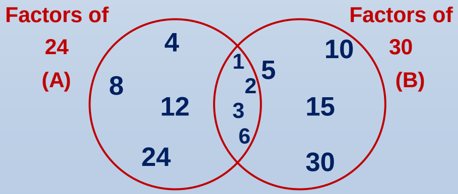


Figure 1.11

$A \cap B = \{1, 2, 3, 6\}$

Exercise 1.E

1. Based on the Venn diagram (Figure 1.12) answer each of the following.

   a) List elements of set P.

   b) List elements of set Q.

   c) List elements of $P \cap Q$.

   d) List elements of $P \cup Q$.

   e) Is $P \subseteq Q$? *is* $Q \subseteq P$?

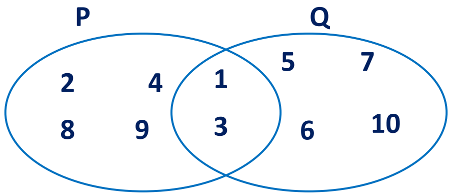


Figure 1.12

2. From the given Venn diagram below what can you say about

   a) A ?

   b) B?

   c) $A \cup B$?

   d) $A \cap B$?

   e) Is it true that $A \subseteq B$?

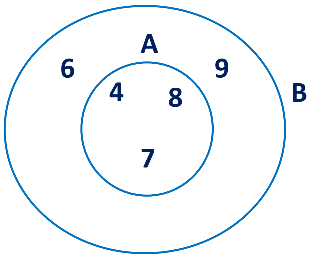


Figure 1.13

3. Use the Venn diagram to represent the following sets:

   G = {triangle, rectangle, circle, trapezium, kite}

   L = {square, parallelogram, kite, rectangle, trapezium, pentagon}

4. In a certain school the members of math club are Ujulu, Almaz, Mamo and Rukia and the members of Minimidia club are Mohammed, Ujulu, Urgessa and Mamo. Use Venn diagram to represent the situation.

5. Use Venn diagram to represent the following sets:

   M = The set of even numbers less than 11

   N = The set of whole numbers less than 11

6. Use the Venn diagram to answer each of the following:
   a) How many elements are there in set A?
   b) How many elements are there in set B?
   c) How many elements are there in the set A⋂B?
   d) How many elements are there out side the union of the two sets A and B?

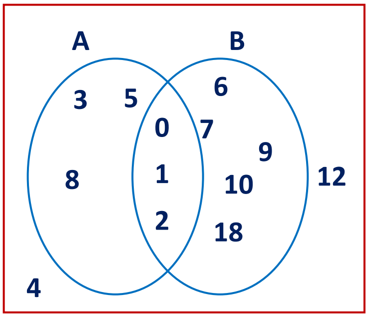


Figure 1.14

7. Let G = {2, 4, 6, 8, 10, 12} and H = {1, 3, 4, 6, 9}.
   a) Draw a Venn diagram that shows the relationship between the two sets.
   b) Shade the region common to both sets and find their common elements.

UNIT SUMMARY

Important facts you should know:

- **A set is a collection of well defined object.**
- **Each object in a set is called element of the set.**
- **A set with no element is called empty or null set. It is denoted by {} or ϕ.**
- **If all elements of set A belong to set B, then set A is called a subset of set B. This is denoted by $A \subseteq B$.**
- **If all elements of set A belong to set B and number of elements of set B is greater than number of elements set A, then set A is called a Proper subset of set B. This is denoted by $A \subset B$.**
- **If two sets, A and B, have equal number of elements, then the two sets are said to be equivalent sets. This is denoted by $A \leftrightarrow B$.**
- **If two sets, A and B, have identical elements, then they are called equal sets. This is denoted by $A = B$.**
- **The set of elements which belong to both set A and set B is called the intersection of set A and set B.**

This is denoted by $A \cap B$.

Two sets are said to be disjoint if their intersection is empty set.

- **The set of elements which belong to either set A or set B or both sets A and B is called the Union of set A and set B. This is denoted by $A \cup B$.**
- **Venn-diagram is a pictorial representation of a set. This name is given after an English mathematician John Venn.**
- **Venn-diagram is a pictorial representation of sets and their relationship.**

Review Exercise

1. Identify whether each of the following statements is true or false.
   a. $2 \in \{1, 2,3,4\}$
   b. $0 \in \{1,5,8,10\}$
   c. $\{3\} \in \{3,6,9,13\}$
   d. $\{1,3,5,7\} \subseteq \{5,7,9,11\}$
   e. $\phi \subseteq A$ for any set A
   f. $4 \notin A$ if $A = \{0,2,24,26\}$
   g. The set of multiples of 16 is a subset of the set of multiples of 8.
   h. The set of divisors of 20 is a proper subset of the set of divisors of 40.
2. Determine all
   a. Subsets of set A, $A = \{0, 4, 6\}$
   b. Proper subsets of set A, $A = \{0, 4, 6\}$
3. Based on the Venn diagram given below, answer each of the following
   a. List elements of set A
   b. List elements of set B
   c. List elements of $A \cap B$
   d. List elements of $A \cup B$

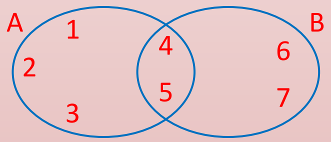


Figure 1.15

4. Let A = The set of multiples of 7.
   B = The set of divisors of 30.
   a. Are they disjoint sets?
   b. Find a set which is a subset of set A.
   c. Find a set which is a proper subset of set B.

5. Let P = The set of odd whole numbers.
   Q = The set of even whole numbers.
   a. Are they disjoint sets?
   b. What is the intersection of set P and set Q?
   c. What is the union of set P and set Q?
6. Consider the Venn-diagram given below. List elements of set
   a. A
   b. B
   c. C
   d. $A \cup B$
   e. $A \cup C$
   f. $B \cup C$
   g. $A \cap B$
   h. $B \cap C$
   i. $A \cap C$
   j. $A \cup (B \cup C)$
   k. $A \cap (B \cap C)$
   l. $A \cup (B \cap C)$
   m. $A \cap (B \cup C)$

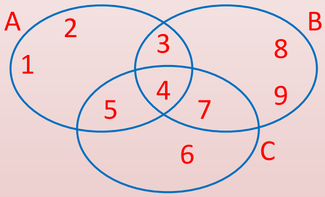


Figure 1.16

7. Shade the region which represents $A \cap (B \cap C)$ in the Venn-diagram below.

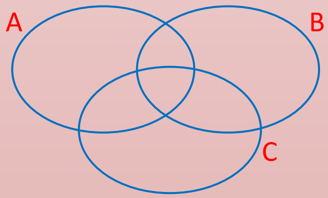


Figure 1.17

8. Find the total number of subsets of the set P, where P = {x| x is a letter in the word 'STAR'}.

9. Let A = {1, 3, 5, 7, 9},

   B = {1, 2, 4, 7, 8}, and

   C = {2, 4, 6, 8} . Show that

   (a) $A \cup (B \cup C) = (A \cup B) \cup C$

   (b) $A \cap (B \cap C) = (A \cap B) \cap C$

   (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

10. Refer to the accompanying figure and find the points that belong to each of the given sets.

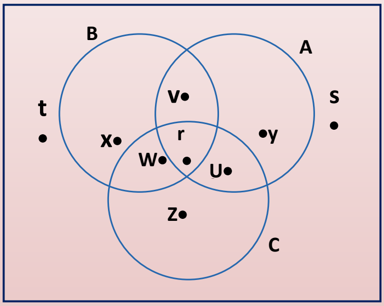


Figure 1.18

a. $A \cup B$

b. $A \cap B$

c. $A \cap (B \cup C)$

d. $B \cap C$

e. $A \cap (B \cap C)$

f. $A \cup B \cup C$

g. The points that do not belong to $A \cup B \cup C$

UNIT 2 THE DIVISIBILITY OF WHOLE NUMBERS

Unit outcomes: After completing this unit you should be able to:

- know the divisibility tests.
- identify prime and composite numbers.
- write prime factorization of a given whole number.

Introduction

You have some knowledge about divisibility of whole numbers from grade 4 mathematics. After a review of your knowledge about divisibility, you will continue studying divisibility tests, prime and composite numbers, and their properties. Also, you will discuss about prime factorization of a given whole number.

2.1 The Notion of Divisibility

Activity 2.1

Use long division to determine whether the following numbers are divisible by 3

(a) 2,781 (b) 7,020 (c) 10, 561

Suppose you are interested in sharing biscuits for your three class mates. To avoid arguments while the students share biscuits, you decide to divide up a total of 48 biscuits. Can you divide the biscuits among the three students?

48÷4 = 12 since the quotient is a **factor** of 48.

Thus, the biscuits can be evenly divided with each student receiving 16 biscuits.

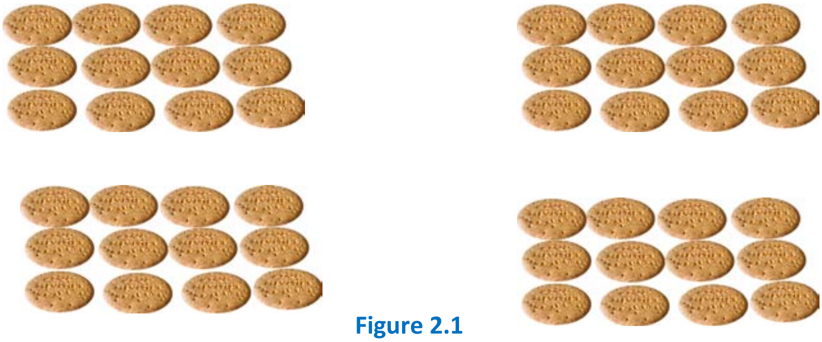


Figure 2.1

Notice that 48 is also divisible by 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

To check whether a number is divisible by a certain number, you need not perform the long division. There are some short and quick tests for divisibility. You have learnt some of the tests for divisibility in your previous mathematics lessons. Here you will deal with divisibility tests in more detail.

Divisibility Tests

- ✓ **Divisibility by 2**

  A number is divisible by 2 if the digit at the units place is even (that is 0, 2, 4, 6, or 8).

Example 1

450 ÷ 2, 29,332 ÷ 2, 45, 794 ÷ 2, 1, 156 ÷ 2 and 77, 638 ÷2.

- ✓ **Divisibility by 3**

  A number is divisible by 3 if the sum of its digits is divisible by 3.

Example 2

a) **576 is divisible by 3 because 5+7+6 = 18 and 18 is divisible by 3.**

b) **425 is not divisible by 3 because 4+ 2+5 = 11 and 11 is not divisible by 3.**

Divisibility by 4

A number is divisible by 4 if the digits at the units and ten's place (last two digits) are divisible by 4, or if the last two digits are 00.

Example 3

a) **3728 is divisible by 4 because 28 is divisible by 4.**

b) **573 is not divisible by 4 because 73 is not divisible by 4.**

✓ Divisibility by 5

A number is divisible by 5 if the digit at the unit's place is 0 or 5.

Example 4

1420 and 4325 are divisible by 5 but 6362 is not divisible by 5 (why?)

✓ Divisibility by 6

A number is divisible by 6 if it is even, and the sum of its digits is divisible by 3. Any number divisible by both 2 and 3 is divisible by 6 because $2\times3 = 6$.

Using the divisibility rules 2,3, 4 and 5 Can you say if: 94926 is divisible by 3?

Is 36700 divisible by 4? Is 6508 divisible by 2? 356075 divisible by 5

Example 5

The number 43182 is divisible by 2 because it is even. It is divisible by 3 because 4+3+1+8+2 = 18; and 18 is divisible by 3. Therefore 43,182 is also divisible by 6. But 63,047 is not divisible by 6 (why?)

✓ Divisibility by 8

A number is divisible by 8 if its last three digits (unit's, tens and hundreds) are zero, or are divisible by 8.

Is the number 3,2482, 938 divisible by 2 and 3? Is it divisible by 6?

Example 6

10, 000 and 41,256 are divisible by 8.
Observe that 256 ÷ 8 = 32. But 10, 309 is not divisible by 8 (why?)

Is the number 36,496 divisible by 8? Check your answer. Is it divisible by 6?

✓ Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

Example 7

1206 is divisible by 9 because 1+2+0+6= 9 and 9 is divisible by itself. But 1345 is not divisible by 9 (why?)

✓ Divisibility by 10

A number is divisible by 10 if the units digit is 0.

Example 8

74,360 and 663,350 are divisible by 10 because their unit's digit is 0. But 70, 463 is not divisible by 10 (why?)

Activity 2.2

1. Tell why every number that is divisible by 10 is also divisible by 5.
2. Draw pictures showing how 48 dots can be equally divided into 4 rows, 6 rows, or 8 rows.
3. Tell how to pick a number that is divisible by 3 but not divisible by 9. Then give two examples of such a number.

Example 9

Determine whether 126 is divisible by 2,3,4,5,6,9, or 10.

Solution

2: The ones digit, 6, is even, so 126 is divisible by 2.
3: The sum of the digits, 9, is divisible by 3, so 126 is divisible by 3.
4: The number formed by the last two digits, 26, is not divisible by 4, so 126 is not divisible by 4.
5: The ones digit is not 5 or 0, so 126 is not divisible by 5.
6: The number is divisible by both 2 and 3, so 126 is divisible by 6.
9: The sum of the digits, 9, is divisible by 9, so 126 is divisible by 9.
10: The ones digit, 6, is not 0, so 126 is not divisible by 10.

✓ **Divisibility by 7**

a) A two digit number is divisible by 7 if the sum of 3 times the tens digit and the unit's digit is divisible by 7. For example, 84 is divisible by 7 because $8 \times 3 + 4 = 28$ is divisible by 7.

b) A three or more digit number is divisible by 7 if the sum of the number formed by the last two digits and twice the number formed by the last two digits and twice the number formed by the remaining digits is divisible by 7. For example,

(i) 672 is divisible by 7 because $72 + 2 \times 6 = 72 + 12 = 84$ is divisible by 7.

(ii) 1512 is divisible by 7 because $12 + 2 \times 15 = 12 + 30 = 42$ is divisible by 7.

Does the divisibility test for 7 seem not easy (or time consuming) to you? Can you give a simpler test than this of your own? You may also use the long division to check whether a number is divisible by 7.

Example 10

Use long division to determine whether 397 is divisible by 7.

Solution: $397 \div 7 = 56.714286$ (check!)

Since the quotient is not a whole number, 397 is not divisible by 7.

Group work 2.1

What number can be divided exactly by 7 and by 8, and is different from 40 by 16?

Example 11

Find a number that is divisible by 3, 9, 5 and 10.

Solution: The ones digit must be 0 in order for the number to be divisible by 10 (which means the number will also be divisible by 5), and the sum of the digits must be divisible by 9(which means the sum is also divisible by 3).

The numbers 1,260 9,990 333,000, and 123,210 are just a few of the numbers that meet these requirements.

Exercise 2.A

1. Apply the tests for divisibility you have learnt, and put a tick mark (√) against the factors each number given is divisible by.

Number	Divisible by						
	2	3	4	5	6	8	9	10
178, 620								
6, 348,025								
179,600								
163,245								
118,224								
712,800								
125,046								

2. Identify whether each of the following statements is true or false.
   a) Odd numbers are divisible by 2.
   b) 8,529 is not divisible by 2. But it is divisible by 3.
   c) 39, 120 is divisible by 2, 3 or 5.
   d) 40,924 is divisible by 4.
   e) 21,408 is divisible by 9.
   f) 27,488 is divisible by 9.
   g) If a whole number is divisible by 9, then it is divisible by 3.
   h) If a whole number is divisible by 3, then it is divisible by 9.
   i) If a whole number is divisible by 6, then it is divisible by 3.
   j) If a whole number is divisible by 8, then it is divisible by 4.

3. What are the possible numbers in the missing digit if 8 – 19 is divisible by 3?

2.2 Multiples and Divisors

2.2.1 Revision on Multiples and Divisors

Group work 2.2

When a mystery number is doubled, then doubled again, the answer is 144. What is the number?

Do you remember?

Recall the facts you have studied in earlier grades mathematics lessons about multiples and divisors before we continue our discussion on prime and composite numbers.

- ✓ A number, which divides the dividend completely leaving no remainder, is known as the number's **divisor** (or **factor**).
- ✓ A dividend into which a factor can divide is called the **Multiple** of that factor. For example, $28 \div 4 = 7$. Therefore 28 is a multiple of 4.
- ✓ Numbers, which are multiples of 2 {0, 2, 4, 6, 8, ---} are called **even** numbers.
- ✓ Numbers which are not multiples of 2 or {1, 3, 5, 7, 9, ---} are called **odd** numbers.

Do you remember how we find multiples of 3?

The first ten multiples of 3 are 0, 3, 6, 9, 12, 15, 18, 21, 24, 27.

Do these numbers look familiar?

The multiples of 3 will go on further than 27. You can write the set of multiples of 3 as M = {0, 3, 6, 9, 12, ---}.

You can find the set of multiples for any number.

$$\left.\begin{array}{l} 1\times 10 = 10 \\ 2\times 10 = 20 \\ 3\times 10 = 30 \\ 4\times 10 = 40 \\ 5\times 10 = 50 \\ 6\times 10 = 60 \end{array}\right\} \text{These are the first six multiples of 10.}$$

Activity 2.3

Write each number as a product of two whole numbers in as many ways as possible.

a) 6 b) 16 c) 19 d) 36

In how many ways can you multiply two numbers together to get 10? You can list them like this.

$$\left.\begin{array}{l} 10 = 10\times 1 \\ 10 = 5\times 2 \\ 10 = 2\times 5 \end{array}\right\} \begin{array}{l}\text{These are all the factors of 10.} \\ \text{Factors of 10 are 1, 2, 5, and 10.} \\ \end{array}$$

Exercise 2 B

1. Find the first six multiples of each of the following numbers.

   a) 7 b) 9 c) 13 d) 20 e) 32

2. Find all divisors of each number.

   a) 30
   b) 37
   c) 40
   d) 19
   e) 45
   f) 50
   g) 64

2.2.2 Prime and Composite Numbers and Prime Factorization

Did you know that on average a hen lays 300 eggs per year? That is 25 dozen eggs. The numbers 25 and 12 are **factors** of 300 because $25\times12= 300$. Just as 300 can be written as 25×12, the number 25 and 12 can also be written as the products of a pair of factors: $5\times5 = 25$ and $3\times4 = 12$. You could also have chosen $2\times6 = 12$.

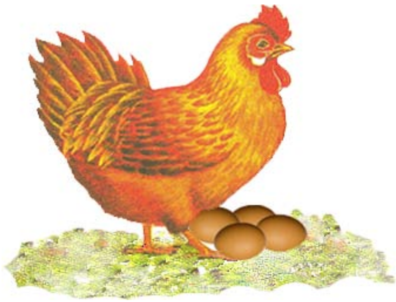


Figure 2.2

When you multiply two factors together, you form a **multiple**. There are many numbers that can be formed in more than one way.

$$\left.\begin{array}{l} 15 = 15\times 1 \\ 15 = 3\times 5 \\ 15 = 5\times 3 \\ 15 = 1\times 15 \end{array}\right\} \text{Factors of 15 are 1, 3, 5 and 15.}$$

A **composite number** is a number that has three or more factors. Can you list the first four composite numbers?

Definition 2.1: A composite number is a whole number that has more than two factors.

Example 12

Six is a composite number because it has more than two factors: 1, 2, 3 and 6.

Consider the factors of 3 and factors of 7:

$$\left\{\begin{array}{l} 3 = 3 \times 1 \\ 3 = 1 \times 3 \end{array}\right. \quad \text{thus factors of 3 are 1 and 3.}$$

$$\left.\begin{array}{l} 7 = 7\times 1 \\ 7 = 1\times 7 \end{array}\right\} \text{factors of 7 are 1 and 7.}$$

What do you notice?

In each case, the only factors of the number are 1 and the number itself.

Numbers which have only two factors (one and the number itself) are called **prime numbers**.

Definition 2.2: A **Prime number** is a whole number greater than 1 that has exactly two factors, 1 and itself.

Is 1 a prime number? What are the factors of 1?

Since 1 has only one factor, 1 is not a prime number.

Is 2 a prime number? What are the factors of 2?

Prime factorization is the process of expressing numbers as a product of prime numbers. Can you express 6 as a product of prime numbers?

Activity 2.4

1. Tell whether each number is prime or composite.
   a) 11 b) 18 c) 23
2. Which is the prime factorization of 75?
   a) $3^2\times5$ b) 3×5^2 c) $3^2\times5^2$ d) 3×5^3
3. Write the prime factorization of each number.
   a) 27 b) 28 c) 52 d) 108

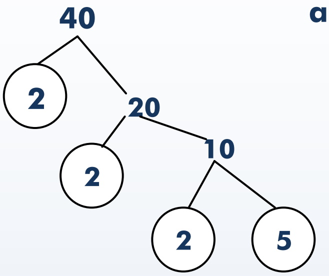
We can use a **factor tree** to show the prime factors of a number. The process ends when the 'branch' finishes with a prime number.

Example 13

Write the prime factorization of

a) 40 **b) 72** **c) 300**

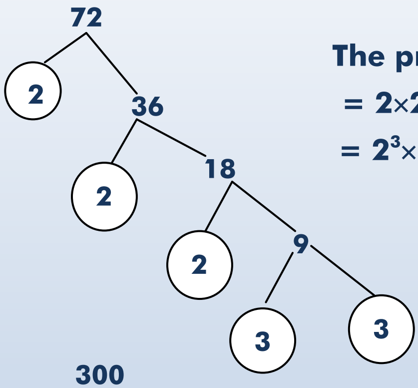
Solution



a) To find the prime factors of 40, we divide by 2 over and over again until the result was not divisible by 2. The result was 5, so the prime factors are 2 and 5. But we had to divide by 2 three times so we can see that the prime factorization of

$$40 = 2\times2\times2\times5 = 2^3 \times 5$$

b)

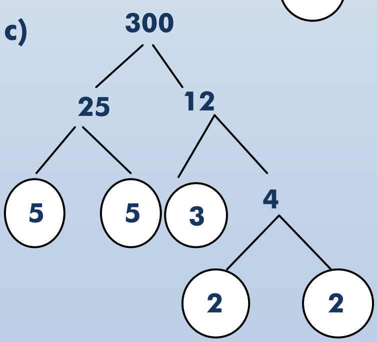


The prime factorization of 72

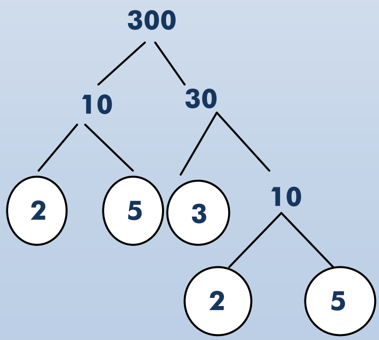
$$= 2\times2\times2\times3\times3$$

$$= 2^3\times3^2$$

c)



Or



The prime factorization of 300 $= 2\times2\times3\times5\times5 = 2^2 \times 3 \times 5^2$

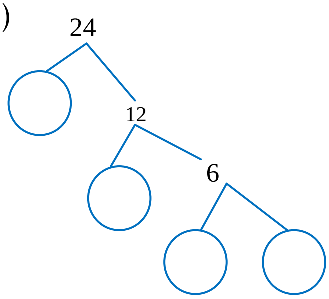
Note. The figure formed by the steps of the factorization (of 40, 72 and 300 shown above) is called a factor tree.

Exercise 2.C

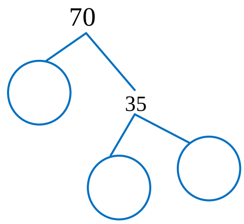
1. Identify whether each of the following statements is true or false.
   a) The smallest prime number is 1.
   b) 2 is the only even prime number.
   c) Among the natural numbers 4 is the smallest composite number.
   d) 1 is neither prime nor composite.
   e) Any whole number is divisible by one and itself.
   f) Any whole number which is greater than one has at least two factors.
   g) The prime factorization of $120 = 2^3 \times 3^2 \times 5$.

2. Fill the gaps of the factor trees.

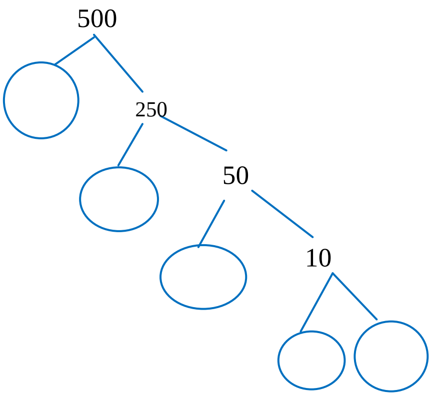
a)



b)



c)



3. Write the prime factorization of the following numbers.
   a) 100 b) 144 c) 150 d) 225 e) 300

4. Write down the largest prime number

   a) less than 20 b) less than 30 c) less than 40

5. List

   a) All composite numbers between 19 and 41.

   b) All prime numbers between 82 and 99.

6. Find a pair of prime numbers that are consecutive whole numbers.

7. Colour the prime numbers

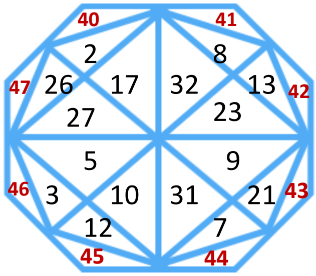


Figure 2.3

8. Determine whether each number is composite or prime.

   a) 75 b) 17 c) 6,453 d) 10,101

9. Find the missing factor: $2^3 \times \square \times 3^2 = 360$.

10. Find the least whole number n for which the expression n^2+n+11 is not prime.

11. Birtukan found a pair of prime numbers, 5and7, that differed by 2. These numbers are called twin primes. Find all the other **twin primes** that are less than 100.

2.2.3 Common Divisors

By using the following process described in the group work given below, you can discover the greatest common divisor (GCD) or the greatest common factor (GCF) of two or more numbers.

Group work 2.3

Work with a partner.

Materials: 4 coloured pencils

- **Copy the array of numbers shown below.**

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

- **Use a different coloured pencil for each step below.**

Follow this process.

Step 1. **Circle 2, the first prime number. Cross out every second number after 2.**

Step 2. **Circle 3, the second prime number. Cross out every third number after 3.**

Step 3. **Circle 5, the third prime number. Cross out every fifth number after 5.**

Step 4. **Circle 7, the fourth prime number. Cross out every seventh number after 7.**

Discuss

a. **In the process, 30 was crossed off when using which primes?**

b. **In the process, 42 was crossed off when using which primes?**

c. **What prime factors do 30 and 42 have in common?**

d. **What is the greatest common factor of 30 and 42?**

Definition 2.3: The greatest common divisor (GCD) or greatest common factor (GCF) of two or more numbers is the greatest number that is a divisor of both (or all) the given numbers.

Example14

Determine the common divisor and the GCD of 24 and 60

Solution: Divisors of 24 = {1, 2, 3, 4, 6, 12, 24}

Divisors of 60 = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

Common divisors = {1, 2, 3, 4, 6, 12}

GCD of 24 and 60 = 12

You may also use the long division method to determine GCD as follows.

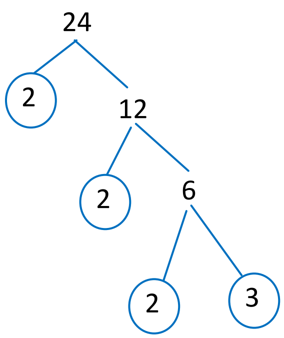
Step 1. Divide the bigger number by the smaller number (that is, 60 divided by 24). The remainder is 12.

Step 2. 12 divides 24 exactly. So the last divisor, 12 is the GCD.

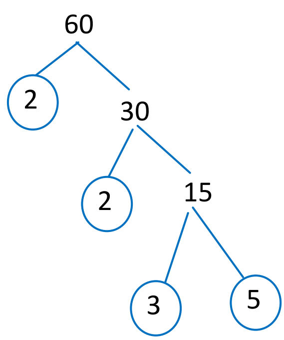
i) $\begin{array}{r|l} & 2 \\ \hline 24 & 60 \\ & \underline{48} \\ & 12 \end{array}$

ii) $\begin{array}{r|l} & 2 \\ \hline 12 & 24 \\ & \underline{24} \\ & 0 \end{array}$

Note: You may also use the factor tree method as follows:



$24 = 2^3 \times 3$



$60 = 2^2 \times 3 \times 5$

Common prime factors: 2, 2, 3.

Thus, the GCD of 24 and 60 is $2 \times 2 \times 3 = 12$.

Activity 2.5

The Maths clubs from 3 schools agreed to a competition. Members from each club must be divided into teams, and teams from all clubs must be equally sized. What is the greatest number of members that can be in a team if school A has 16 members, school B has 24 members, and school C has 72 members?

Example 15

Determine the GCD of 96, 144, and 160

Solution: Divisors of 96 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96}

Divisors of 144 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 36, 48, 72, 144}

Divisors of 160 = {1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160}

Common divisors = {1, 2, 4, 8, 16}

Therefore, GCD = 16

Let us use the long division method to determine GCD

i)
```
       3
48 ) 160
     144
      16
```

ii)
```
      3
16 ) 48
     48
      0
```

iii)
```
       1
96 ) 144
      96
      98
```

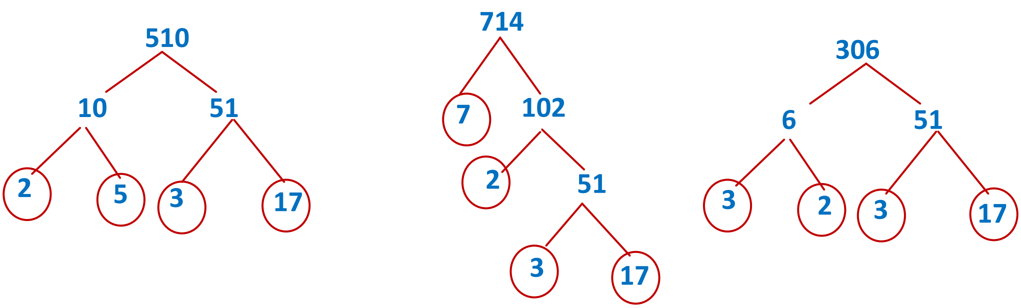
iv)
```
      2
48 ) 96
     96
      0
```

Note

If the GCD of two whole numbers is 1, then the numbers are called **relatively prime.** Can you find two numbers which are relatively prime?

Exercise 2.D

1. Find the common divisors of the following numbers.
   a) 24 and 30 c) 32 and 48
   b) 60 and 80 d) 35 and 57
2. Determine the GCD of the following numbers.
   a) 20 and 45 d) 100, 120 and 150
   b) 32 and 80 e) 180, 210 and 270
   c) 144 and 249
3. Which of the following numbers are relatively prime?
   a) 8 and 9 c) 80 and 90
   b) 10 and 15 d) 100 and 111
4. What is the GCD of all the numbers in the sequence 15, 30, 45, 60, 75,...?
5. Find the GCD of 510, 714 and 306 by using the following factor trees.



6. Name two different pairs of numbers whose GCD is 28.
7. Find the two least composite numbers that are relatively prime.

2.2.4 Common Multiples

When you multiply a number by the whole numbers 0, 1, 2, 3, 4, and so on, you get **multiples** of the number.

Consider multiples of 3 and multiples of 4

Multiples of 3 = {0, 3, 6, 9, 12, 15, 18, 21,24, 27, 30, . . . }.

Multiples of 4 = {0, 4, 8, 12, 16, 20, 24, 28, 32, . . .}.

Common multiples of 3 and 4 = {0, 12, 24, 36, 48, . . .}.

Definition 2.4: The least non- zero common multiples of two or more numbers is called the **least Common multiple (LCM)** of the numbers.

For example LCM of 3 and 4 is 12. Can you find the LCM of 4 and 5?

Activity 2.6

(i) List multiples of 5 and multiples of 6.

(ii) List common multiples of 5 and 6.

What is the least of all the common multiples which is different from zero?

Let us study the different methods of finding LCM of numbers:

One of the following two methods is usually used to find the least common multiple of two or more numbers.

Method 1. List several multiples of each number. Then identify the common multiples and choose the least of these common multiples. (The LCM).

Method 2. Write the prime factorization of each number. Identify all common prime factors. Then find the product of the prime factors using each common prime factor only once and any remaining factors. The product is the **LCM**.

Example 16

Determine the LCM of 18 and 24

Solution: Method 1. Set Intersection Method

Multiples of 18 = { 0, 18, 36 , 54, 72, 90, - - -}

Multiples of 24 = { 0, 24, 48, 72, 96, - - -}

The least non zero whole number that belongs to both of them is 72 and therefore LCM of 18 and 24 is 72.

Method 2. **Prime factorization method**

Step 1. Determine a prime factorization of each of the given numbers.

Step 2. Find the LCM as the product of each prime factor taking the greatest number of times it occurs in any of the prime factorization of the given numbers.

$$18 = 2 \times 3 \times 3 = 2 \times 3^2 \text{ and}$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

LCM of 18 and 24 is $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 = 8 \times 9 = 72$

Method 3. **Division method**

This method of finding the LCM is given below

3	18, 24
2	6, 8
2	3, 4
2	3, 2
3	3, 1
	1, 1

LCM (18, 24) = $2^3 \times 3^2$

Can you find LCM of 12 and 20? You may use one of the method we discussed above. Did you find 60 as the LCM of 12 and 20?

Group work 2.4

A man is catering a party for 152 people. He wants to seat the same number of people at each table. He also wants more than 2 people at a table. How many people can he seat at each table?

Note: There is also an important relation between two numbers, their LCM and GCD.

Study the following example:

Example 17

LCM of 12 and 20 = 60 (Why?) and GCD of 12 and 20 = 4 (Why?)
Notice that 12×20 = (LCM of 12 and 20) × (GCD of 12 and 20)

That is 12× 20 = 60× 4

Can we conclude for two numbers a and b that

a × b = LCM (a, b) × GCD (a, b)?

We can also determine the LCM of more than two numbers. Here is one such example.

Example 18

Determine the LCM of 15, 18 and 20 by the method of division.

Solution:

2	15, 18, 20
2	15, 9, 10
3	15, 9, 5
3	5, 3, 5
5	5, 1, 5
	1, 1, 1

Therefore, LCM of 15, 18 and 20 = 2 ×2×3×3×5 = 180.

Exercise 2.E

1. Find the LCM.

   a) 10 and 16

   b) 12 and 18

   c) 24 and 60

   d) 32, 40 and 50

   e) 24, 30 and 45

2. The product of two numbers is 300. Their LCM is 75. Find their GCD.
3. The LCM of 18 and x is 72. The GCD of 18 and x is 12. Find the value of x.
4. If a and b are two prime numbers, is it true that LCM (a, b) = a. b?

5. Two numbers have GCD = 9 and LCM = 18 . One number is twice as big as the other number. Find the two numbers.
6. The LCM of two numbers is 36 and their GCD is 1. Neither of them is 36. Can you find the numbers?
7. In a school, the duration of a period in the primary section is 30 minutes and in the secondary section it is 40 minutes. Two bells ring differently for each section. When will the two bells ring together next if the school begins at 9:00 a. m?
8. When will the LCM of two numbers be one of the numbers?
9. When will the LCM of two numbers be their product?
10. Write a set of three numbers whose LCM is the product of the numbers?

UNIT SUMMARY

Important facts you should know:

- ***Divisibility Tests:*** **There are some short and quick tests for divisibility. They are as follows.**

  Divisibility by 2: A number is divisible by 2 if the unit's digit is even.

  Divisibility by 3: A number is divisible by 3 if the sum of its digits is divisible by 3.

  Divisibility by 4: A number is divisible by 4 if the digits at the unit's and ten's place (last two digits) are divisible by 4; or if the last two digits are 00.

  Divisibility by 5: A number is divisible by 5 if the unit's digit is 0 or 5.

  Divisibility by 6: A number is divisible by 6 if it is divisible by both 2 and 3.

  Divisibility by 8: A number is divisible by 8 if its last three digits are zero, or are divisible by 8.

  Divisibility by 9: A number is divisible by 9 if the sum of its digits is divisible by 9.

  Divisibility by 10: A number is divisible by 10 if the digit at the units place is 0.

- The greatest number which is a common factor of two or more than two numbers is called the **Greatest common divisor (GCD)** of the given numbers.
- The smallest number which is a common multiple of two or more than two numbers is called the **Least common multiple (LCM)** of the given numbers.
- Zero is a whole number, and a multiple of every number.
- The process of factoring a number into its prime factors is called **prime factorization**.
- Every number has fixed number of factors. However, a given number has unlimited number of multiples.

Review Exercise

1. Identify whether each of the following statements is true or false.
   a. 420,042 is divisible by 6.
   b. 12,357 is divisible by 4.
   c. If a whole number is divisible by 10, then it is also divisible by 5.
   d. If a whole number is divisible by 8, then it is also divisible by 4.
   e. If a whole number is divisible by 26, then it is also divisible by 13.
   f. 1,000 is divisible by 2,4,5 and 10.
   g. If two numbers are relatively prime, then one of the numbers must be prime.
2. Find the GCD of
   a. 360 and 600
   b. 2500 and 750
   c. 100 and 49
   d. 252, 180, 96 and 60
3. Find the LCM of
   a. 750 and 1000
   b. 1200 and 2000
   c. 2700 and 3000
   d. 2960, 6400 and 2000
4. The product of two numbers is 3150. Their LCM is 630. Find their GCD.
5. You are planning a picnic. You can purchase paper plates in packages of 30, paper napkins in packages of 50, and paper cups in packages of 20. What is the least number of each type of package that you can buy and have an equal number of each?

6. Which pair of numbers has a GCD that is a prime number, 48 and 90 or 105 and 56?
7. Which pair of numbers has a GCD that is not a prime number?
   a) 15 and 20 c) 24 and 75
   b) 18 and 30 d) 6 and 10
8. Match the items in column A with the items in column B.

Column A	Column B
a. LCM of 30 and 75	i. 20
b. GCD of 30, 70, 65 and 100	ii. $3^3 \times 5^2$
C. Prime factorization of 780	iii. 5
d. GCD of 20, 40, 80 and 100	iv. 300
e. LCM of 15, 30, 50 and 100	v. 150
f. Prime factorization of 675	vi. $2^3 \times 3 \times 37$
g. Prime factorization of 888	vii. $2^2 \times 3 \times 5 \times 13$
	viii. $2^3 \times 3^3 \times 5^3$
	x. 75

9. If the prime factor of a number are all the prime numbers less than 10 and no factor is repeated, what is the number?
10. Find the smallest number that is divisible by 2, 3, 4, 5, 6, 7, 8 , 9 and 10.
11. Solve this riddle: I am a number whose prime factors are all the prime numbers between 6 and 15. No factor is repeated. What number am I?
12. A baker expects to use 126 eggs is one week. He can either order cartons which contain 8 eggs or cartons which contain 18 eggs, but not both. If he does not want any eggs left over at the end of the week, which size carton should he order?

UNIT 3 FRACTIONS AND DECIMALS

Unit outcomes: After completing this unit you should be able to:

- understand fractions and decimals and realize that there are two ways to represent the same numbers.
- develop skill in ordering, adding, subtracting, multiplying and dividing fractions and decimals.
- work with problems represented by fractions and decimals.

Introduction

Much of what you have learned in earlier grades will be useful in your study of algebra. In particular, you will use what you know about fractions and decimals.

This unit begins with a brief review of simplification of fractions and continues with conversion of fractions, decimals and percentages, and comparing and ordering fractions. The unit also deals with addition, subtraction, multiplication and division of fractions and decimals in more detail.

3.1. The Simplification of Fractions

Do you remember what you have learnt about fractions in grade 5 mathematics lessons? In order to help you recall about fractions, attempt the following Activity.

Activity 3.1

1. What is the numerator in the fraction $\frac{6}{55}$? What is its denominator?
2. Which of the following are proper fractions? Which of them are improper fractions? Which ones are mixed numbers?
   $3\frac{1}{2}$, $\frac{1}{10}$, $\frac{7}{6}$, $\frac{12}{13}$, $\frac{22}{19}$, $33\frac{5}{6}$, $\frac{8}{8}$, $\frac{10}{1}$
3. Convert $\frac{160}{9}$ to a mixed fraction.
4. Find x if $\frac{4}{7} = \frac{x}{28}$.
5. Convert $9\frac{5}{8}$ to an improper fraction.
6. Which of the following fractions is equal to $\frac{1}{2}$?
   $\frac{3}{4}, \frac{10}{40}, \frac{8}{24}, \frac{50}{100}$

Operation on fraction, decimals and percents is the focus of this unit. The process of factoring numbers can be used to reduce fractions to lowest terms. A fractional portion of a whole can be represented by infinitely many fractions. For example, Figure 3.1 below shows that $\frac{1}{2}$ is equivalent to $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, and so on.

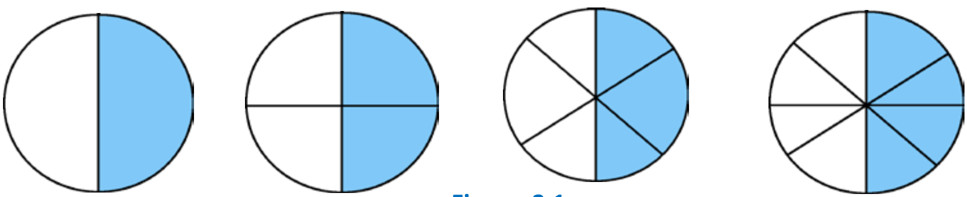


Figure 3.1

The fraction $\frac{1}{2}$ is said to be in lowest terms because the numerator and denominator share no common factor other than 1. The fraction $\frac{2}{4}$ is not in lowest terms because the numerator and denominator are both divisible by 2. To reduce a fraction to lowest terms is to "divide out" common factors from both the numerator and denominator.

Can you reduce $\frac{2}{4}$ to its simplest form? Let us factor both the numerator and denominator as follows: $\frac{1 \times 2}{2 \times 2}$. We can see that the numerator and the denominator have common factor.

If we divide both the numerator and the denominator by the greatest common divisor, we get $\frac{1}{2}$. Can you see that $\frac{1}{2}$ is the simplest form of $\frac{2}{4}$?

To express a fraction in simplest form:

- ✓ Find the greatest common divisor (GCD) of the numerator and denominator,
- ✓ Divide the numerator and the denominator by the GCD, and
- ✓ Write the resulting fraction.

Note: If the numerator and denominator of a given fraction do not have any common factor other than 1, then the fraction is in simplest form.

Study the following example:

Example 1

1. Reduce the following fractions in lowest terms

a) $\frac{150}{120}$ b) $\frac{600}{120}$

Solution: $\frac{150}{120} = \frac{15}{12} = \frac{5}{4}$ or $\frac{150}{120} = \frac{5}{4}$ because G CD of 150 and 120 is 30.

Then we see if we can simplify it further.

$\frac{5}{4}$ is an improper fraction. $\frac{5}{4} = \frac{4+1}{4} = 1\frac{1}{4}$. This gives a mixed number.

b) $\frac{600}{120} = \frac{60}{12} = \frac{15}{3} = \frac{5}{1} = 5$ (Why?)

It is important to remember that a whole number can be expressed as a fraction.

$$4=\frac{4}{1} \qquad 18=\frac{18}{1}$$

You can also find equivalent fractions for whole numbers in the usual way.

Example 2

$$4=\frac{4}{1}=\frac{8}{2}=\frac{12}{3}=\frac{36}{9}=\frac{400}{100} \qquad 18=\frac{18}{1}=\frac{36}{2}=\frac{54}{3}=\frac{72}{4}=\frac{90}{5}=\frac{180}{10}$$

Similarly, 1 can be expressed as a fraction with the same number in the numerator and the denominator.

Example 3

$$1=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{15}{15}=\frac{26}{26}=\frac{45}{45}=\ldots$$

x2

x2

Exercise3 A

Express each of these fractions in its simplest form.

a) $\frac{4}{16}$ c) $\frac{45}{20}$ e) $\frac{128}{224}$ g) $\frac{128}{384}$

b) $\frac{8}{12}$ d) $\frac{72}{48}$ f) $\frac{28}{140}$ h) $\frac{2160}{270}$

3.2 The Conversion of Fractions, Decimals and Percentages

Activity 3.2

1. Identify whether each of the following statements is true or false.

a) $\frac{1}{10}=0.1$ c) $\frac{7}{25}=\frac{28}{100}$ e) $\frac{3}{20}=0.15$

b) $\frac{3}{5}=0.6$ d) $\frac{4}{50}=0.8$ f) $\frac{6}{100}=0.03$

2. Write <, > or = in the box to compare the given fractions

   a) $\frac{2}{5}$ $\boxed{}$ $\frac{1}{3}$

   b) $\frac{5}{6}$ $\boxed{}$ $\frac{7}{9}$

   c) $\frac{3}{8}$ $\boxed{}$ $\frac{4}{7}$

Do you remember?

In your grade 5 mathematics lessons you have learnt that in a place value number system each digit in a numeral has a particular value determined by its location in the numeral (Figure 3.2).

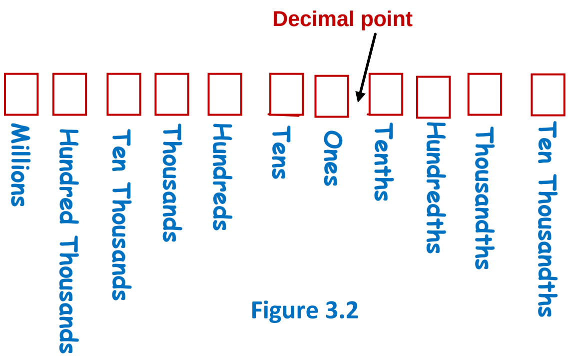


Figure 3.2

For example, the number 396.215 represents

$$(3 \times 100) + (9 \times 10) + (6 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{100}\right) + \left(5 \times \frac{1}{1000}\right)$$

Each of the digits 3, 9, 6, 2, 1 and 5 is multiplied by 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$ and $\frac{1}{1000}$, respectively, depending on its location in the numeral 396.215.

By obtaining a common denominator and adding fractions, we have

$$396.215 = 300 + 90 + 6 + \frac{200}{1000} + \frac{10}{1000} + \frac{5}{1000}$$

$$= 396 + \frac{215}{1000} \text{ or } 369\frac{215}{1000}$$

Because 396.215 is equal to the mixed number $396\frac{215}{1000}$, we read 396.215 as three hundred ninety-six and two hundred fifteen over thousand (or three hundred ninety-six point two, one, five).

If there are no digits to the right of the decimal point, we usually omit the decimal point. For example the number 8257.0 is written simply as 8257.
You have seen that decimals are expressed as fractions over multiples of 10, with each place to the right of the decimal point corresponding to further division by 10. Thus: $0.3=\frac{3}{10}$, $0.07=\frac{7}{100}$, $0.009=\frac{9}{1000}$.

Group work 3.1

Fill in the blank space

Fraction	Decimal
$\frac{32}{10,000}$	**0.0032**

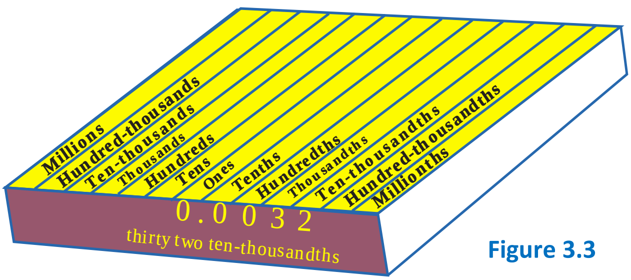


Figure 3.3

a) **The 3 in the decimal is in the _________ place and means ________**
b) **The 2 is in the _____________ place and means ___________**

3.2.1. Conversion of Fractions to Decimals and Percentage

Let us indicate the place value of each digit in the number 379. 468

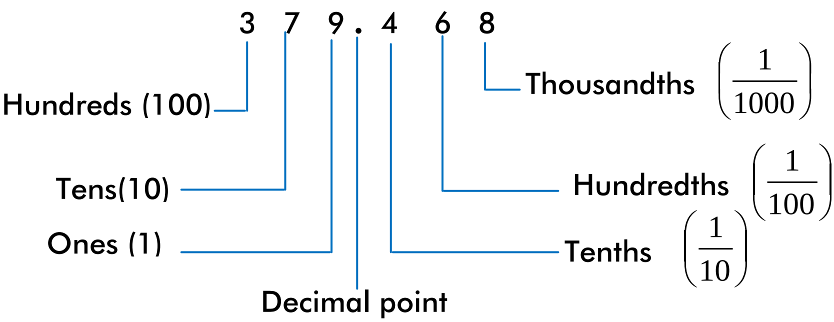


Figure 3.4

Activity 3.3

Tell the place value of each digit in the number 3418.297

Converting a fraction to a decimal: To convert a fraction to a decimal use long division and divide the numerator of the fraction by the denominator of the fraction.

Given a fraction of the form if we divide a by b using the long division method, the only possible remainders are 0, 1, 2, 3, …, b- 1. If 0 is obtained as a remainder, the process of the division is terminated and the decimal expression in this case, is called a **terminating decimal.**

Example 4

Convert the fractions to decimals

a) $\frac{4}{5}$ b) $\frac{3}{8}$ c) $\frac{5}{4}$

Solution a)

$$\begin{array}{r|l} & 0.8 \\ \hline 5 & 40 \\ & \underline{-40} \\ & 0 \end{array}$$

Therefore $\frac{4}{5} = 0.8$. The number 0.8 is said to be a terminating decimal

b)

$$\begin{array}{r|l} & 0.375 \\ \hline 8 & 30 \\ & \underline{-24} \\ & 60 \\ & \underline{-56} \\ & 40 \\ & \underline{-40} \\ & 0 \end{array}$$

Therefore $\frac{3}{8} = 0.375$

c)

$$\begin{array}{r|l} & 1.25 \\ \hline 4 & 5 \\ & \underline{-4} \\ & 10 \\ & \underline{-8} \\ & 20 \\ & \underline{-20} \\ & \underline{0} \end{array}$$

Therefore, $\frac{5}{4} = 1.25$

It is possible to express $\frac{3}{8}$, and $\frac{5}{4}$ as terminating decimals? Why?

When we divide a by b it is also possible that the non- zero possible remainders may occur and recur again in the process of division and the division becomes an endless process. Such a decimal is called a **repeating decimal.** Study the following example:

Example 5

Convert the fractions to decimals.

a) $\frac{4}{9}$ b) $\frac{5}{3}$ c) $\frac{133}{99}$

Solution a)

$$\begin{array}{r|l} & 0.444 \\ \hline 9 & 40 \\ & \underline{-36} \\ & 40 \\ & \underline{-36} \\ & 40 \\ & \underline{-36} \\ & 4 \end{array}$$

There fore, $\frac{4}{9} = 0.444\ldots$
$= 0.\dot{4}$

0.444 … may be denoted as $0.\dot{4}$ that is, $\frac{4}{9} = 0.\dot{4}$

It is possible to round 0.444 … to one decimal place as 0.4 or round 0. 444 … to two decimal places as 0. 44 or round 0. 444 …. to three decimal places as 0.444 and so on.

Rounding to one decimal place:
If the hundredth digit is 5 or more we increase the tenth digit by one. If it is less than 5, leave the hundredth digit unchanged

b) $3 \overline{)5}$ = 1.666...

-3
20
-18
20
-18
2

Therefore, $\frac{5}{3} = 1.666... = 1.\dot{6}$

Rounding 1. 666 ... to one decimal gives 1.7 or rounding 1. 666 .... to two decimals we get 1. 67 rounding 1. 666 .... to three decimals we get 1. 667.

Rounding to two decimal places: If the thousandth digit is 5 or more we increase the hundredth digit by one. If it is less than 5, leave the hundredths digit unchanged

c) $99 \overline{)133}$ = 1.3434...

-99
340
297
430
396
340

Therefore, $\frac{133}{99} = 1.3434 ... = 1.\dot{3}\dot{4}$

Can you convert 12/25 to decimals?. Is it termina-ting or repea-ting?

- ✓ rounding 1.3434 ... to one decimal place we get 1.3.
- ✓ rounding 1.3434 ... to two decimal place we get 1.34.
- ✓ rounding 1. 3434 ... to three decimal places we get 1.343 .

Fractions with 100 as denominator are called **percents**. Thus, $\frac{70}{100}$ is denoted as 70% and is read as 'seventy percent'.

Percent means per hundred or for every hundered or hundredths.

Example 6

Convert the fractions to percents.

a) $\frac{2}{5}$ b) $\frac{1}{16}$ c) $\frac{5}{3}$

Solution: a) $\frac{2}{5}=\frac{?}{100}$ $\frac{2\times 20}{5\times 20}=\frac{40}{100}=40\%$

b) $\frac{1}{16}=\frac{1\times 100}{16\times 100}=\frac{100}{16\times 100}=\frac{100}{16}\%=6.25\%$

c) $\frac{5}{3}=\frac{5\times 100}{3\times 100}=\frac{500}{3\times 100}=\frac{500}{3}\%=166.\dot{6}\%$

Activity 3.4

Name a pair (in Figure 3.5) that have signs showing the same discount.

a.


b.


c.


d.


e.


f.


Figure 3.5

Can you convert $\frac{3}{5}$ to percents?

In daily life we often use the word percent. Working with percentages often makes it easier to compare quantities.

Example 7

Kuma got 13 marks out of 20, and Maritu got 7 marks out of 10. Whose mark was better?

Solution: **To express these as percentages, we need to find the equivalent fractions with denominator 100.**

$$\frac{13}{20}=\frac{?}{100}\ , \qquad \frac{7}{10}=\frac{?}{100}$$

$$\frac{13\times 5}{20\times 5}=\frac{65}{100}\text{ or } 65\,\% \text{ and } \frac{7\times 10}{10\times 10}=\frac{70}{100}\text{ or } 70\%$$

Maritu's mark was 70 out of 100, so her mark was higher than Kuma's mark.

Note

When we change a fraction to a percentage, we usually multiply by 100% and leave out the denominator 100.

Example 8

There are 40 students in a class and 30 of these are girls. Express the number of girls as a percentage of the number of students in the class.

Solution: **To find the solution, we simply find what fraction of 100% are girls.**

Step 1 **Find the fraction of girls. Fraction of girls** $=\frac{30}{40}=\frac{3}{4}$

Step 2 **Change this to a percentage by multiplying by 100%**

$\frac{3}{4}\times 100\% = 75\%$

Therefore 75% of students are girls .

Group work 3.2

Kemal spent $\frac{2}{5}$ of his Birr 200 birthday check on clothes. How much did Kemal's new clothes cost? What percent of kemal's money is left?

Example 9

Compare

$\frac{1}{5}, \frac{2}{3}$ and $\frac{5}{7}$

Solution.

$$\frac{1}{5} = \frac{1}{5} \times \frac{100}{100} = \frac{100}{5} \times \frac{1}{100} = \frac{100}{5}\% = 20\%$$

$$\frac{2}{3} = \frac{2}{3} \times \frac{100}{100} = \frac{200}{3} \times \frac{1}{100} = \frac{200}{3}\% = 66\frac{2}{3}\%$$

And $\frac{5}{7} = \frac{5}{7} \times \frac{100}{100} = \frac{500}{7} \times \frac{1}{100} = \frac{500}{7}\% = 71\frac{3}{7}\%$

From the above calculation,

$20\% < 66\frac{2}{3}\% < 71\frac{3}{7}\%$

Thus $\frac{1}{5} < \frac{2}{3} < \frac{5}{7}$

Exercise 3.B

1. Convert each of the following fractions in to decimals rounded to two decimal places whenever it is repeating. Identify each as terminating or repeating.

a) $\frac{3}{4}$	d) $\frac{11}{32}$	g) $\frac{13}{11}$
b) $\frac{2}{5}$	e) $\frac{8}{125}$	h) $\frac{5}{9}$
c) $\frac{7}{25}$	f) $\frac{5}{6}$	i) $\frac{20}{99}$

2. Express the fractions given in question number 1 as percentages.
3. Complete the table by rounding off the decimals to the indicated places.

Decimal	One decimal place	Two decimal places	Three decimal places
0.121212…			
2.3636…			
4.257257…			

4. There are 13 boys and 12 girls in a class.
   a) Express the number of boys as a fraction of the class's membership
   b) Express the number of girls as a fraction of the class's membership
   c) Express the number of boys as a percentage of the class.
   d) Express the number of girls as a percentage of the class.

5. Belaynesh had 14 marks out of 25, Tewabech had 10 marks out of 40 and Awol had 23 out of 50.
   a) Express each one's score as a fraction.
   b) Express each one's score as a percentage.
   c) Arrange the three candidates in order of performance from the lowest to the highest.

3.2.2 Conversion of Terminating Decimals to Fractions and Percentages

Activity 3.5

Which of the following fractions is equal to $\frac{1}{50}$?

A 0.2 B. 0.02 C. 0.002 D. 0.05

To convert a terminating decimal to a fraction, we return to the basis of the place value system.

Converting a terminating decimal to a fraction:

1. Write the digits to the left of the decimal point as a whole number.
2. Write each digit to the right of the decimal point as a fraction over 10, 100, 1000 and so on, depending on its place position.
3. Convert to a single fraction by adding fractions.

Example 10

Convert each decimal to a fraction.

a) 0.0027 b) 40.08

Solution: a) $0.0027 = 0\times\frac{1}{10}+0\times\frac{1}{100}+2\times\frac{1}{1,000}+7\times\frac{1}{10,000}$

$= \frac{2}{1,000}+\frac{7}{10,000}$ simplify

$= \frac{20}{10,000}+\frac{7}{10,000}$ find a common denominator

$= \frac{27}{10,000}$ add the fractions

Or $0.0027 = 0.0027\times\frac{10,000}{10,000}=\frac{27}{10,000}$

b) $40.08 = 40+0\times\frac{1}{10}+8\times\frac{1}{100}$

$= 40+\frac{8}{100}$ simplify

$= \frac{4000}{100}+\frac{8}{100}$ find a common denominator

$= \frac{4008}{100}$ add the fractions

$= \frac{1002}{25}$ reduce to lowest terms

Notice that it is also possible to convert terminating decimals to fractions and percentages.

Consider the following example:

Example 11

$0.3 = 0.3 \times \frac{100}{100} = \frac{30}{100} = 30\% = \frac{3}{10}$

and $0.32 = 0.32 \times \frac{100}{100} = \frac{32}{100} = 32\% = \frac{8}{25}$. **You can see that it is possible to convert decimals to fractions and percentages using the method of multiplying and dividing decimals by powers of 10.**

Can you convert 0.25 to fractions and percentages? Study some more examples given below.

Group work 3.3

What percent of 800 is 20? Express the percentage in decimal.

Example 12

Convert the following decimals to fractions and percentages

a) 0.24 **b) 0.534** **c) 0.075**

Solution a) $0.24 = 0.24 \times \frac{100}{100} = \frac{24}{100} = 24\% = \frac{6}{25}$

b) $0.534 = 0.534 \times \frac{1000}{1000} = \frac{534}{1000} = \frac{267}{500} = \frac{267}{5}\% = 53\frac{2}{5}\%$

c) $0.075 = 0.075 \times \frac{100}{100} = \frac{7.5}{100} = \frac{15}{2}\% = 7\frac{1}{2}\% = \frac{3}{40}$

Exercise 3.C

Convert the following decimals to fractions and percentages.

a) 0.36	c) 0.23	e) 0.032	g) 0.751	i) 1.25
b) 0.82	d) 0.465	f) 0.345	h) 0.259	j) 24.3

3.2.3 Conversion of Percentage to Fractions and Decimals

Activity 3.6

Write a fraction that represents the number of sections shaded. Then express each fraction as percentage.

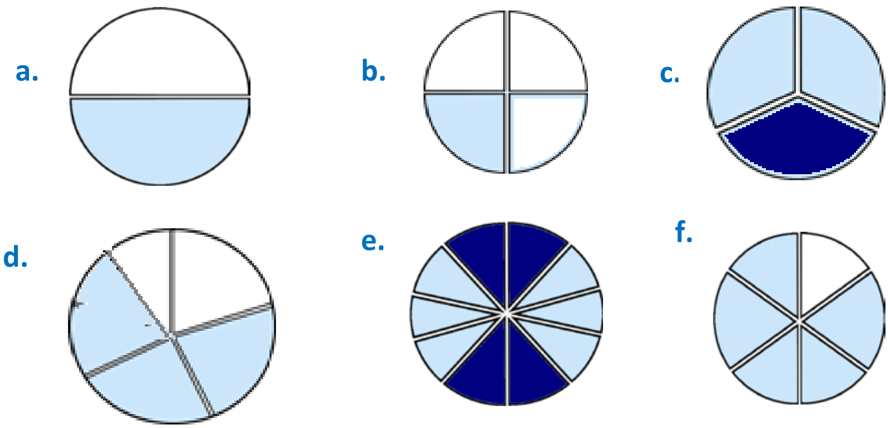


Figure 3.6

Since percentages are already fractions with denominator 100, they can be expressed as equivalent fractions, but in this case the denominator cannot be omitted.

Example 13

Express 20% as a) a fraction b) a decimal

Solution: a) $20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$

b) $20\% = \frac{20}{100} = 0.2$

Can you convert 30% to fraction and decimal?

Group work 3.4

Rain forests are home to 90,000 of the 250,000 identified plant species in the world. What percent of the world's identified plant species are found in rain forests?

Example 14

Express 12 ¼% as

a) a fraction **b) a decimal**

Solution: a) $12\frac{1}{4}\% = 12\frac{1}{4} \times \frac{1}{100} = \frac{49}{4} \times \frac{1}{100} = \frac{49}{400}$

b) $12\frac{1}{4}\% = \frac{49}{400} = 0.1225$ (check by long division)

Exercise 3.D

1. Express these percentages as common fractions and as decimals.

a) 15%
b) 28%
c) 72%
d) $2\frac{1}{4}\%$
e) $8\frac{1}{2}\%$
f) $12\frac{1}{6}\%$

2. Fill in the table with equivalent forms.

Fraction	Decimal	Percent
$\frac{2}{5}$	0.40	40%
		28%
$\frac{1}{25}$		
	0.375	
		62.5%
$3\frac{3}{5}$		

3.3 Comparing and Ordering Fractions

Recall that equivalent fractions are fractions that amount to the same part of the whole, even if they are expressed in different terms.

Example 15

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24}$$

$$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24}$$

Activity 3.7

Which of the following fractions are equivalent to $\frac{1}{8}$?

$\frac{3}{5}, \frac{2}{16}, \frac{5}{40}, \frac{10}{800}, \frac{6}{48}$

In mathematics class, Alemitu has earned 30 points out of a possible 35 points on tests, and home take assignments. In English class she worked hard writing her short story and listening, earning 42 out of a possible 48 points. In which class has Alemitu earned a greater portion of the possible points?

That is, which faction is grater $\frac{30}{35}$ or $\frac{42}{48}$?

Recall, from your previous study on fractions, that one way to compare these two fractions is first write each fraction in simplest form.

$$\frac{30}{35} \overset{\div 5}{=} \frac{6}{7} \qquad \frac{42}{48} \overset{\div 6}{=} \frac{7}{8}$$

To compare 6/7 and 7/8, rewrite each fraction using the same denominator. Then you need only compare the numerators.

To rewrite 6/7 and 7/8 with the same denominators, first find the LCM. Did you get 56?

So, rewrite each fraction using a denominator of 56.

$\frac{6}{7}=\frac{6\times 8}{7\times 8}=\frac{48}{56}$ $\qquad$ $\frac{7}{8}=\frac{7\times 7}{8\times 7}=\frac{49}{56}$

Now, compare 49/56 and 48/56. Since 49> 48, then 49/56 > 48/56, that is $\frac{7}{8}>\frac{6}{7}$ and Alemitu has earned a greater portion of the possible points in English than in mathematics.

Thus, inorder to compare fractions one of the strategies is to express each fraction in simplest form. Then write equivalent fractions using the LCM as we have discussed above.

Group work 3.5

The table shows the times that it takes four of the tower's elevators to travel various distances. The speed of each elevator is the distance divided by the time. Which elevator is the fastest? Slowest? Explain your reasoning.

	Distance (metre)	Time (second)	Speed (m/s)
Elevator A	238	28	$\frac{238}{28}$
Elevator B	195	26	$\frac{195}{26}$
Elevator C	187	10	$\frac{187}{10}$
Elevator D	$20\frac{4}{5}$	1	$20\frac{4}{5}$

Study the following example.

Example 16

Arrange the fractions in an ascending order.

$\frac{2}{3}, \frac{7}{8}$ and $\frac{12}{27}$

Solution: **LCM of 3, 8 and 27 is 8×27**

= 216 (check)

Also $\frac{8\times 27}{3} = 72, \frac{8\times 27}{8} = 27, \frac{8\times 27}{27} = 8$

Now $\frac{2}{3} = \frac{2\times 72}{3\times 72} = \frac{144}{216}, \frac{7}{8} = \frac{7\times 27}{8\times 27} = \frac{189}{216}$

$\frac{12}{27} = \frac{12\times 8}{27\times 8} = \frac{96}{216}$

96<144 and 144<189

Arranging in increasing order, we get

$\frac{96}{216}, \frac{144}{216}, \frac{189}{216}$ or $\frac{12}{27}, \frac{2}{3}, \frac{7}{8}$

Can you compare and arrange $\frac{1}{3}, \frac{1}{2}$ and $\frac{4}{5}$?

We may also use another method to compare fractions as explained in the following example:

Example 17

Arrange these fractions in descending order.

$\frac{1}{4}, \frac{3}{5}, \frac{1}{8}, \frac{17}{50}$ and $\frac{1}{2}$

Solution

Express each of them as a percentage.

$\frac{1}{4} \times 100\% = 25\%$

$\frac{3}{5} \times 100\% = 60\%$

$\frac{1}{8} \times 100\% = 12\frac{1}{2}\%$

$\frac{17}{50} \times 100\% = 34\%$

and $\frac{1}{2} \times 100\% = 50\%$

Now arrange them, starting with the biggest to the smallest.

$\frac{3}{5}, \frac{1}{2}, \frac{17}{50}, \frac{1}{4}, \frac{1}{8}$

Exercise 3.E

1. Write three fractions equivalent to each of the following.

   a) $\frac{1}{6}$ b) $\frac{2}{5}$ c) $\frac{3}{7}$ d) $\frac{9}{8}$ e) $\frac{11}{3}$

2. Find in the blanks in the numerator $\frac{2}{5} = \frac{.....}{15} = \frac{....}{20} = \frac{....}{25} = \frac{....}{30}$.

3. Fill in the blanks in the denominator.

   $\frac{9}{10} = \frac{18}{....} = \frac{27}{....} = \frac{36}{....} = \frac{63}{....}$

4. Identify whether each of the following statements is true or false.

   a) $\frac{9}{10} > \frac{8}{9}$ c) $\frac{9}{11} < \frac{7}{8}$ e) $\frac{5}{7} = \frac{16}{21}$

   b) $\frac{3}{15} = \frac{5}{25}$ d) $\frac{4}{5} < \frac{16}{20}$ f) $\frac{13}{30} < \frac{19}{40}$

5. Arrange these fractions in ascending order.

a) $\frac{3}{5},\frac{7}{8},\frac{2}{25},\frac{7}{10},\frac{3}{4}$

b) $\frac{7}{8},\frac{1}{2},\frac{11}{20},\frac{4}{5},\frac{9}{10}$

c) $\frac{14}{25},1\frac{1}{2},\frac{1}{4},\frac{47}{50},\frac{3}{10}$

d) $\frac{1}{2},\frac{1}{4},\frac{3}{4},\frac{4}{5},\frac{7}{25},\frac{7}{8}$

6. Arrange these fractions in descending order.

a) $\frac{4}{5},\frac{1}{2},\frac{3}{4},\frac{1}{3},\frac{9}{10}$

b) $\frac{12}{25},\frac{13}{20},\frac{7}{40},\frac{4}{15},\frac{49}{50}$

c) $\frac{3}{7},\frac{2}{3},\frac{4}{5},\frac{6}{25},\frac{5}{9}$

d) $2\frac{1}{4},2\frac{1}{2},2\frac{3}{5},2\frac{2}{25},1\frac{49}{50}$

7. Which is cheaper?

a) 9 books for Birr 5 or 15 books of the same kind for Birr 8?

b) 3 lemons for 50 cents or 9 lemons for 100 cents?

8. You buy an item for Birr 24.75 and pay 6.25% sales tax. Another man buy the same kind of item for Birr 20.25 and pay 7.5% sales tax. Which one is expensive?

3.4 Further on Addition and Subtraction of Fractions and Decimals

In this subunit you will deal with addition and subtraction of fractions and decimals in more detail.

3.4.1. Addition of Fractions and Decimals

Activity 3.8

Perform the indicated operation

a) $\frac{3}{2}+\frac{1}{2}$

b) $\frac{7}{15}+\frac{2}{15}$

c) $\frac{4}{8}+\frac{1}{8}$

d) 0.2 + 0.4

e) 0.61 + 0.25

f) 0.87 + 0.31

Can you add fractions and decimals such as $\frac{1}{2} + 0.8$? What is the sum in decimal? What is the sum in fraction? . Let us study the following example.

Example 18

Find the sum

a) $\frac{1}{8}+0.4$ b) $\frac{4}{5}+2.4$ c) $2\frac{1}{4}+6.35$

Solution

a) $\frac{1}{8}+0.4=0.125+0.400=0.525$

or $\frac{1}{8}+0.4=\frac{1}{8}+\frac{4}{10}=\frac{1}{8}+\frac{2}{5}=\frac{5}{40}+\frac{16}{40}=\frac{21}{40}$

there fore, $\frac{1}{8}+0.4=0.525$ or $\frac{1}{8}+0.4=\frac{21}{40}$

Observe that 0.525 = $\frac{21}{40}$

b) $\frac{4}{5}+2.4=0.8+2.4=3.2$

or $\frac{4}{5}+\frac{24}{10}=\frac{4}{5}+\frac{12}{5}=\frac{16}{5}=3\frac{1}{5}$

Therefore, $\frac{4}{5}+2.4=3.2$ or $\frac{4}{5}+2.4=3\frac{1}{5}$

Observe that 3.2 = $3\frac{1}{5}$

c) $2\frac{1}{4}+6.35=\frac{9}{4}+6.35=2.25+6.35=8.60$

or $2\frac{1}{4}+6.35=\frac{9}{4}+\frac{635}{100}=\frac{9}{4}+\frac{127}{20}=\frac{45}{20}+\frac{127}{20}=\frac{172}{20}=\frac{43}{5}$

Observe that 8.60 = $\frac{43}{5}$

Note: To add fractions and decimals:

1. Convert the decimal to fraction (or the fraction to decimal).
2. Add the fractions (or the decimals).
3. Simplify .

Exercise 3F

1. Find the sum and write the sum in decimals and in fractions.

   a) $\frac{3}{5}+0.1$

   b) $\frac{5}{8}+0.6$

   c) $2\frac{1}{2}+5.6$

   d) $5\frac{1}{4}+1.375$

   e) $4\frac{7}{8}+3.4$

   f) $14\frac{1}{2}+7.2$

2. Almaz purchased $2\frac{1}{2}$ kg of potatoes, and 0.75kg of onion. How many kilograms of vegetable did she buy in all?
3. A boy walked $3\frac{3}{4}$km one day, 2.5km the second day and 6.875 km the third day. How many kilometers did he walk in all on these three days?
4. Hadas makes 2 cakes and puts them into the oven at 3:15 P.M. Each must bake for $1\frac{1}{4}$ hours. At what time should she remove them from the oven?

3.4.2 Subtraction of Fractions and Decimals

Activity 3.9

Subtract $\frac{1}{2}-0.1$

We use similar method that we used for addition when subtracting fractions and decimals. Can you subtract 0.2 from $\frac{4}{5}$? Study the following example:

Example 19

Find the difference

a) $\frac{3}{4} - 0.38$ b) $4\frac{3}{5} - 2.1$

Solution

a) $\frac{3}{4} - 0.38 = 0.75 - 0.38 = 0.37$

Therefore, $\frac{3}{4} - 0.38 = 0.37$ or $\frac{3}{4} - 0.38 = \frac{37}{100}$

b) $4\frac{3}{5} - 2.1 = 4.6 - 2.1 = 2.5$

or $4\frac{3}{5} - 2.1 = \frac{23}{5} - \frac{21}{10} = \frac{46-21}{10} = \frac{25}{10} = \frac{5}{2}$

Therefore, $4\frac{3}{5} - 2.1 = 2.5$ or $4\frac{3}{5} - 2.1 = \frac{5}{2}$

Check whether the results are equal. What do you conclude?

Note: To subtract fractions and decimals:

1. Convert the decimal to fraction (or the fraction to decimal).
2. Subtract the fractions (or the decimals).
3. Simplify.

Exercise 3.G

1. Find the difference and write the difference in decimals and in fractions.

a) $\frac{4}{5} - 0.32$ c) $2\frac{3}{8} - 1.75$ e) $7\frac{15}{16} - 2.375$

b) $\frac{1}{2} - 0.125$ d) $4\frac{1}{2} - 1.375$ f) $13.125 - \frac{17}{10}$

2. Shashitu cut 2.125 cm material off of the bottom of $21\frac{1}{4}$ cm skirt .Howlong is the skirt now?

3. Of the $5\frac{1}{2}$ hours school time, the brake for lunch takes up 1.25 hours. Find the number of hours available for actual teaching work?

4. Abdu had to travel a distance of $7\frac{1}{2}$ km. He traveled 6.75km by bus and walked the rest of the distance. How many kilometers did he walk?

5. Is $\frac{3}{4}+0.875-\frac{5}{8}=\frac{7}{8}+\frac{5}{8}-0.75$?

6. A string is cut in half and one half is used to bundle news papers. Then one fifth of the remaining string is cut off and used to tie a ballon. The piece left is 2 metre long. How long was the string originally?

3.5 Further on Multiplication and Division of Fractions and Decimals

In this sub-unit you will deal with multiplication and division of fractions and decimals in more detail.

3.5.1. Multiplication of Fractions and Decimals

Do you remember how to obtain product of two fractions or two decimals?

Activity 3.10

Multiply

a) $\frac{3}{4}\times\frac{1}{2}$ b) $\frac{5}{12}\times\frac{6}{25}$ c) 0.2×0.4

Multiplying Fractions

Step 1. Convert mixed numbers (if any) to improper fractions.

Step 2. Multiply the numerators and denominators.

Step 3. Reduce the answer to lowest terms.

$$2\frac{1}{3}\times\frac{1}{2}=\frac{7}{3}\times\frac{3}{2}=\frac{7\times3}{3\times2}=\frac{21}{6}=\frac{7}{2}=3\frac{1}{2}$$

Multiplying Decimals

Step 1. Multiply numbers, ignoring decimal points.

Step 2. Count and total number of decimal places in multiplier and multiplicand.

Step 3. Starting at right in the product, count to the left the number of decimal places totaled in step 2. Insert decimal point. If number of places is greater than space in answer, add zeros.

$$\begin{array}{rl} 2.48 & \text{2 decimal places} \\ \underline{.018} & \text{3 decimal places} \\ 1984 & \\ 248 & \\ \underline{000} & \\ .04464 & \text{5 decimal places} \end{array}$$

Here are some examples:

Example 20

Multiply

a) $\frac{8}{9} \times \frac{3}{4}$ c) 0.2×0.6 e) 28.1×0.73

b) $\frac{2}{5} \times \frac{4}{7}$ d) 2.5×3.5

Solution a) $\frac{8}{9} \times \frac{3}{4} = \frac{8 \times 3}{9 \times 4} = \frac{24}{36} = \frac{2}{3}$

b) $\frac{2}{5} \times \frac{4}{7} = \frac{2 \times 4}{5 \times 7} = \frac{8}{35}$

c) $0.2 \times 0.6 = \frac{2}{10} \times \frac{6}{10} = \frac{12}{100} = \frac{3}{25} = 0.12$ (by long division)

$$\begin{array}{rl} \text{or } 0.2 & \text{one decimal place} \\ \underline{\times\ 0.6} & \text{one decimal place} \\ 0.12 & \text{two decimal places} \end{array}$$

Step 1. **Multiply as with whole numbers.**

Step 2. **Count the number of decimal places in both factors**

Step 3. **Place the decimal point. The number of decimal places in the product is the total number of decimal places in the factors.**

d) $2.5 \times 3.5 = \frac{25}{10} \times \frac{35}{10} = \frac{875}{100} = \frac{35}{4} = 8.75$

or

$$\begin{array}{rl} 2.5 & \text{one decimal place} \\ \times\ \underline{3.5} & \text{one decimal place} \\ 125 & \\ \underline{75\ \ } & \\ 0.875 & \text{two decimal places} \end{array}$$

e)

$$\begin{array}{rl} 28.1 & \text{one decimal place} \\ \times\ \underline{0.73} & \text{two decimal places} \\ 843 & \\ \underline{1967} & \\ \underline{20.513} & \text{three decimal place} \end{array}$$

We now look at an example that deals with multiplication of fractions and decimals.

Example 21

Multiply

a) $0.2 \times \frac{2}{5}$ b) $2\frac{3}{4} \times 1.625$

Solution: a) $0.2 \times \frac{2}{5} = 0.2 \times 0.4 = 0.08$

or $0.2 \times \frac{2}{5} = \frac{2}{10} \times \frac{2}{5} = \frac{4}{50} = \frac{2}{25}$

$0.2 \times \frac{2}{5} = 0.08$ or $0.2 \times \frac{2}{5} = \frac{2}{25}$

b) $2\frac{3}{4} \times 1.625 = 2.75 \times 1.625 = 4.46875$ (why?)

or $2\frac{3}{4} \times 1.625 = \frac{11}{4} \times \frac{1625}{1000} = \frac{11}{4} \times \frac{13}{8} = \frac{143}{32}$

Therefore $2\frac{3}{4} \times 1.625 = 4.46875$ or $2\frac{3}{4} \times 1.625 = \frac{143}{32}$

Note: The following steps explain how to multiply fractions and decimals.

Step 1. Convert the decimal to fraction (or the fraction to decimal).

Step 2. Multiply the fractions (or the decimals).

Step 3. Simplify.

Exercise 3.H

1. Multiply

a) 0.8×0.5

b) 0.12×0.3

c) 0.042×0.4

d) 0.153×0.5

e) 2.235×1.35

f) $\frac{3}{4}\times\frac{2}{5}$

g) $1\frac{3}{4}\times 3\frac{1}{2}$

h) $1\frac{3}{7}\times\frac{7}{10}$

2. Find the product. Write the product either in fraction form or decimal form.

a) $\frac{5}{8}\times 0.4$

b) $2.6\times\frac{1}{4}$

c) $7.5\times 2\frac{3}{4}$

d) $7\frac{3}{4}\times 3.2$

e) $12\frac{1}{8}\times 0.25$

f) $4\frac{1}{5}\times 0.375$

3. The width of a poster measures 38 cm. If a photocopy machine is used to make a copy of 0.6 of the original size, what is the width of the copy?

4. Debele ran 2.5 laps every morning for 5 days. How many laps did he run in all?

5. Derartu runs 8.25 kilometers every day in practice. How far does she run in 7 days?

3.5.2 Division of decimals

Remember that when multiplying by 10, 100, 1000, and so on, we move the decimal point in multiplicand the same number of places to the right as we have zeros in the multiplier. And for division, we move the decimal point to the left.

The following steps explain how to divide decimals:

Step 1. Convert the dividend and divisor to natural numbers by multiplying with powers of 10 (i.e.10, 100, 1000,…).

Step 2. Simplify.

Activity 3.11

Divide

a. $0.2 \div 0.1$

b. $0.4 \div 0.01$

c. $0.5 \div 0.001$

d. $0.01 \div 0.004$

Can you divide decimals? Here are some examples.

Example 22

Divide

a) 0.8÷ 0.2

b) 19.6÷ 0.14

c) 6.15÷0.5

d) 9. 718÷ 0.226

Solution

a) $0.8 \div 0.2 = \frac{0.8}{0.2} = \frac{0.8 \times 10}{0.2 \times 10} = \frac{8}{2} = 4$

b) $19.6 \div 0.14 = \frac{19.6}{0.14} = \frac{19.6 \times 100}{0.14 \times 100} = \frac{1960}{14} = 140$

c) $6.15 \div 0.5 = \frac{6.15}{0.5} = \frac{6.15 \times 100}{0.5 \times 100} = \frac{615}{50} = \frac{123}{10} = 12.3$

d) $9.718 \div 0.226 = \frac{9.718}{0.226} = \frac{9.718 \times 1000}{0.226 \times 1000} = \frac{9718}{226} = 43$

We can also divide fractions by decimals as follows:

Example 23

Divide

a) $\frac{1}{4} \div 0.08$ b) $3.2 \div \frac{1}{5}$ c) $20\frac{5}{8} \div 6.875$

Solution:

a) $\frac{1}{4} \div 0.08 = \frac{1}{4} \div \frac{8}{100} = \frac{1}{4} \times \frac{100}{8} = \frac{100}{32} = \frac{25}{8} = 3.125$

b) $3.2 \div \frac{1}{5} = \frac{32}{10} \div \frac{1}{5} = \frac{32}{10} \times \frac{5}{1} = \frac{160}{10} = 16$

c) $20\frac{5}{8} \div 6.875 = \frac{165}{8} \div \frac{6875}{1000} = \frac{165}{8} \times \frac{1000}{6875} = \frac{165{,}000}{55{,}000} = 3$

Do you remember?

To divide a number or a fraction by a fraction, we should multiply the first by the reciprocal of the second.

Note

We can not change order in division but we can do so in multiplication. That means $a \times b = b \times a$ but $a \div b \neq b \div a$.

Scientific Notation

Did you know that the planet Mercury is about 36, 000,000 miles from the sun? Large numbers like 36,000,000 can be expressed in scientific notation.

Definition 3.1: A number in scientific notation (standard notation) is written as the product of a number greater than or equal to 1 and less than 10 and a power of ten.

For example, let us write 36,000,000 in scientific notation.

3.6000000 (move the decimal point to get a number between 1 and 10).

36,000,000 = 3.6×10^7 (The decimal point was moved 7 places).

Mercury is about 3.6×10^7 miles from the sun.

Example 24

Write 387,000 in scientific notation.

Solution: 3.87000 (move the decimal point to get a number between 1and 10)

3.87×10^5 (The decimal point was moved 5 places).

Group work 3.6

Mercury is 9.17×10^7 kilometers from Earth. Jupiter is 6.287×10^8 kilometers from Earth. Which planet is closer to Earth?

Example 25

The planet Mars is an average distance of 141,710,000 miles from the sun. Express this number in scientific notation.

Solution: 1.41710000 (move the decimal point to get a number between 1 and 10).

1.4171×10^8 (The decimal point was moved 8 places)

Notice that: the number in example 25 has more digits to the right of decimal point than the numbers in the previous examples. Usually,

the decimal part of a number written in scientific notation is rounded to the hundredths place.

$1.4171 \times 10^8 \rightarrow$ The distance from Mars to the sun is about 1.42×10^8 miles.

To change a number from scientific notation to ordinary decimal notation, reverse the steps taken at the left.

Example 26

Write $3.5 \text{x} 10^6$ in ordinary decimal notation.

Solution: $3.5 \text{x} 10^6 = 3{,}500{,}000$

Exercise 3.I

1. Divide

   a) $0.24 \div 0.3$

   b) $0.725 \div 0.5$

   c) $0.12 \div 1.5$

   d) $2.9 \div 0.25$

   e) $\frac{7}{8} \div 1\frac{3}{4}$

   f) $3 \div 12\frac{1}{2}$

   g) $12\frac{1}{2} \div 0.25$

   h) $0.04 \div \frac{5}{2}$

2. A nurse was to give each of her patients at a 1.32 unit dosage of a prescribed drug. The total remaining units of the drug at the hospital pharmacy were 53.12. The nurse has 38 patients. Will there be enough dosages for all her patients?

3. Write in scientific notation (standard notation).

   a) 9, 600 b) 80,700 c) 500, 000 d) 8,300,000

4. Write in ordinary decimal notation.

   a) 2.38×10^3 c) 8.11×10^2 e) 4.321×10^7

   b) 4.917×10^5 d) 8.007×10^1

UNIT SUMMARY

Important facts you should know:

- **To express a fraction in simplest form:**

  1. Find the greatest common divisor (GCD) of the numerator and denominator,
  2. Divide the numerator and denominator by the GCD, and
  3. Write the resulting fraction.

- **To convert a fraction to a decimal,** divide the numerator of the fraction by the denominator of the fraction.

- Fractions with 100 as denominator are called **percents.**

- While **converting a terminating decimal to a fraction**:

  1. Write the digits to the left of the decimal point as a whole number.
  2. Write each digit to the right of the decimal point as a fraction over 10, 100,1000 and so on, depending on its place position.
  3. Convert to a single fraction by adding fractions.

- It is also possible **to convert decimals to fractions and percentages** using the method of multiplying and dividing decimals by powers of 10.

- In order **to compare fractions** one of the strategies is to express each fraction in simplest form. Then write equivalent fractions using LCM.

- **To add or subtract fractions and decimals:**
  1. **Convert the decimal to fraction (or the fraction to decimal)**
  2. **Add or subtract the fractions (or decimals)**
  3. **Simplify**
- **The following steps explain how to multiply fractions and decimals.**
  1. **Convert the decimal to fraction (or the fraction to decimal)**
  2. **Multiply the fractions (or the decimals)**
  3. **Simplify**
- **In order to divide decimals**
  1. **Convert the dividend and divisor to natural numbers by multiplying with powers of 10.**
  2. **Simplify**
- **A number in scientific notation is written as the product of a number greater than or equal to 1 and less than 10 and a power of ten.**

Review Exercise

1. Simplify

   a) $\frac{800}{1000}$ b) $\frac{450}{4050}$ c) $\frac{2160}{2880}$ d) $\frac{3150}{5040}$

2. Convert the following fractions to decimals and percentages.

   a) $\frac{33}{12}$ b) $\frac{37}{5}$ c) $14\frac{3}{5}$ d) $\frac{9}{10}$

3. Convert the following decimals to fractions and percentages.

   a) 0.45 b) 0.65 c) 3.2 d) 10.25

4. Arrange the following numbers in ascending order.

   a) $\frac{2}{7}$, 0.5 and $\frac{1}{3}$
   c) $\frac{1}{10}, \frac{5}{4}$ and $\frac{3}{5}$
   b) 0.83, $\frac{17}{10}$ and $\frac{5}{2}$
   d) $0.4, \frac{1}{6}$ and $\frac{5}{9}$

5. Perform the indicated operations.

   a) $\frac{3}{2} + 0.8$
   e) $\frac{21}{8} \times 0.4$
   i) $0.224 \div 1.6$
   b) $\frac{3}{8} + 0.625$
   f) $\frac{10}{3} \times 3\frac{1}{2}$
   j) $0.0032 \div 0.4$
   c) 45.5 - $\frac{4}{15}$
   g) $1.5 \div 2\frac{1}{10}$
   d) 28.1 – 0.25
   h) $12 \div 2.5$

6. Write in scientific notation (standard notation).

   a) 8,900 b) 400,000 c) 1,290,000 d) 98,000,000

7. Write in ordinary decimal notation.

   a) $6.03 \text{x} 10^5$ b) $3.89 \text{x} 10^6$ c) $5.66 \text{x} 10^9$ d) $9.9923 \text{x} 10^{10}$

8. If 2000x 2.14 = 4280, then find 0.2×21.4.

9. Abeba and Saba went to a party. Cake was served. Abeba ate $\frac{1}{8}$ of the cake. Saba had 2 smaller pieces. She ate $\frac{2}{16}$ of the cake. Who ate more cake? What fraction of the cake left uneaten?

10. Order $\frac{1}{5}$%, 20%, 200% and 1 from least to greatest.

11. Which of the following numbers is the greatest? The least?

    0.9, 63%, 7, $\frac{3}{8}$%

12. Demis pays Birr 73.86 per month to pay back a loan of Birr 1772.64. In how many months will the loan be pair off?
13. Find the value of each expression.
    a) $\frac{11}{5} - 2(0.8)$
    b) $\left(\frac{1}{10}\right)^2 + (1.6)(2.1)$
    c) $\frac{3}{8}\left(5.9 - \frac{47}{10}\right)$
14. If a = 4.8 and b = 3.2, find the value of $\frac{6a}{2b} + \frac{3a^2b}{2b} + ab$.
15. A store sells an item for Birr 486.50. If that price is 3.5 times what the store paid, what was the store's cost?
16. In which school does about 73% of the students own computers?

Students who own computers

School	Number of students
A	90 out of 270
B	56 out of 100
C	110 out of 150
D	125 out of 500

17. Which one is the largest?
    a) 30% of 80
    b) 7% of 200
    c) 27% of 900
    d) 60 % of 150

UNIT 4

INTEGERS

Unit outcomes: After completing this unit you should be able to:

- understand the concept of integers
- represent integers on a number line
- perform the operations addition and subtraction on integers

Introduction

In earlier grades, you have learnt about whole numbers, their properties and basic mathematical operations upon them. In the present unit you will continue studying a new number system, the set of integers. Here you will learn about integers, their properties and the operations addition and subtraction on integers.

4.1. Introduction to Integers

Activity 4.1

Place 4,5,6,7,8 and 9 in the empty circles so that each side has the same sum.

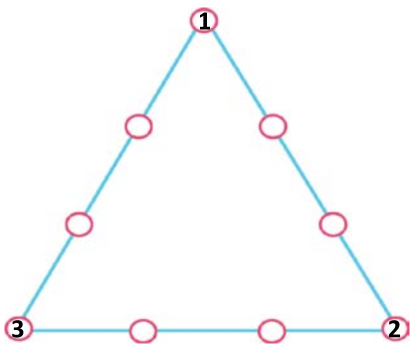


Figure 4.1

Figure 4.2

Do you like cold weather? If so, you might want to move to Debre Birhan. In some days of the year 2002 E.C the temperature in Debre Birhan was 5 degrees centigrade below zero. Have you ever heard of temperature below zero degree centigrade? Although the whole numbers have many uses, they are not adequate for describing such situations. But you can express this temperature using the **negative number written as -5**. This number is a member of the **set of integers**. In this sub-unit you will study about the set of integers. Integers are used to represent real-world quantities such as temperatures below zero.

Definition 4.1: The set of integers = $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

- The set of integers is usually denoted by Z.
- Integers greater than 0 are **positive integers**. Integers less than zero are **negative integers**. Zero is neither positive nor negative. Positive integers are written without the +sign, so +6 and 6 are the same.

Activity 4.2

Use the Venn diagram below to express relations between the sets N, W and Z. Which one of the following relations is correct?

a) $Z \subset W \subset N$ or

b) $N \subset W \subset Z$?

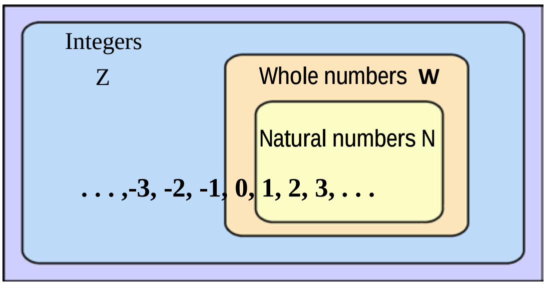


Figure 4.3

- You can represent integers as points on a number line. On a horizontal number line, positive integers are represented as points to the right of 0, and negative integers as points to the left of 0.

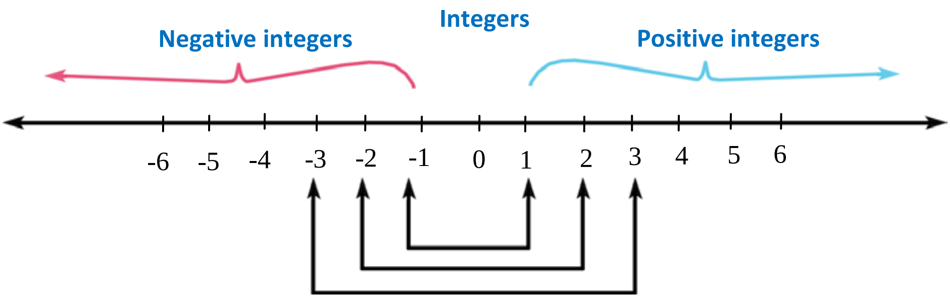


Figure 4.4

Definition 4.2: Two numbers are **opposites** of one another if they are represented by points that are at the same distance from zero, but on opposite sides of zero.

Example 1

a) -3 is the opposite of 3.

b) 7 is the opposite of -7.

Note: For any integer x, -(-x)=x. That is opposite of an opposite is itself.

Example 2

a) – (-4) = 4

b) – (-10)= 10

Exercise 4.A

1. Represent each of the following integers on a number line.

   a) -5 b) -7 c) -10

2. What integers are represented by the variables on the number line given below?

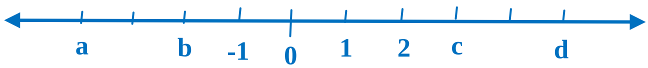


Figure 4.5

3. Write the opposites of the following integers.

   a) 10
   b) -8
   c) 120
   d) -1000
   e) 0

4. Write an integer for each situation.

   a) A profit of birr 4.
   b) A withdrawal of birr 5.
   c) 12 days behind.
   d) 9 steps forward.
   e) A loss of 20 points.
   f) 10°c below zero
   g) The opposite of 16
   h) The opposite of -20
   i) -(-30)

5. What is the greatest negative integer? And what is the least positive integer?

6. Write a number the opposite of which is

   a) Positive.
   b) Negative.
   c) Neither positive nor negative.

4.2. Comparing and Ordering Integers

Here you will study how you compare integers and the way of ordering integers.

Activity 4.3

1. What is the predecessor of 4? What is its successor?
2. Write down the three integers that come before the integers listed below.
   a) ______, ______, ______, -2, -1, 0, 1,2,3,4.
   b) ______, ______, ______, -10,-9,-8,-7,-6,-5,-4,-3.
   c) ______, ______, ______, -23, -22,-21,-20,-19,-18,-17.

Assegid and Bahiru were playing a game resulting in loss and gain. In such a game a player can end up with negative scores. Assegid's final score was -1000, and Bahiru's final score was -400. Whose score was greater?

You can use a number line to answer this question. On a number line, values increase as you go right and decrease as you go left.

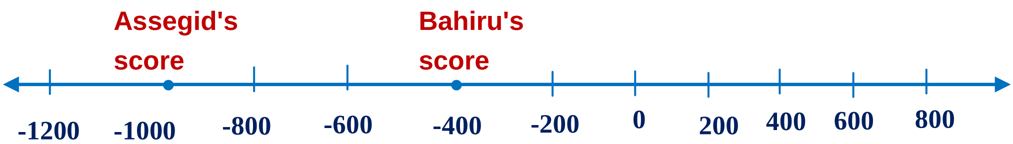


Figure 4.6

Since Bahiru's score is to the right of Assegid's score on the number line, Bahiru's score is greater than Assegid's. You can write -400 > -1000 or -1000 < -400.

Note: **On a number line, any positive integer is to the right of a negative integer. So when comparing a positive and negative integer, the positive integer will always be greater. For example, 1> -100 or -100 <1.**

Example 3

Replace $\square$ with <, >, or = in -5 $\square$ -1 to make a true sentence.

Solution: **plot a point for each integer on a number line.**

Figure 4.7

Since -5 is to the left of -1 on the number line, -5 <-1. Can you compare -6 and -2? Which is greater? -6 or -2?

Example 4

Order the integers -5,6,3,-2 and 0 from least to greatest.

Solution: **plot the point for each integer on a number line.**

Figure 4.8

Order the integers by reading from left to right.

-5, -2, 0, 3, 6

Can you order the integer -6, 4, 2, -1, and 0 from least to greatest?

Group work 4.1

The table shows the average temperatures of a certain city, for several months. In which month is the average temperature lowest?

Monthly	Temperatures
January	-12° C
March	-13° C
May	20° C
July	23° C

Example 5

Since -9<-4 and -4<1, you see that -9<1.

Example 6

7>4 but -4>-7 or -7 < -4

- **Properties of order of integers:**

1. The greater the integer, the smaller is its opposite, i.e. if a and b are integers such that a>b, then –a<-b.
2. In general, for any two integers a and b, a<b means the point corresponding to a is on the left of the point corresponding to b on the number line.
3. For any two integers a and b, either a=b or a<b or a>b.
4. For three integers a,b and c, if a<b and b<c, then a<c.

Exercise 4.B

1. Identify whether each of the following statements is true or false.
   a. For two integers on a number line, the integer which is to the right of the other is larger.
   b. Every positive integer is greater than every negative integer.
   c. Zero is less than every negative integer.
   d. -8 is the predecessor of -7.

2. Replace each $\boxed{}$ with >, <, or = to make a true statement.
   a) 2 $\boxed{}$ -10 c) -5 $\boxed{}$ 5 e) -20 $\boxed{}$ -90
   b) 3 $\boxed{}$ - 87 d) -101 $\boxed{}$ -1,001 f) -81 $\boxed{}$ -51

3. Order the integers from least to greatest.
   a) 7, -4, 0, -11, 29, -78 c) 17, 5, -9, 0, -1, -44,44
   b) 2, -13, -67, -91, 35, 18, -6

4. Order the integers from greatest to least.
   a) 10, 8, -3, 18, -70, -5 c) 17, -40, 31, -28, 26, -52
   b) 6, -77, -55, -3, 15, -19

5. Complete the table.

Predecessor	Integer	Successor
	29	
-16		
		-73
	-152	

6. List a) All negative integers greater than -9.
   b) Integers between -20 and -15.

4.3. Addition and Subtraction of Integers

Here you will be introduced how to find the sum of integers and also the difference of integers.

Addition of Integers

Activity 4.4

1. Find an integer which is 3 more than -4.
2. What is the sum of an integer and its opposite?
   e.g. -3+3=?

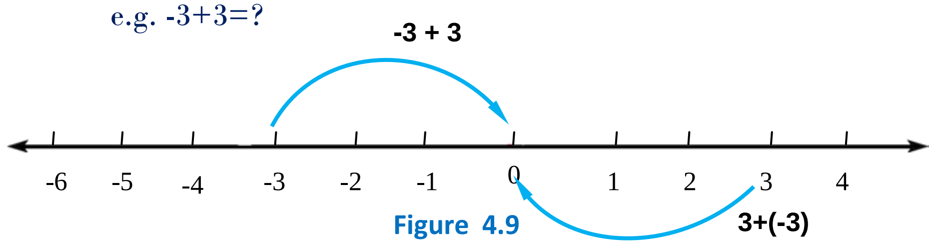


Figure 4.9

You know how to add two whole numbers on the number line. Addition of integers can be done in the same way. The only difference is that when you add a negative integer, you move to the left of the number.

Example 7

Add 5 and -7.

Figure 4.10

Solution: Start from 0, move 5 units to the right and turn in opposite direction. Move 7 units to the left and reach -2 which is the answer. Thus, we obtain 5+(-7)= -2. We may think of, for better understanding, adding 5 and -7 as follows: suppose a man made a gain of Birr 5 followed by a loss of Birr 7. Does he have profit or loss in the whole? The answer to this question will lead us to the result of adding 5 and -7.

Note: In the addition, 5+(-7), 5 and -7 are called addends and the result -2 is called the sum. We say 5 plus -7 is equal to -2.

Example 8

Add -4 and 6.

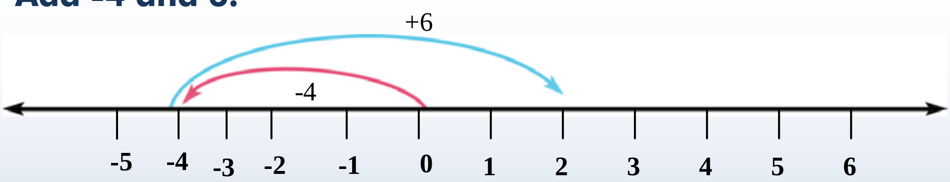


Figure 4.11

Solution: We draw a number line, we start from 0, move 4 units to the left of 0 and reach at -4. Now we move 6 units to the right and reach at 2. Therefore,

-4+6= 2. Can you add -3 and 7? What is the sum?

Example 9

Add -2 and -3.

Solution: On the number line, we start from 0. Move 2 units to the left of 0 and reach at-2. Now we move further 3 units to the left and reach at -5. Therefore -2+(-3)=-5. Can you add -6 and -9? What is the sum?

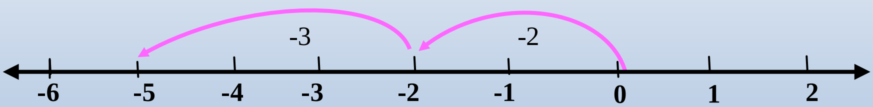


Figure 4.12

Subtraction of Integers

Before we can find difference, let's see how addition and subtraction of integers are related. Let's compare the subtraction 5-2 to the addition 5+(-2).

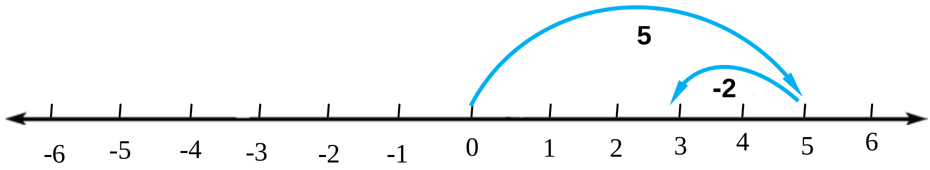


Figure 4.13

The figure shows that 5-2=5+(-2). This example suggests that adding the opposite of an integer produces the same result as subtracting it.

Note: To subtract an integer, add its opposite. That is, if a and b are two integers, then a - b= a+(-b).

Example 10

Subtract

a) 27-35 b) 43-(-37) c) -60-45

Solution

Subtracting 35 from 27 is the same as adding -35 to 27.

That is, 27-35=27+(-35)= -8

Remember that for any integer x, -(-x)=x.

Thus, 43-(-37) = 43+(-(-37))= 43+37=80

-60-45= -60+(-45)= -105

Group work 4.2

Discuss

(i) **Will 8 + (-3) and (-3) +8 give the answer? Why or why not?**

(ii) **Is the sum of any two integers an integer?**

(iii) **What will happen to the sum when you add 0 to an integer?**

The answers to this group work may lead you conclude the following important properties of addition of integers.

Properties of addition of integers: we have learnt the properties of addition of whole numbers. All those properties hold true for integers also.

Property I : The sum of two integers is always an integer. For example,

3+4=7, 7 is an integer.

3+(-4)=-1 ; -1 is an integer.

-3+(-4)= -7; -7 is an integer.

-3+4=1 ; 1 is an integer.

PropertyII (Commutative Property of Addition): For any two integers a and b, a+b= b+a.

For example, -3+5=5+(-3)=2

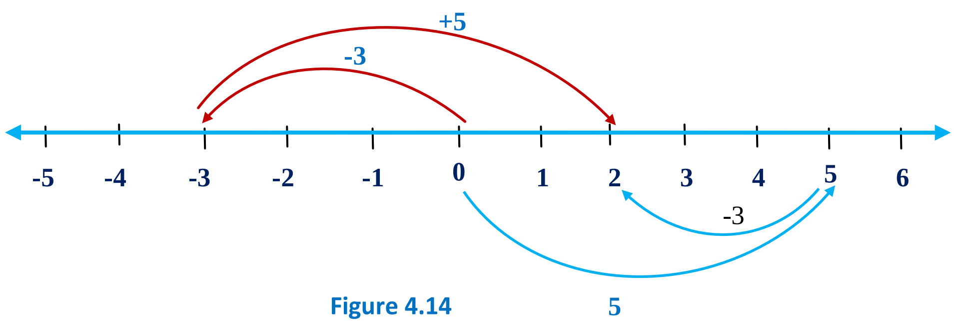


Figure 4.14

Property III(Associative Property of Addition)

If a, b and c are three integers, then (a+b)+c= a+(b+c).

For example, $(2+3)+(-5)= 5+(-5)=0$ and
$2+(3+(-5))= 2+(-2)=0$
Therefore, $(2+3)+(-5) = 2+(3+(-5))$

Property IV : When 0 is added to any integer, the sum remains unchanged.

For example, $3+0=0+3=3$; $0+(-2)=-2+0=-2$. In general, for any integer a, $a+0=0+a=a$.

Property V: The sum of an integer and its opposite is zero. That is, for any integer a, $a+(-a)=0$

For example, $3+(-3)= (-3)+3=0$

Exercise 4.C

1. Identify whether each of the following statements is true or false.
   a) If a is an integer, a-1 is its predecessor.
   b) $a–a= -a+a$ for any integer a.
   c) $3–(4 – 6)=5$
   d) $-3 – 5 = 5 – 3$
   e) The opposite of a negative integer is positive.
2. Subtract
   a) 10-23 c) 35-(-61) e) (10-(-41))-28
   b) -18-(-34) d) -69-(-31) f) 40-(12-49)
3. If a and b are two integers such that
   i) a is the successor of b, then what is the value of a-b?
   ii) a is the predecessor of b, then what is the value of a-b?
4. Use >, < or = to compare the following.
   a) $-8– (-12)$ $\boxed{}$ 0 c) $-79+36$ $\boxed{}$ $-28– (-28)$
   b) $-27+30$ $\boxed{}$ 0 d) $-45-(-45)$ $\boxed{}$ $54 – 54$
   e) $-38–57$ $\boxed{}$ $57–38$
5. Evaluate $x + y$ for $x = 42$ and $y = -71$
6. The Maths club's income from a cake sale was Birr 286. Expenses were Birr 198. Use integer addition to find the club's total profit or loss.

UNIT SUMMARY

Important facts you should know:

- **The set of integers =Z**
  Z = $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.
- **1, 2, 3,…are positive integers.**
- **…-4, -3, -2, -1 are negative integers.**
- **0 is neither positive nor negative.**
- **Every positive integer is greater than every negative integer.**
- **Two numbers are opposites of one another if they are represented by points that are the same distance from zero but on opposite sides of zero.**
- **For any integer x, -(-x)=x.**
- **For any two integers a and b.**

i) a<b means the point corresponding to a is on the left of the point corresponding to b on the number line.

ii) a=b or a<b or a>b.

- **For three integers a, b and c, if a<b and b<c, then a<c.**
- **The greater the integer, the smaller is its opposite, i.e. if a and b are integers such that a>b, then–a<-b.**
- **If a,b and c are integers, then**
  - **i) a+b is an integer**
  - **ii) a+b= b+a**
  - **iii) (a+b)+c= a+(b+c)**
  - **v) a+(-a)=0**
  - **iv) a+0=0+a=a**
- **If a, b, and c are integers, then**
  - **i) a-b is an integer**
  - **ii) a-b ≠ b-a**
  - **iii) (a-b)-c≠ a-(b-c)**
  - **iv) a-0≠0-a**

Review Exercise

1. Identify whether each of the following statements is true or false.
   a) If a and b are integers, then a-b is always an integer.
   b) There does not exist any smallest integer.
   c) If a is an integer, then a+1 is its successor.
   d) 4 – 2 = 2– 4.
   e) If a and b are opposites, then a + b = 0.
   f) If x is an integer, then x is always greater than –x.
2. Draw a number line and write all integers between -8 and 3.
3. Draw a number line and mark the following points on it:
   -6, 0, 5, -10, 6 and 11.
4. Complete the sequence given below.
   a) -18, -14, -10, _______, _______, ______, ________
   b) ______, ______, ______, ______, -17, -12, -7
   c) -100, -82, -64,_______, ______, ______, _______
5. Order the integer from least to greatest.
   -123, -541, -120, -97, 0, -58, 2
6. Subtract
   a) 30 from the sum of 12 and 13.
   b) - 420 from the sum of 354 and 147.
7. Perform the indicated operations.
   a) 3+ (-24)+13-8
   b) 1-10+63-(-21)+(-54)-27
   c) 10- [10-[10-(10-1)]]
   d) -10- [-10-[-10-(-10-10)]]
8. Use a number line to find the sum or difference.
   a) -7 + (-5) b) -13 + 15 c) 4 – 8 d) -2 – 7
9. Evaluate
   a) x + y for x = 8 and y = -12.
   b) x – y for x = -9 and y = - 13.
   c) x + y – z for x = -2, y = -5 and z = 7.
10. At 3 P.M the temperature was 9°C. By 11p.m, it had dropped 21°C. What was the temperature at 11 p.m?

UNIT 5

LINEAR EQUATIONS, LINEAR INEQUALITIES AND PROPORTIONALITY

Unit outcomes: After completing this unit you should be able to:

- develop your skills in solving linear equations and inequalities (of the form $x+a=b$, $x+a>b$).
- understand the concept of direct and inverse proportionalities and represent them graphically.

Introduction

In grade 4 mathematics lessons, you have learnt about a mathematical expression. In the present unit, you will learn about solution of simple linear equations and inequalities. You shall also learn about the concept of direct and inverse proportionalities and representing them graphically.

5.1 Solution of Simple Linear Equations and Inequalities

5.1.1 Solution of One-step Linear Equations

Activity 5.1

1. Write a mathematical expression for each phrase.

   i) 6 plus k __________

   ii) x more than 18__________

   iii) Y less than 10 _____

   vi) 54 divided by d ___________

   v) 16 times h___________

2. Evaluate each expression, if a=6, b=3, and c=2.

   i) $a+b+c$
   ii) $4ab-c$
   iii) $2(a+b)-c$
   v) $\frac{6(a+c)}{b}$
   vi) $c(b+a)-a$

3. Match the terms in column A with its simplified expression in column B

Column A	Column B
i. $2x + 7 + 5x - 4 - x$	a) $5y + 9x + 12$
ii. $5 + 7x + 2x - 3 + 6$	b) $12y + 6x + 24$
iii. $x + y + 4x - 3x + 2y + 3y$	c) $9x+8$
iv. $3x^2 + 5x - 17 + 6x + 20$	d) $3x^2$
v. $4x + x^2 + 12 - 4 + 2x$	e) $6x + 3$
vi. $12y + 12x + 12 - 6x + 12$	f) $3x^2 + 11x + 3$
vii. $12y + 4 + x - 7y + 8 + 8x$	g) $x^2 + 6x + 8$
viii. $5x + x^2 + 2x + 4 - 4 - x^2$	h) $7x$
ix. $5x^2 + 8x + 7x^2 + 6 - 12x^2$	i) $2x + 6y$
x. $x^2 + 3 + 2x^2 + 4 - 7$	j) $8x+6$
	k) $9x + 18$
	l) $3x + 6$

4. Identify each of the following as equation or inequality.

   a) $x + 1 = 3$
   b) $2x > 5$
   c) $5a = 10$
   d) $x-1 < 4$

Can you tell the difference between a mathematical expression, equation and inequality? You have learnt about a mathematical expression, an equation and an inequality in grade 4 mathematics lessons. Here you will study about solution of one-step linear equations.

Definition 5.1: An equation that can be written in the form $ax+b=0$, $a \neq 0$ is called a **linear equation**.

Example 1

Equations such as 2x+3=0, 3x-5=0, 10x=10 and x+7=0 are linear equations in one variable.

In the linear equation x+2=6, for example, the variable x represents the number or unknown for which we are solving.

We **solve** the equation when we replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution or a root of the equation**. The solution to x+2=6 is 4 because 4+2= 6 is true.

Example 2

Which of the numbers 8,9 or 10 is the solution of 9+x=19?

Solution:

Replace x with 8	Replace x with 9	Replace x with 10
9+x=19	9+x=19	9+x=19
9+8=19	9+9=19	9+10=19
$17\neq19$	$18\neq19$	19=19
This sentence is false.	This sentence is false.	The sentence is true.

Therefore, the solution is 10.

Definition 5.2: The set whose elements are considered as possible replacement for the variable in a given equation or inequality is called the **domain of the variable**.

For example, in Example 2 above, the domain of the variable ={8,9,10}. Usually in solving for the unknown, we place variable(s) on the left side of the equation and constants on the right.

Activity 5.2.

By trial –and –error, find a value of the variable.

i) $x+3=8$ ii) $y+6=0$ iii) $z-5=0$

iv) $7=x-1$ v) $5a=20$ vi) $23-n=20$

The trial-and-error method can be used only for simple equations. For large value of x it is not suitable. Even for simple equation it is time-consuming and tedius too. So, it is better to find a simple and systematic method of solving an equation.

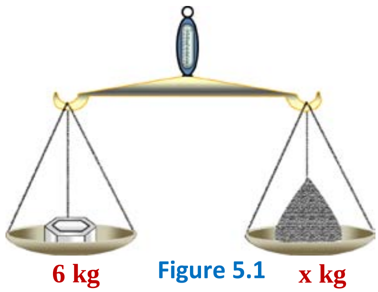


Figure 5.1

To visualize how equations work, think of the balancing scale shown in Figure.5.1. We know that when equal weights are placed on two pans, the beam of the balance remains horizontal. Similarly, two sides of an equation are equal when we have equal numbers on both sides. Suppose, we have x kg of sugar on the right hand pan and to keep it in a balance position we put 6 kg of weight to the left. Then we can say that x=6.

I) Consider Figure 5.2

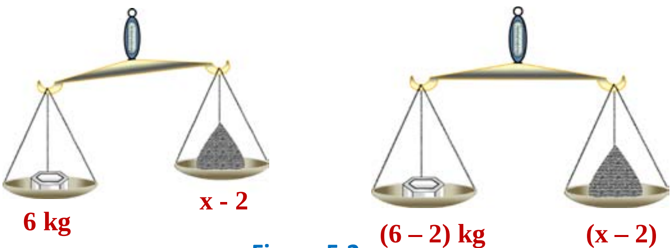


Figure 5.2

If we take away 2kg of sugar, the balance would dip to the left. Clearly, if we remove 2kg of weight from the left pan then only the balance will return to the horizontal position.

In equation form, we can write.

$x = 6$

$x-2 = 6-2$ (Taking away 2 from both sides)

or $x-2=4$

II) Consider Figure 5.3

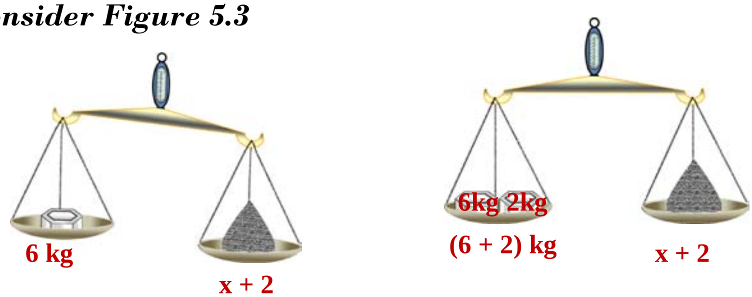


Figure 5.3

If we add 2 kg more sugar to the right pan, the balance would dip to the right. To bring it back to the horizontal position, we have to put 2 kg of weight on the left pan.

In equation form, we can write this as:

$x = 6$

$x+2 = 6+2$

or $x+2 = 8$

III) Consider Figure. 5.4

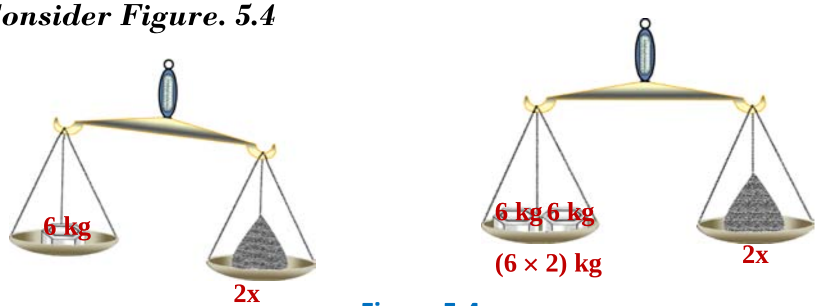


Figure 5.4

If we double the quantity of sugar, the balance would dip to the right pan due to heavy mass and clearly we have to put double weight in the left pan to keep the balance in horizontal position.

In equation form, we can write this as:

$x = 6$

Or $2x = 6 \times 2$

Or $2x = 12$

IV) Consider Figure 5.5

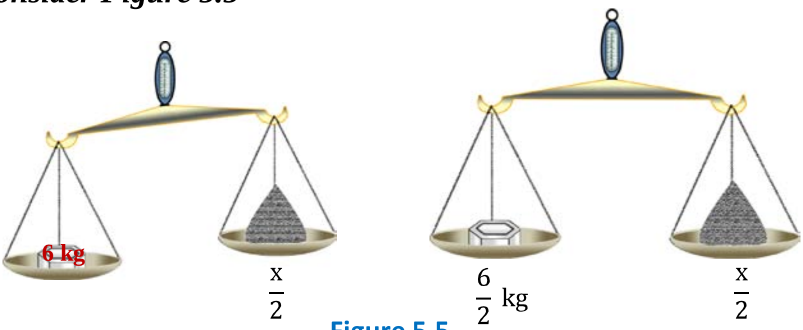


Figure 5.5

If we take away half of the sugar, the quantity of sugar in the right pan becomes less. As a result, the balance will dip to the left. To bring the balance in horizontal position, we have to take half of the weight from the left pan. Therefore, we write $\frac{x}{2} = \frac{6}{2}$ or $\frac{x}{2} = 3$.

In all the above four cases, we make the balance in horizontal position by adjusting the weight with corresponding quantity of sugar. Same process can be applied to an equation, too, under the following **rules**.

1. If we add or subtract the same number to or from both sides of an equation, then the equality symbol will not change.

   For example, a) If x-2=3, then x-2+2=3+2.
   Or x=5
   b) If x+2=3, then x+2-2=3-2 or x=1.

Since multiplication is repeated addition and division is repeated subtraction, we can also apply the following rules:

2. The equality symbol will not be changed in the equation if we multiply both sides of the equation by the same non-zero number.

   For example, if $\frac{x}{2} = 3$, then $\frac{x}{2} \times 2=3\times2$ or x=6.

3. The equality symbol will not be changed in the equation, if we divide both sides of the equation by the same (non-zero) number.

   For example, if 4x=8, then

   $\frac{4x}{4} = \frac{8}{4}$ or $x = 2$.

The following examples will illustrate how the rules are used in solving the given equations.

Example 3

Solve a) $x+5=13$ c) $3x=24$

b) $x-7=19$ d) $\frac{x}{8} = 5$

Solution:

a) $x+5=13$ ....Given

$x+5-5=13-5$ ...... Subtracting 5 from both sides of the equation

Or $x = 8$

Check: $8+5= 13$(True)

Therefore, $x=8$ is the root or solution of the given equation.

b) $x - 7=19$..............Given

$x-7+7=19+7$........Adding 7 to both sides of the equation.

$x = 26$

Check: $26-7=19$ (True)

Therefore, $x=26$ is the root or solution of the given equation.

c) $3x=24$.................. Given

$\frac{3x}{3} = \frac{24}{3}$ .... Dividing both sides of the equation by 3 or $x=8$

Check: $3\times 8 =24$ (True)

Therefore, $x=8$ is the root or solution of the given equation.

d) $\frac{x}{8} = 5$

8$\left(\frac{x}{8}\right) = 8(5)$**...............Multiplying both sides of the equation by 8**

Or x=40

Check: $\frac{40}{8} = 5(\text{True})$

Therefore, x=40 is the root or solution of the given equation.

Example 4

Write and solve the equation

a) **A number less three is 14. Find the number.**
b) **The product of a number and 6 is 84. Find the number.**

Solution

a) **Let x represent the number. Then, the equation can be given by x-3=14.**

x-3+3=14+3...... Adding 3 to both sides of the equation.

Or x=17

Check: 17-3=14(True)

Therefore, x=17 is the solution.

b) **Let x represent the number. Then, the equation can be given by**

6x=84

$\frac{6x}{6} = \frac{84}{6}$**...... Dividing both sides of the equation by 6**

Or x=14

Check: 6(14)=84(True)

Therefore, x=14 is the solution.

Exercise 5 A

1. Solve

a) x-3=9

b) x-7=14

c) x+9=37

d) 16-y=5

e) 34-x=0

f) 2= y-17

g) $\frac{y}{17} = 4$

h) 13-m=59

i) $\frac{10}{3}x = 20$

j) $\frac{2}{5}y = 4$

k) $\frac{11}{6}m = 3$

l) $\frac{4}{7}n = 8$

m) $\frac{n}{10} = 4$

n) 200 x=0.1

o) 0.01n=10

2. Write and solve an equation.

a) 7 more than a number is 34.

b) 3 less than a number is 19.

c) Twice a number is 26.

d) 5 subtracted from 8 times a number gives 0.

e) The number of grade six students at a school is 316. This is 27 more than the number of grade eight students. How many grade eight students are enrolled?

f) An object on the moon weighs one sixth as much as it does on earth. If a person weighs 12 kg on the moon, how much does he weigh on earth?

g) A designer dress costs Birr 225. This is three times the cost of another dress. What is the cost of the less expensive dress?

5.1.2 Solution of One-step Linear Inequalities

Activity 5.3

Write an inequality for each situation.

a) There are at least 20 people in the waiting room.

b) No more than 150 people can occupy the room.

Do you remember what you have studied about inequality in your previous mathematics lessons? You have learnt that mathematical expressions which contain the relation symbols $<$, $\leq$, $>$, $\geq$ or $\neq$ are called **inequalities**. Here you will learn more about inequalities and find out rules for solving linear inequalities.

Definition 5.3: A linear Inequality is inequality of the form $ax + b > 0$, or $ax + b \geq 0$ or $ax + b < 0$, or $ax + b \leq 0$, where $a \neq 0$.

Example 5

Inequalities such as $x+3>0$, $x-4\leq0$, $2x-1<0$ and $3x+7\geq0$ are linear inequalities.

Before we discuss the rules of transformation of inequalities, let us solve a linear inequality by substituting values from a given list of numbers.

Example 6

Which of the following numbers satisfy the inequality $x+3<5$?

a) -4 b) -1 c) 0 d) 2 e)3

Solution

a) $x= -4$, $-4+3<5$
$-1<5$ (True)

b) $x=-1$, $-1+3<5$
$2<5$ (True)

c) $x=0$, $0+3<5$
$3<5$ (True)

d) $x=2$, $2+3<5$
$5<5$ (False)

e) $x=3$, $3+3<5$
$6<5$ (False)

Thus, -4, -1 and 0 satisfy the given inequality.

Group work 5.1

Study each of the following pairs of inequalities given below carefully.

	Left	Right
a)	$5+3<9$	$(5+3)+\frac{1}{2} < 9+\frac{1}{2}$
b)	$\frac{2}{3} < \frac{5}{3}$	$\frac{2}{3}-\frac{1}{3} < \frac{5}{3}-\frac{1}{3}$
c)	$4 > 2$	$4 \times 3 > 2 \times 3$
d)	$10 < 15$	$10 \div 5 < 15 \div 5$

What do you observe? Is the one on the left true? Is the one on the right true?

What was done to the one on the left to obtain the one on the right?

If you have observed carefully, you see the following **rules of transformation**.

1. **Adding or subtracting the same number to or from each side of an inequality keeps the inequality sign remain as it is.**
2. **Multiplying or dividing both sides of an inequality by the same positive number keeps the inequality sign as it is.**

Let us use the above rules of transformation in order to solve inequalities.

Example 7

Solve

$x+4<7$ if the domain is

i) The set of whole numbers

ii) The set of counting numbers

Solution: $x+4<7$

i) $x+4-4<7-4$ .... Subtracting 4 from both sides of the inequality

Or x<3

i) The solution of the inequality x+4<7, when the domain is the set of whole numbers, consists of all whole numbers less than 3. That is, 0,1 and 2 are the solutions of the given inequality. Thus, solution set= $\{0, 1, 2\}$.

ii) The solution of the inequality x+4<7, when the domain is the set of counting numbers, consists of all counting numbers less than 3. Thus, 1 and 2 are the solutions of the given inequality. That is, solution set=$\{1, 2\}$.

Example 8

Solve

a) x-2>5 on the set of whole numbers.

b) 2x<10 on the set of natural numbers.

c) $\frac{1}{4}x > 3$ if x$\in \{0, 1, 2 \ldots, 20\}$.

d) x + $\frac{7}{8}$ <1 on the set of natural numbers.

Solution: a) x-2>5

x-2+2>5+2..... Adding 2 to both sides of the inequality

or x>7 and x $\in$ W

Thus, solution set = $\{8, 9, 10, \ldots\}$.

b) 2x<10

$\frac{2x}{2} < \frac{10}{2}$ ... Dividing both sides of the inequality by 2.

Or x <5 and x$\in$N

Thus, solution set = $\{1, 2, 3, 4\}$.

c) $\frac{1}{4}x > 3$

$4\left(\frac{1}{4}x\right) > 4(3)$ … **Multiplying both sides of the inequality by 4**

Or x>12 and x$\in\{0, 1, 2, \ldots, 20\}$

Thus, solution set =$\{13, 14, 15, 16, 17, 18, 19, 20\}$

d) **x+**$\frac{7}{8} < 1$

x+$\frac{7}{8} - \frac{7}{8} < 1 - \frac{7}{8}$ …. **Subtracting** $\frac{7}{8}$ **from both sides of the inequality**

x< $\frac{1}{8}$ **and x is a counting number**

But there is no counting number which is less than $\frac{1}{8}$**.**

Therefore, solution set = {}.

Note: **The above example illustrates that the existence of the solution set of an inequality depends on the domain of the variable.**

Remember also that the solutions of an inequality are the values that make the inequality true. They can be indicated on a number line.

Study the following examples.

Example 9

Solve and indicate the solution on the number line.

a) **x-** $\frac{3}{4} < \frac{9}{4}$, **domain = The set of whole numbers**

b) **x+2<0, domain =The set of integers**

Solution: a) x - $\frac{3}{4} < \frac{9}{4}$

x< $\frac{9}{4} + \frac{3}{4}$ **Why?**

x<3, x is a whole number

Therefore, solution set= $\{0, 1, 2\}$

Its graph can be seen from Figure 5.6 below.

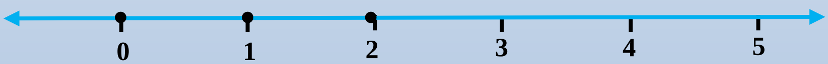


Figure 5.6

b) x+2<0

x+2-2<0-2 (why?)

Or x<-2, x is an integer

Therefore, solution set= $\{\ldots, -5, -4, -3\}$.

Its graph can be seen from figure 5.7 below

Figure 5.7

Exercise 5-B

1. Solve each of the following inequalities on the given domain.
   a) x+4 < 8, Domain = The set of whole numbers.
   b) y-2 < 7, Domain = The set of integers.
   c) y-3 >-5, Domain = The set of natural numbers.
   d) $\frac{1}{5}$y > 2, Domain = The set of negative integers.
   e) $\frac{2}{3}a < 4$, Domain = The set of whole numbers.
   f) $\frac{3}{4}x > 2$, Domain = The set of positive integers.
2. Draw the graph of the solution set of each inequality on a number line.
   a) x<6, Domain =The set of whole numbers.
   b) 2y>5, Domain = The set of integers.
   c) $\frac{1}{4}a < 2$, Domain = The set of natural numbers.
   d) y+3< -3, Domain = The set of integers.
   e) $\frac{1}{6}x < \frac{3}{4}$, Domain = The set of whole numbers.
3. Which of the following pairs of inequalities have the same solution on the set of integers?
   a) 2x>5; x>$\frac{5}{2}$
   b) $\frac{1}{4}x < 3$; x<12
   c) $\frac{3}{5}x > 2$; x<$\frac{10}{3}$
   d) x-7<1; x<8
   e) $\frac{1}{7}x - 1 > 0$; $\frac{1}{4}x > \frac{7}{4}$

4. If six times a whole number is less than 18, then find the solution and indicate the solution on a number line.

5.2 Coordinates

Activity 5.4

The grid shown has a horizontal number line and a vertical number line that meet in the middle, called the **origin.**

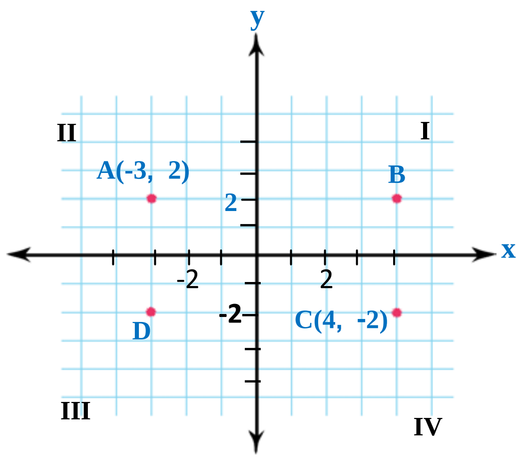


Figure 5.8

Point A is located 3 units to the left of the vertical number line and 2 units above the horizontal number line. Point A can be represented with the coordinates (-3,2).

Point C is located 4 units to the right of the vertical number line and 2 units below the horizontal number line. Point C can be represented with the coordinates (4,-2).

1. Name the coordinates that represent point B.
2. Name the coordinates that represent point D.

In mathematics, a **coordinate system**, or coordinate plane, is used to plot points in a plane. It is made up of a horizontal number line and a vertical number line that intersect at O. On the vertical number line, positive integers are represented as points above O and negative integers as points below O.

The horizontal number line is called the **x-axis**, and the vertical number line is called the **y-axis**. They intersect at their zero points. This point is called the **origin**. Together they make up a coordinate system that separates the plane into four regions called **quadrants**.

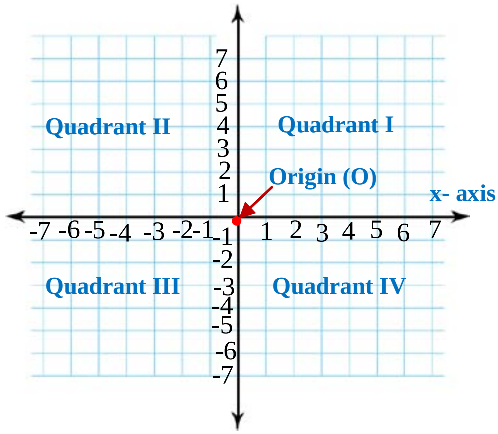


Figure 5.9

Example 10

Identify the quadrant that contains each point.

a) P

P lies in quadrant II.

b) Q

Q lies in quadrant IV.

c) R

R lies on the x-axis, between Quadrants II and III.

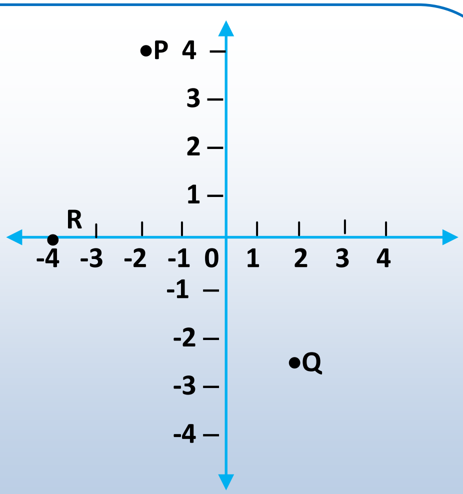


Figure 5.10

Points plotted on a coordinate system are identified by using **ordered pairs**. The first number in an ordered pair is called the **x-coordinate** (or **abscissa**), and the second number is called the **y-coordinate** (or **ordinate**).

Example 11

In the ordered pair (4,3) the x-coordinate (or abscissa) is 4 and the y-coordinate (or ordinate) is 3. What is the abscissa of the ordered pair (5,-2)? What is its ordinate?

Group work 5.2

Write down the letters at the following coordinates. What does the massage say?

(1, -4) (3, 5) (-2, 1) (4, -2)
(-5, -3) (3, 0) (3, 5) (-6, 3)
(0, -5) (1, -2) (-5, 0)
(-3, 6) (-6, 3)

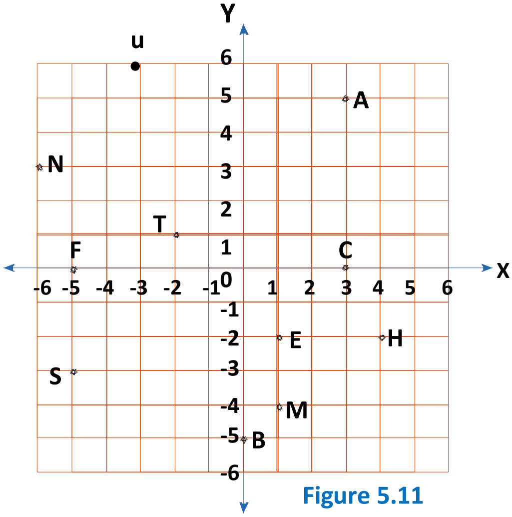


Figure 5.11

Example 12

Name the ordered pair for points A and B and identify their quadrants.

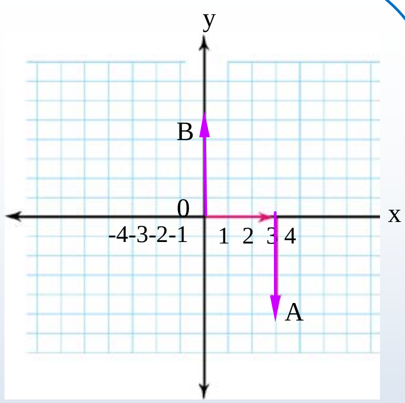


Figure 5.12

Solution: i) Start at the origin,0. Locate point A by moving right 3 units along the x-axis. The x-coordinate is +3. Now move down 5 units along the y-axis.

The y-coordinate is -5. The ordered pair for point A is (3,-5). Point A is in quadrant IV.

ii) We can see that point B is located on the y-axis, 4 units from the origin. The ordered pair is (0, 4). Point B is not in any quadrant because it is on an axis. What is the ordered pair for the origin?

Group work 5.3

Consider a point on the y-axis. What is the value of its abscissa? What is the value of the abscissa of any point on the y-axis?

Now consider a point on the x-axis? What is the value of its ordinate? What is the value of the ordinate of any point on the x-axis?

In order to plot a point in a coordinate system, draw a dot at the location named by its order pair. Study the following example.

Example 13

Plot the points C(3,5) and D(-3.5,0).

Solution: **i) First draw a coordinate system. Start at the origin 0. Move 3 units to the right. Then move 5 units up to locate the point. Draw a dot and label it C(3,5).**

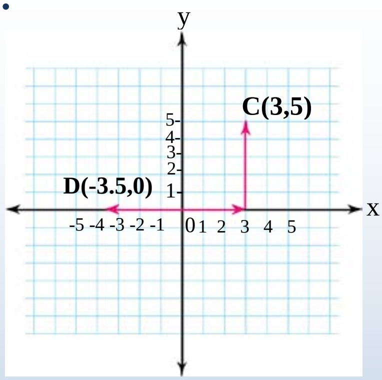


Figure 5.13

ii) To Plot the point D(-3.5,0), start at 0. Move 3.5 units to the left. Do not move up or down. Draw a dot and label it D(-3.5,0). Can you locate (-4, 2) on the coordinate plane? In which quadrant do you find it?

Exercise 5.C

1. Identify whether each of the following statements is true or false.
   a) The point p(-4, $\frac{-3}{2}$) is located in quadrant III.
   b) Q(0,-6) is a point on the x-axis.
   c) The point (3,1) satisfies the equation y=2x-5.
   d) If an ordered pair has negative x-coordinate and positive y-coordinate, then it is located in quadrant IV.
2. Write the ordered pairs corresponding to the points A,B,C,D,E and F shown in Figure 5.14.

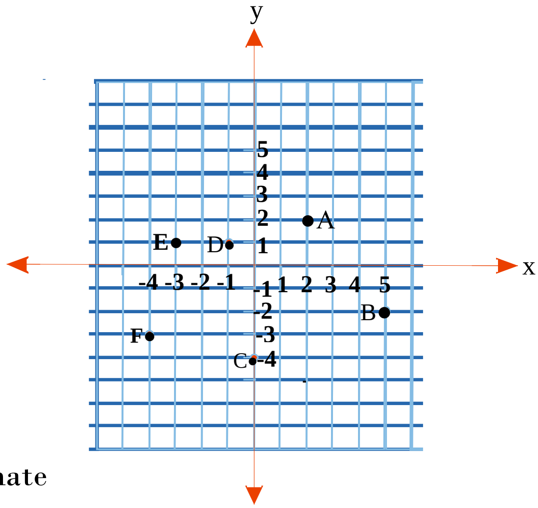


Figure 5.14

3. On graph paper, draw a coordinate plane. Then draw the graph and label each point.
   a) P(-3,6)
   b) Q(2.5,4)
   c) R(1,-4)
   d) S($\frac{-1}{2}$, -5)
   e) T(0,-3)
   f) U(4,0)
4. Name the ordered pair for each point on the city map as shown in Figure.5.15.
   a) grocery
   b) library
   c) bank
   d) city hall
   e) theater
   f) gas station

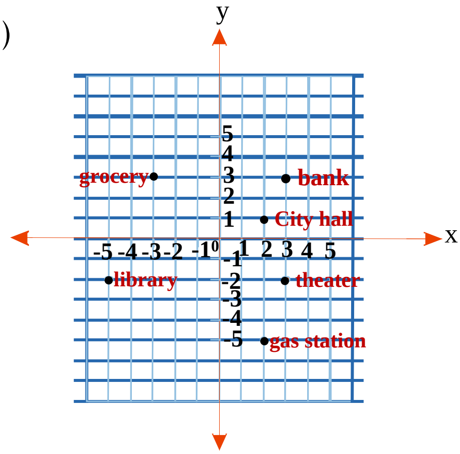


Figure.5.15

5.3. Proportionality

Activity 5.5

If y=3x, then the value of y depends on the value of x. As x varies, so does y. Simple relationships like y= 3x are customarily expressed interms of variation. How does y vary with x?

a) What will happen to y when x increases?

b) What will happen to y when x decreases?

Here you will learn the language of variation (proportionality) and how to write formulas from verbal descriptions.

5.3. 1. Direct Proportion

Suppose a car moves 60 km in one hour. The distance, d, that the car travels depends on the amount of time, t, the car takes.

Using the formula d=kt, we can write d= 60t.

Consider the possible values for t and d given in the following table.

t(hours)	1	2	3	4	5	6
d(kms)	60	120	180	240	300	360

The graph of d= 60 t is shown in figure 5.15. Note that as t gets larger, so does d.

In this situation we say that **d varies directly with t, or d is directly proportional to t.**

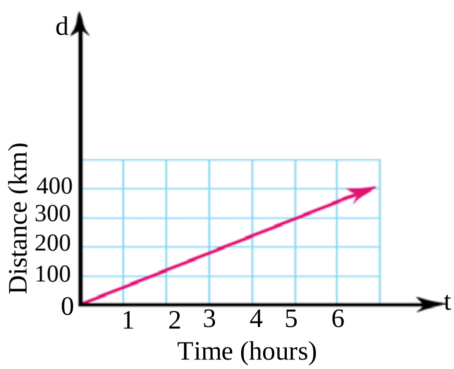


Figure 5.16

The constant rate, 60 km per hour, is called the **proportionality constant**. Join the ordered pairs (1,60), (2,120), (3,180), (4,240), (5,300) and (6,360) to get a line.

Notice that d=60 t is simply a linear equation. We are just introducing some new terms to express an old idea.

Definition 5.4: y is said to be **directly proportional** to x (written as $y \propto x$) if there is a constant k such that $y = kx$ or $\frac{y}{x} = k$.

Note: In y =kx, k is called the constant of proportionality.

Example 14

Have you observed that as the number of kilos increase the price also increases in case you go to shop to buy sugar? Suppose, in the following table, x represents number of kilos of sugar and y represents the price in Birr, what is the constant of proportionality?

x	1	2	3	4	5
y	12	24	36	48	60

Solution: Because y varies directly with x, there is a constant of proportionality k such that

$y=kx$

Because (for example) y=60 when x=5, we can write

60=k(5) or k=12(the constant of proportionality)

Therefore y=12x.

How much will it cost you if you buy 6 killos of sugar?

Observe that the graph of y=12x is a straight line through the origin (why?)

What do you obtain when you join the ordered pairs (1, 12), (2, 24), (3, 36), (4, 48), (5, 60)? A line? How many kilos of sugar can you buy if you pay Birr 84?

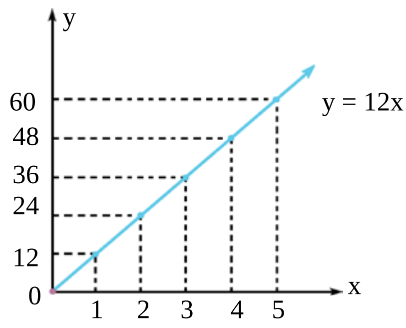


Figure 5.17

Group work 5.4

a. **Fill in the table for this rule: y = x + 3**

x	0	2	-2	-1
y			1	

b. **Fill in the gaps. Four points on the line y = x + 3 are (0, __) and (2, __), (-2, 1), (-1, __)**

c. **Plot these four points on the grid. Draw a line through them and label it y = x + 3.**

d. **Does the point (1, 4) lie on this line? Explain.**

Note. 1. **The slope of a line is a measure of its steepness.**

2. **The graph of a directly proportional relation is a straight line (where the constant of proportionality is the slope of the line) since each ordered pairs of numbers which satisfy the direct proportional relation lies on a straight line.**

Exercise 5.D

1. Find the constant of proportionality and write a formula that expresses the indicated variation.

   a) y varies directly with x, and y=12 when x=3.

   b) m varies directly with w, and m=$\frac{1}{2}$ when w=$\frac{1}{4}$.

2. Use the given formula to fill the missing entries in each table.

a) $y=\frac{3}{4}x$

y	x
$\frac{1}{3}$	
8	
	9
20	

b) $m=\frac{2}{3}w$

m	w
$\frac{1}{2}$	
3	
	6
21	

3. Solve each proportion problem.

   a) y varies directly with x, and y=100 when x=20. Find y when x=5.

   b) n varies directly with q, and n=39 when q=3. Find n when q=8.

4. A car moves at 65 km per hour. The distance traveled varies directly with the time spent traveling. Find the missing entries in the following table. Draw its graph.

Time(hours)	1	2	3	4	5
Distance(kms)					

5. The price of a cloth varies directly with the length. If a 5 metre cloth costs Birr 20, then what is the price of a 6 metre cloth?
6. If Ayele can bicycle 25 km in 2 hours, then how far can he bicycle in 5 hours?
7. If Mamitu can extract 3 kg of butter from 21 litres of milk, then how much butter can she extract from 84 litres of milk?

5.3.2. Inverse Proportion

Activity 5.6

If yx = 10, how does y vary with x?

a) What will happen to y when x increases?

b) What will happen to y when x decreases?

If you plan to make a 400 km trip by car, the time it will take depends on your rate of speed. Using the formula d=vt, we can write $t=\frac{400}{v}$

Consider the possible values for v and t given in the following table:

v(km/hr)	10	20	40	50	80	100
t(hours)	40	20	10	8	5	4

The graph of t= $\frac{400}{V}$ is shown in Figure5.17. You may join the ordered pairs (10,40), (20,20), (40,10), (50,8), (80,5), (100,4) to get such a curve. As your rate increases, the time for the trip decreases. In this situation we say that the time is **inversely proportional** to speed. Note that the graph of t=$\frac{400}{V}$ is not a straight line because t=$\frac{400}{V}$ is not a linear equation.

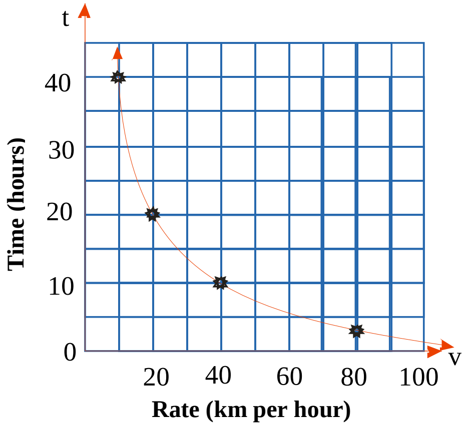


Figure 5.18

Definition 5.5: y is said to be inversely (indirectly) proportional (written $y \propto \frac{1}{x}$) if there is a constant k such that $y=\frac{k}{x}$ or y.x=k.

Example 15

The following situations are some examples of inverse proportional relations.

a) The time taken to finish a piece of work and the number of people doing the work.

b) The volume of a gas at a constant temperature and pressure.

c) The steady speed of a car and the time it takes to cover a fixed distance.

Example 16

A prize of Birr 80,000 is to be shared equally among x winners of a game.

If y represents the share of each winner, then y=$\frac{80,000}{x}$.

Answer the following.

a) If 40 people share the prize, what is the share of each?

b) If the share of each winner is Birr 400, then how many people won the game?

c) Find the constant of proportionality?

Solution: a) x=40, Y= $\frac{80,000}{40} = 2,000$

Therefore, the share of each people will be Birr 2,000.

b) Y= 400, then 400= $\frac{80,000}{x}$. Thus 400x=80,000 (why?)

or x= $\frac{80,000}{400}$ =200

therefore, 200 people won the prize.

c) Y=$\frac{k}{x}$ implies k=80,000

Therefore, the constant of proportionality equals 80,000.

Exercise 5.E

1. Find the constant of proportionality, and write a formula that expresses the indicated variation.
   a) y varies inversely with x, and y=3 when x=2.
   b) c varies inversely with d, and c=5 when d=2.
   c) a varies directly with b, and a=3 when b=4.

2. Solve each variation problem.
   a) y varies directly with x, and y=100 when x=20. Find y when x=5.
   b) a varies inversely with b, and a=9 when b=2.
      Find a when b=6.
3. Use the given formula to fill the missing entries in each table and determine whether b varies directly or inversely with a.

   i) $b=\frac{300}{a}$ ii) $b=\frac{500}{a}$ iii) $b=\frac{3}{2}a$

a	b
$\frac{1}{2}$	
1	
	10
900	

a	b
4	
	$\frac{1}{2}$
250	
$\frac{1}{8}$	

a	b
12	
	24
	$\frac{9}{4}$
15	

4. For each table, determine whether y varies directly or inversely with x and find a formula for y interms of x.

   a)

x	y
10	5
15	7.5
20	10
25	12.5

   b)

x	y
2	10
4	5
10	2
20	1

   c)

x	y
2	7
3	10.5
4	14
5	17.5

5. The time that it takes to complete a 300 km trip varies inversely with your average speed. Fill in the missing entries in the following table.

speed (km/hr)	20	40	50	
time (hours)				2

6. A bag contains sweets. When divided among 20 children each child receives 8 sweets. If the sweets were divided among 32 children, how many would each receive?
7. A train traveling at a speed of 80 km/hr will take 9 hours to cover the distance between two cities. How long will it take a car traveling at 60 km/hr to cover the same distance?

UNIT SUMMARY

Important facts you should know:

- **A mathematical statement of equality which involves one or more variables is called an equation.**
- **An equation that can be written in the form $ax+b=0$, $a \neq 0$ is called a linear equation.**
- **The set whose elements are considered as possible replacement for the variable in a given equation or inequality is called the domain of the variable.**
- **While solving an equation, the following operations may be carried out without changing the equation:**
  1. **Add the same number to both sides of the equation.**
  2. **Subtract the same number from both sides of the equation.**
  3. **Multiply both sides of the equation by the same non-zero number and**
  4. **Divide both sides of the equation by the same non-zero number.**
- **While solving inequalities, use the following rules of transformation.**
  1) **Adding or subtracting the same number to or from each side of an inequality keeps the inequality sign remain as it is.**
  2) **Multiplying or dividing both sides of an inequality by the same positive number keeps the inequality sign as it is.**
- **A coordinate system, or coordinate plane is used to plot points in a plane. It is made up of a horizontal number line and a vertical number line that intersect at the origin.**
- **y is said to be directly proportional to x if there is a constant k such that $y=kx$. K is called the constant of proportionality.**
- **y is said to be inversely (indirectly) proportional to x if there is a constant k such that $y = \frac{k}{x}$**

Review Exercise

1. Which expression has a value of 74 when x = 10, y = 8 and z = 12?
   a. 4 xyz c. x + 5y + 2z
   b. 2xz – 3y d. 6xyz + 8
2. Which expression simplifies to 9x + 3 when you combine like terms?
   a. $10x^2 - x^2 - 3$ c. 3x + 7 – 4 + 3x
   b. 18 + 4x – 15 + 5x d. $7x^2 + 2x + 6 - 4$
3. What is the solution of the equation 810 = x – 625?
   a. x = 185 c. x = 725
   b. x = 845 d. x = 1,435
4. Solve each of the following linear equations
   a. $x - \frac{1}{4} = \frac{3}{5}$ c. $2x = \frac{1}{3}$
   b. $x + \frac{1}{5} = 2$ d. $\frac{3}{4}x = 81$
5. Solve each of the following linear inequalities on the given domain.
   a. $x - \frac{1}{4} < \frac{1}{5}$, domain = the set of whole numbers.
   b. $x + \frac{2}{3} > 4$, domain= the set of counting numbers.
   c. $3x < \frac{3}{7}$, domain = the set of negative integers.
   d. $\frac{1}{2}x > \frac{3}{5}$, domain = the set of negative integers.
6. The perimeter of a square is four times the length of one of its sides. What is the perimeter of a square whose side has length 21 cm?

7. Name the point with the given coordinates (Figure 5.19).
   a) (8, 0)_______
   b) (1, 4)_______
   c) (-5, 6) _______
   d) (6, -5)_______
   e) (8, -8)_______
   f) (-7, -4)_______
   g) (-2, 0)_______
   h) (-5, -3)_______
   i) (-8, 0) _______
   j) (0, -8) _______

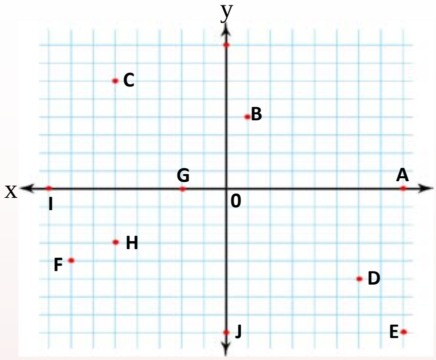


Figure 5.19

8. Without graphing, tell whether (in Figure 5.19) the line containing each pair of points is vertical or horizontal.
   a) I and G   b) H and C   c) A and E

9. When a weight is hung on a spring, the extension produced on the spring is directly proportional to the weight.
   a) Find x(Figure 5.20)
   b) Find the amount that the spring will stretch with a weight of 6 kg (Fig.5.21)

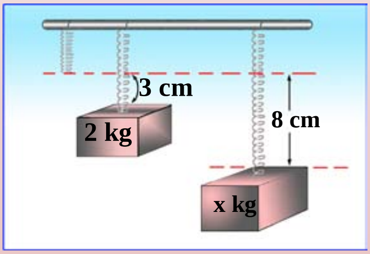


Figure 5.20

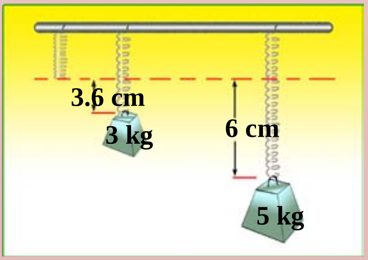
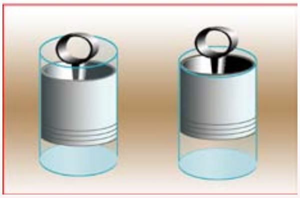


Figure 5.21

10. The volume of a gas in a cylinder is inversely proportional to the pressure on the gas. If the volume is 12 cubic centimeters when the pressure on the gas is 200 kilograms per square centimeter, then what is the volume when the pressure is 150 kilograms per square centimeter? (figure 5.22)



$V = 12\ cm^3$ $\quad V = ?$

$P = 200 \frac{kg}{cm^2}$ $\quad P = 150 \frac{kg}{cm^2}$

Figure 5.22

11. Identify whether the given relation is directly proportional or inversely proportional.
    a) The number of children in a family and the share of their father's fortune.
    b) The speed of a car and the time it takes to cover a fixed distance.
    c) The length of a rectangle of constant area with the width.
12. Wallpaper for a bedroom costs Birr 16 per roll for the walls and Birr 9 per roll for the border. If the room requires 12 rolls of paper for the walls and 6 rolls for the border, find the total cost for decorating the bedroom.

UNIT 6 GEOMETRY AND MEASUREMENT

Unit outcomes: After completing this unit you should be able to:

- identify angles.
- prove congruency of triangles.
- construct triangles.

Introduction

Geometry is an important part of human life. In every day life we refer to the starting point of a race or a point on a map, lines on a paper and lines of longitude. We also refer to planes when we talk about floors and counter tops. From grade 4 mathematics lessons you have learnt that point, line and plane are fundamental undefined terms of geometry. Here, in unit 6, you will learn how to identify angles, prove congruency of triangles and construct triangles. You will also study measurement (or measuring areas, permeters and volume) of some geometic figures.

6.1. Angles

Do you remember that, in your grade 5 mathmatics lessons, you have learned about angles, measurment and classification of angles, and also bisecting an angle? Here, you will learn about angles in more detail.

6.1.1. Related Angles

Activity 6.1

1. Answer each of the following statements as true or false.
   a. Two rays with the same end point form an angle.
   b. In Figure 6.1, points P, and Q are in the interior of ∠AOB, where as points R and S are in the exterior of ∠AOB.

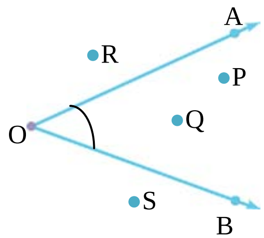


Figure 6.1

   c. We use protractor to measure or draw an angle.
   d. In Figure 6.2, the degree measure of angle BAC is 80º or
      m(∠BAC) = 80º or ∠BAC = 80º
      Or m (∠ A) = 80º

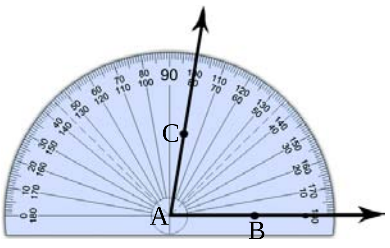


Figure 6.2

   e. The marked angle shown in Figure 6.3 can be named as ∠AOB, ∠BOA or simply ∠O.

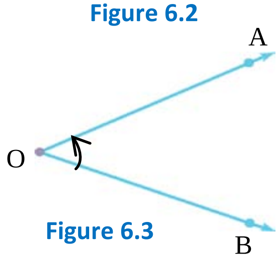


Figure 6.3

2. Name each of the marked angles given below. Also name vertex and each side.

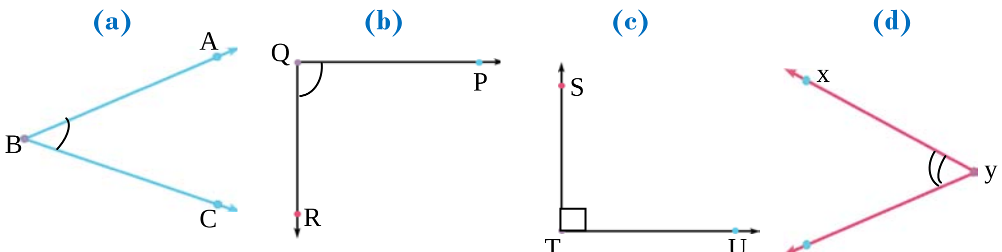


Figure 6.4

3. Study Figure 6.5

   a) Name the points that are in the interior of ∠PQR.

   b) Name the points that are in the exterior of ∠PQR.

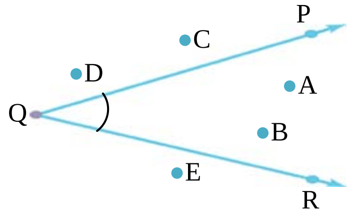


Figure 6.5

4. Name all the angles that you can find out from the following figures.

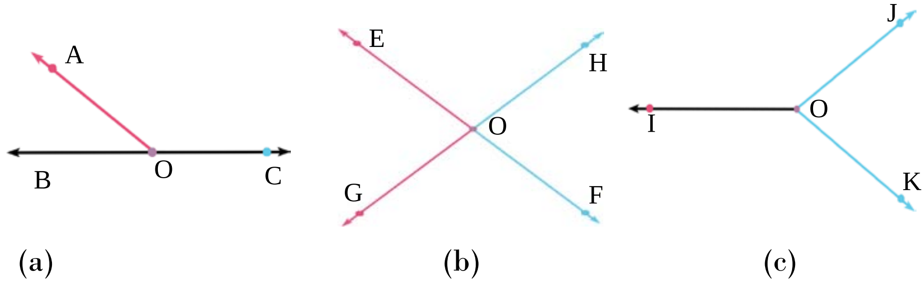


Figure 6.6

5. Measure the following marked angles and fill in the blanks.

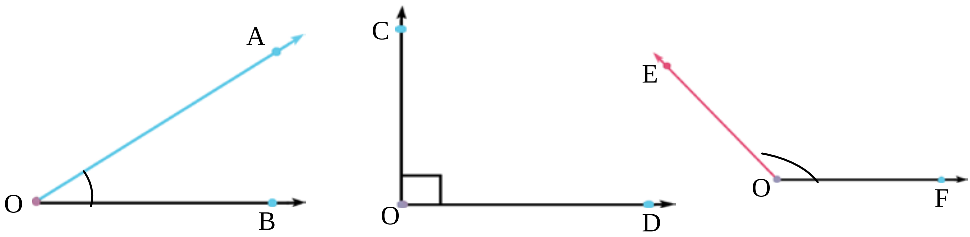
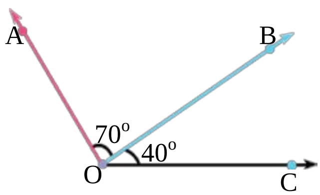


Figure 6.7

(a) m(∠AOB) = ……. (b) m(∠COD) = …… (c) m(∠EOF) = ………

6. Fill in the blanks.

(a)



m(∠AOB) = ………..

m(∠BOC) = ………..

m(∠COA) = ………..

(b)

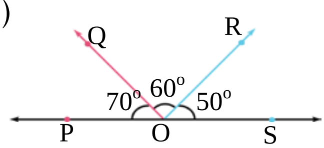


Figure 6.8

m(∠QOP) = ... m(∠SOQ) = ...

m(∠QOR) = ... m(∠ROP) = ...

m(∠SOR) =........... m(∠POS) = ...

7. Use protractor to measure the following angles a, b, c and d.

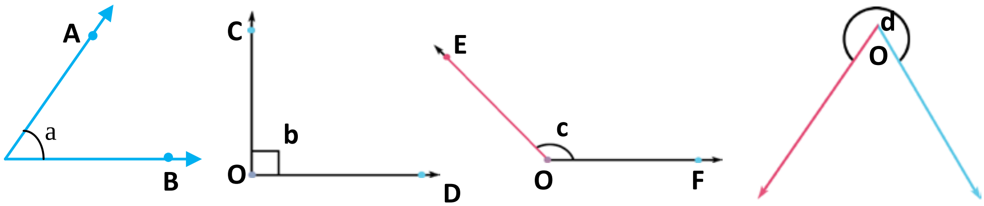


Figure 6.9

a = ... b = ... c = ... d = ...

8. Classify each angle with the given measure as acute, right, obtuse, straight or reflex.

a. 45^o .... c. 90^o .... e. 180^o .... g. 240^o ....

b. 75^o ...... d. 138^o .... f. 119^o .... h. 305^o ....

9. Draw the following angles using a protractor.

a) 90^o b) 60^o c) 120^o d) 180^o

Remember that, in grade 5 mathmatics lessons, you have learnt **classification of angles according to size** as acute angle, right angle, obtuse angle, straight angle and reflex angle. Here you will learn about types of **angles according to position**.

We know that angles are formed when lines intersect at a point. Let us state the definitions of different kinds of angles formed and study their relations.

1. Adjacent Angles

Definition 6.1. Two angles are said to be adjacent if they have the same vertex and a common side between them.

Example 1

In Figure 6.10 angles α and β are adjacent angles (or $\angle$ COB and $\angle$BOA have common side $\overrightarrow{OB}$ and the same vertex, O, thus they are adjacent angles). But $\angle$COA and $\angle$BOA are not adjacent similarly $\angle$COA and $\angle$COB are not adjacent (Why?) Can you name adjacent angles in Figure 6.11?

Are $\angle$ DOE and $\angle$FOG adjacent? Why?

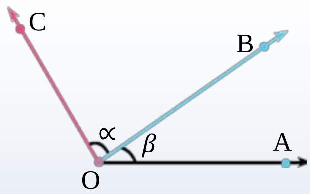


Figure 6.10

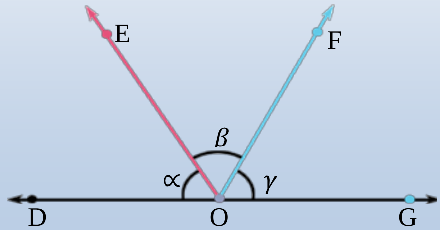


Figure 6.11

The second definition on related angles is given as follows:

2. Vertically opposite angles

Definition 6.2. **Vertically opposite angles** are two non adjacent angles formed by two intersecting lines.

Example 2

$\angle$ABC and $\angle$DBE, in Figure 6.12, are vertically opposite angles formed by intersection lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{CD}$.
(or angles α and γ are vertically opposite).
Can you name another pair of vertically opposite angles?

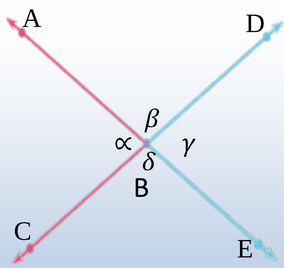


Figure 6.12

The third and fourth definitions on related angles are given respectively as follows:

3. Complementary Angles

Definition 6.3 If the sum of the measures of two angles is 90°, they format are called **complementary angles**. Either of the two complementary angles is said to be the **complement** of the other.

Example 3

In Figure 6.13 below

a) $\angle A$ and $\angle B$ are complementary since $m(\angle A) + m(\angle B) = 40° + 50° = 90°$ Here we can say that the complement of angle 40° is an angle of 50°. What is the measure of a complementary angle to an angle of 70°?

b) $\angle ABC$ and $\angle CBD$ are complementary since $m(\angle ABC) + m(\angle CBD) = 60° + 30° = 90°$

Can you give examples of the angle measures of two complementary angles of your own?

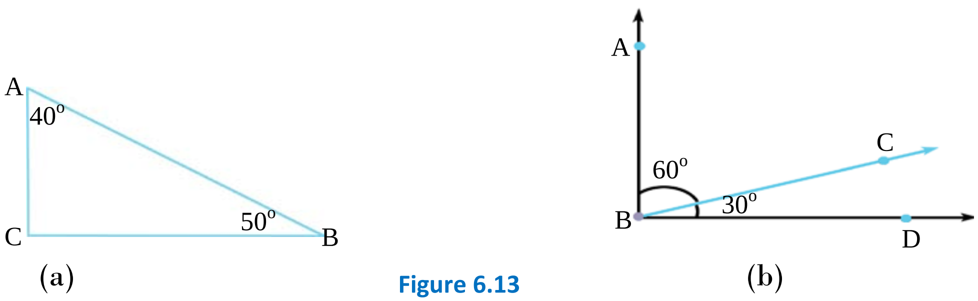
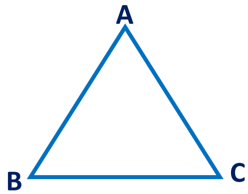


Figure 6.13

Group work 6.1

1. Measure each angle of ΔABC and find the sum $m(\angle A) + m(\angle B) + m(\angle C)$.
2. Use the result you obtain in question 1 to find the value of x.



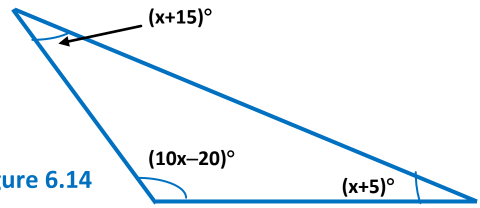


Figure 6.14

4. Supplementary Angles

Definition 6.4. If the sum of the measures of two angles is 180°, they are called **supplementary angles**. Either of the two supplementary angles is said to be the **supplement** of the other.

Example 4

In Figure 6.15 below

a) $m(\angle\alpha) + m(\angle\beta) = 180°$ (Forms a straight angle)

b) P and Q are supplementary since $m(\angle P) + m(\angle Q)$ $= 40° + 140° = 180°$

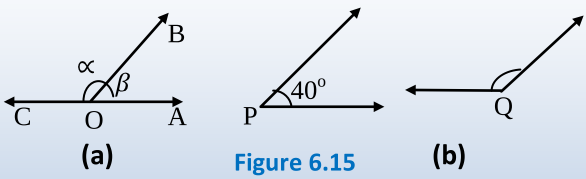


Figure 6.15

Thus, we can say P is the supplement of Q (or Q is the supplement of P).

What is the measure of an angle supplement to an angle of 120°? Can you give your own example of the measures of angles that are supplementary? Remember that a straight angle is an angle whose degree measure is 180°.

Let us study the following theorem on vertically opposite angles.

Theorem 6.1 **Vertically opposite angles are congruent.**

Given: α **and** β **are vertically opposite angles.**

Prove: $\alpha = \beta$

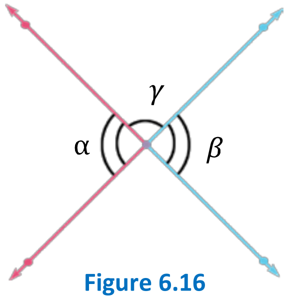


Figure 6.16

Proof

Statements	Reasons
1. $\alpha + \gamma = 180^\circ$	α and γ are angles on a straight line
2. $\beta + \gamma = 180^\circ$	β and γ are angles on a straight line
3. $\alpha + \gamma = \beta + \gamma$	substitution
4. $\alpha + \gamma - \gamma = \beta + \gamma - \gamma$	Subtracting γ form both sides
5. $\alpha = \beta$	step 4

Exercise 6.A

1. Answer each of the following statements is true or false?
   a) The supplementary of an acute angle is obtuse.
   b) The supplementary of a right angle is right.
   c) The supplementary of an obtuse angle is obtuse.
   d) The complementary of an angle with measure 70° is an angle with measure 20°.
   e) A complementary of an acute angle is acute.
   f) Adjacent angles are always complementary.

2. Fill in the blanks

Measure of Angle	Measure of Complementary angle	Measure of Supplementary angle
32^{o}	………	………..
……..	47^{o}	………..
…….	………	150^{o}
54^{o}	………	………..
……..	81^{o}	…………

3. What is the value of x in Figure 6.17

(a)

(b)

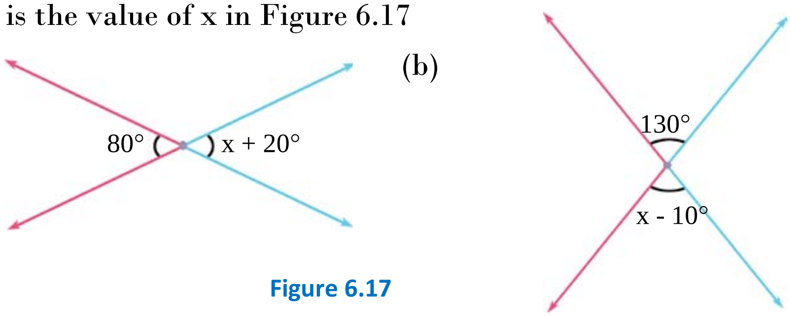


Figure 6.17

4. If, in Figure 6.18, $\alpha = 60^{o}$, then find the angle measures β, γ and δ.

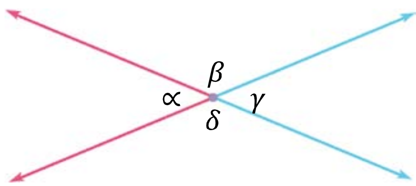


Figure 6.18

5. If the sum of the measures of two angles is equal to the measure of an obtuse angle, then one of the two angles must be

(a) acute (b) right (c) obtuse (d) 180^{o}

6. If α and β are measures of supplementary angles, then fill the blank space for each of the following.

a) $\alpha = 70^{o}$, $\beta =$ __________

b) $\alpha =$………., $\beta = 60^{o}$

c) $\alpha = \beta - 20^{o}$, $\alpha =$ __________ $\beta =$ __________

d) $\propto = \frac{1}{2}\beta$, $\propto =$ __________ $\beta =$ __________

e) $\propto = \beta$, $\propto =$ __________ $\beta =$ __________

7. Many fashion designers use basic geometric shapes and patterns in their textile designs. In the textile design shown, angles 1 and 2 are formed by two intersection lines. Find the measures of $\angle 1$ and $\angle 2$ if the angle adjacent to $\angle 2$ measures 88^o.

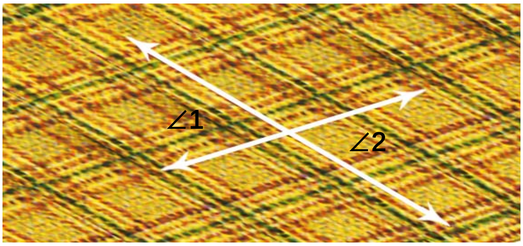


Figure 6.19

6.1.2. Angles and Parallel Lines

Here you will learn about angles formed by parallel lines and a **transversal**. The following terms, which are needed for future theorems about parallel lines, are defined as follows:

Definition 6.5. A **transversal** is a line that intersects two or more other lines in different points.

How many angles are formed if you draw two parallel lines and a transversal?

Example 5

In Figure 6.20

Line t is a transversal of lines h and k. The angles formed (numbered 1, 2, 3, 4, 5, 6, 7 and 8) have special names.

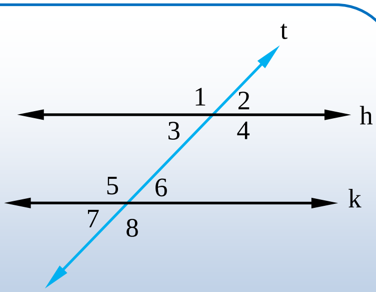


Figure 6.20

The following definition will introduce you the names given to each of these angles.

Definition 6.6. Alternate interior angles (alt. int. ∠s) are two non adjacent interior angles on opposite sides of the transversal.

Example 6

In Figure 6.21

You can see the transversal t intersecting lines m and n in two different points.

a) ∠3, ∠4, ∠5 and ∠6 are called interior angles.

b) ∠3 and ∠6; ∠4 and ∠5 are called alternate interior angles.

c) ∠1, ∠2, ∠7 and ∠8 are called exterior angles.

d) ∠3 and <5; ∠4 and ∠6 are called same side interior angles.

e) ∠1 and ∠8; ∠2 and ∠ 7 and called alternate exterior angles.

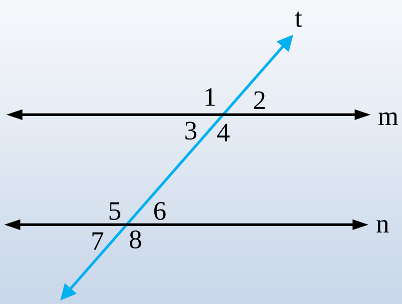


Figure 6.21

Definition 6.7 Corresponding angles (corr. ∠s) are two angles in corresponding positions relative to the two lines.

Example7

In Figure 6.20, ∠1 and ∠5; ∠2 and ∠6; ∠3 and ∠7; ∠4 and ∠8 are corresponding angles.

Activity 6.2

List the alternate interior angles, and alternate exterior angles in Figure 6.22?

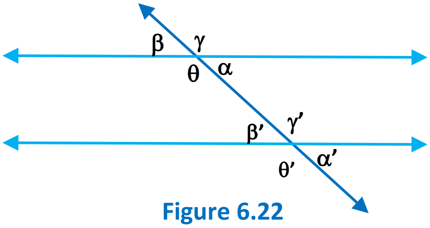


Figure 6.22

Let us study properties of parallel lines when crossed by a transversal. The following postulates (basic agreements) and theorems will illustrate ideas about angles and parallel lines.

Postulate 6.1 **If two parallel lines are crossed by a transversal, then the corresponding angles are congruent.**

Example 8

If, in Figure 6.23, lines m and n are parallel (m||n), then the corresponding angles are congruent. That is, $\propto = \delta$, $\theta = \phi$, $\beta = \gamma$ **and** $\omega = \mu$.

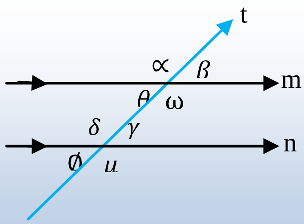


Figure 6.23

Activity 6.3

Let, in Figure 6.24, $\ell_1 \| \ell_2$ and m(∠1) = 60º then can you tell measure of angle 5?

What is the measure of angle 2? Why?

Can you tell the degree measures of other angles?

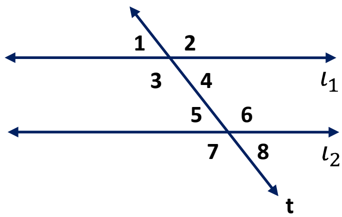


Figure 6.24

From postulate 6.1 we can easily prove the following theorems.

Theorem 6.2 If two parallel lines are crossed by a transversal, then the alternate interior angles are congruent.

Given: n∥ m; transversal t crosses lines n and m. prove: $\angle 1 = \angle 2$

Proof.

	Statements	Reasons
1.	n∥m; t is a transversal	Given
2.	<1 = <3	vertically opposite angles are congruent
3.	<3 = <2	postulate 6.1
4.	<1 = <2	Steps 2 and 3

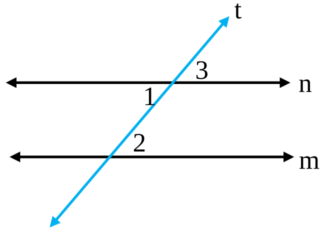


Figure 6.25

Example 9

Let, in Figure 6.26, P∥q m(∠1) = 70°, then m(∠3) = 70° (vertically opposite angles are congruent), and m(∠2) = 70° (Theorem 6.2). What is, then the measure of <4?

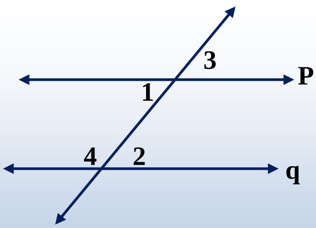


Figure 6.26

Theorem 6.3 If two parallel lines are crossed by a transversal, then interior angles on the same side are supplementary.

Given: k∥m; transversal t crosses lines k and m.

Prove: ∠1 is supplementary to ∠4.

Proof

Statements	Reasons
1. $m(\angle 4) + m(\angle 2) = 180^o$	Angles on a straight line.
2. k∥ m	Given.
3. $m(\angle 1) = m(\angle 2)$	Theorem 6.2.
4. $m(\angle 4) + m(\angle 1) = 180^o$	Substitution.
5. ∠1 is supplementary to ∠4	Definition of supplementary angles.

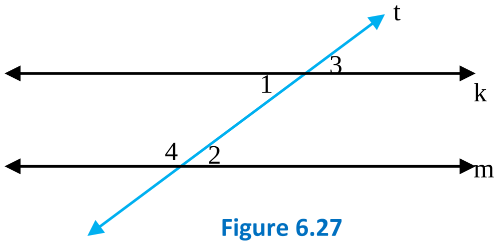


Figure 6.27

Example 10

Let, in Figure 6.28, $\ell \| m$, $m(\angle\alpha) = 100°$, then

$m(\angle\beta) = 180° - m(\angle\alpha)$… Theorem 6.3

$= 180° - 100°$

$\therefore m(\angle\beta) = 80°$

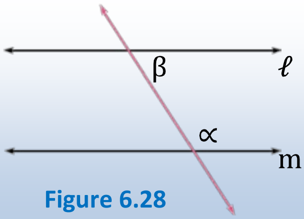


Figure 6.28

Group work 6.2

Can you complete the proof of the following theorem?

Theorem 6.4: If a transversal is perpendicular to one of the two parallel lines, then it is perpendicular to the other one also.

Given: Transversal t crosses lines k and ℓ;
$t \perp k$ (lines t and k are perpendicular); $k \| \ell$

Prove: $t \perp \ell$

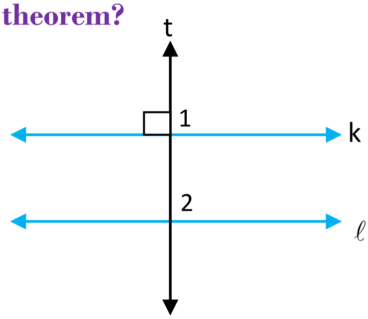


Figure 6.29

Proof

Statements	Reasons
1. $t \perp k$	Given.
2. $m(\angle 1) = 90°$	Definition of perpendicular lines and definition of a right angle.
3. $k \| \ell$	_________________
4. $m(\angle 2) = m(\angle 1)$	_________________
5. $m(\angle 2) = 90°$	Substitution.
6. _________________	Definition of a right angle and definition of perpendicular lines.

Note: At this point in our study of geometry, pairs of arrow heads (and double arrow heads when necessary) will be used to indicate parallel lines, as shown in the following example.

Example 11

In Figure 6.30, p ∥ q and k∥m, then find the values of x and y.

Solution: x = 70°(measures of corresponding angles) since k ∥ m.

y + 70° = 180° since p∥q.

Thus y = 110°.

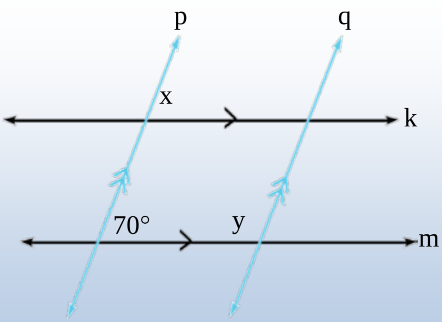


Figure 6.30

Activity 6.4

If in Figure 6.31, $\ell \parallel m$, then what can you say about the measure of $\propto + \beta$?

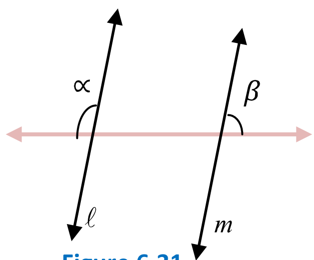


Figure 6.31

Note: The **angle – tests** used to prove that the two lines are parallel that may be stated as follows:

Two lines are parallel if and only if the angles determined by them and any transversal have the following properties.

- **Any pair of corresponding angles are congruent.**
- **Any pair of alternate interior angles are congruent.**
- **Any pair of the same side exterior angles are supplementary.**
- **Any pair of alternate exterior angles are congruent.**

Example 12

Is ℓ_1 parallel ℓ_2?

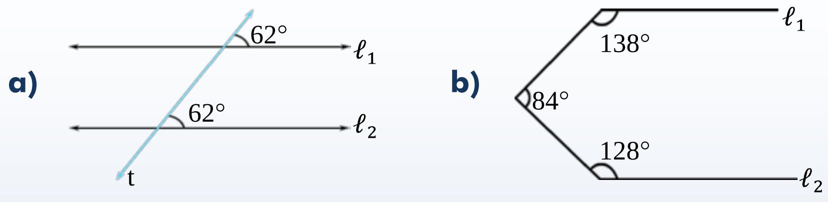


Figure 6.32

Solution

a) $\ell_1 \parallel \ell_2$ since the given corresponding angles have equal measure and hence are congruent.

b)

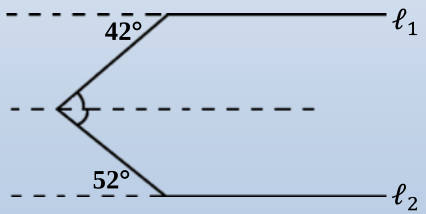


Figure 6.33

Draw a line through the vertex of the angle with 84 degrees

$42° + 52° = 94° \neq 84°$
Therefore ℓ_1 is not to parallel ℓ_2

Exercise 6.B

1. Lines ℓ and m are parallel. ∠6 and ∠10 are
   A. Alternate exterior angles.
   B. Alternate interior angles.
   C. Consecutive interior angles.
   D. Corresponding angles.
   E. Vertical angles.

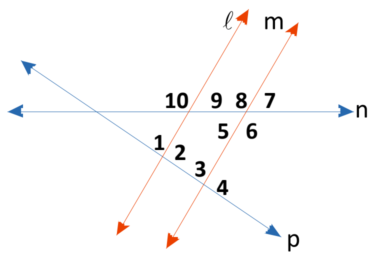


Figure 6.34

2. If, in Figure 6.35, $\ell \parallel m$, p and q are two transversals, then find the values of $\propto, \beta, \gamma$ and θ.

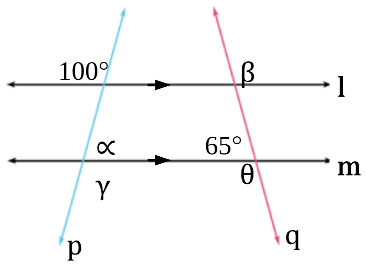


Figure 6.35

3. Except for (a) and (d) below assume, in each case of Figure 6.36, that $\ell_1 \parallel \ell_2$ and find the value of x.

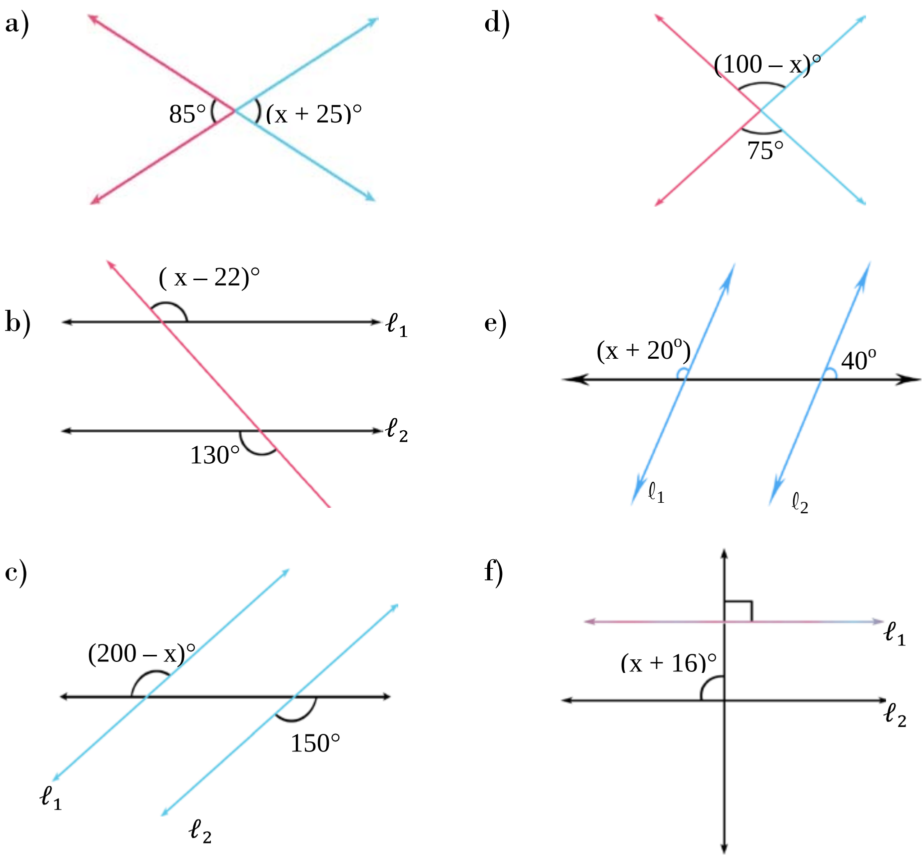


Figure 6.36

4. If, in Figure 6.37, $\ell \parallel m$ and t is a transversal, then complete the table given below.

θ	α	β	γ	δ	ϵ
35^o					
76^o					
138^o					

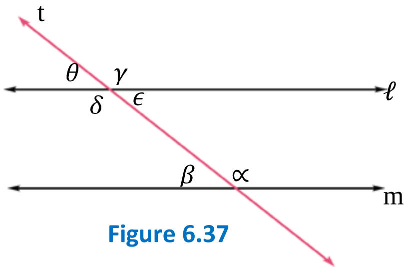


Figure 6.37

5. If, in Figure.6.38, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $m(\angle GBH) = (x + 14)^\circ$ and $m(\angle ADC) = 80^\circ$, then find

   a) the value of x

   b) $m(\angle BAD)$

   c) $m(\angle JCI)$

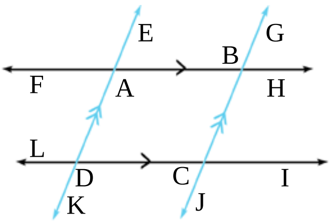


Figure 6.38

6. In Figure 6.39, which segments are parallel? $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ or $\overleftrightarrow{BC}$ and $\overleftrightarrow{DF}$ Why?

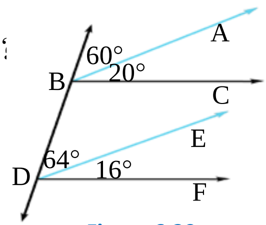


Figure 6.39

6.2 Construction of Triangles

Do you remember that a triangle is a three sided closed figure made of three line segments? Here you will learn about construction of triangles in more detail. Consider, in Figure 6.40 below, triangle ABC has three sides namely $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. It has three vertices A, B and C. The angles included between two sides are angles of the triangle. $\angle ABC$, $\angle BAC$ and $\angle ACB$ are three angles of ΔABC.

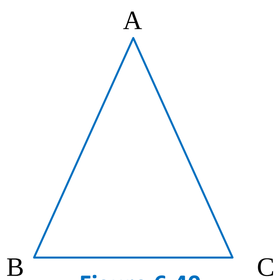
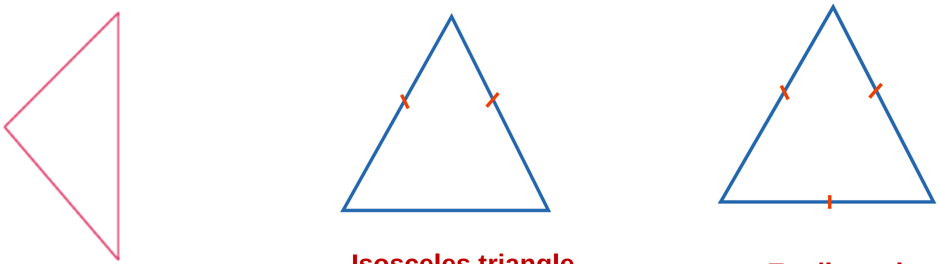


Figure 6.40

You have learnt that a triangle is sometimes classified by the number of congruent sides it has as follows:



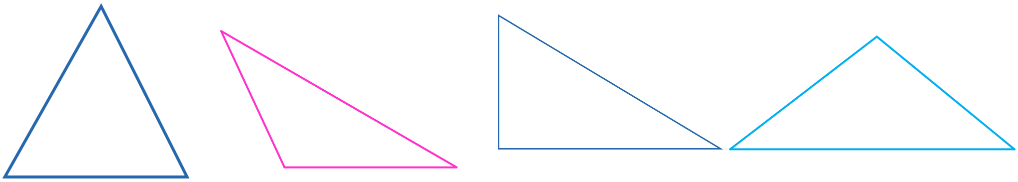
Scalene triangle
(No sides congruent)
(a)

Isosceles triangle
(At least two sides
Congruent)
(b)

Equilateral
(All sides congruent)
(c)

Figure 6.41

Triangles can also be classified by their angles:



Acute angled triangle
(Three acute angles)
(a)

Obtuse angled triangle
(one obtuse angle)
(b)

Right angled triangle
(one right angle)
(c)

Equiangular triangle
(All angles are congruent)
(d)

Figure 6.42

Group work 6.3

Work with a partner

Materials: ruler, pencil, paper, protractor measure the angles and sides of each triangle and classify each triangle as acute angled, right angled, obtuse angled, scalene, isosceles, or equilateral.

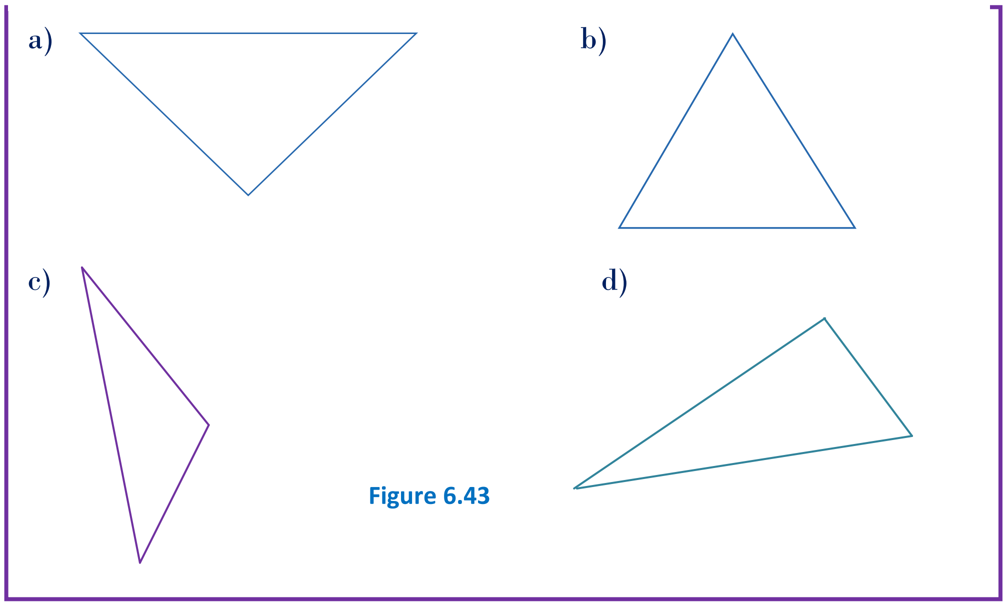
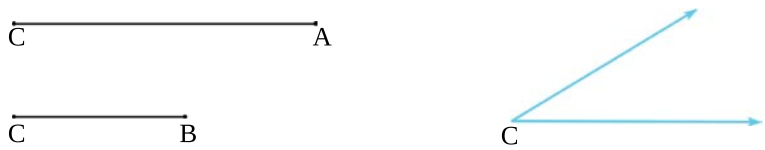


Figure 6.43

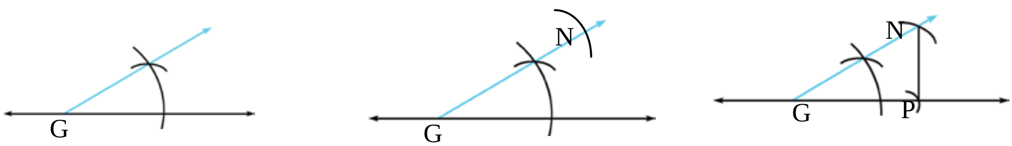
Constructing triangles

Let us construct a triangle by using a straight edge, compasses and protractor.

Given sides CA and CB and ∠C.



Solution:



1. Draw a line
Lable point G. At point G
construct ∠ G ≅ ∠C

2. On one side of ∠G
Construct $\overline{GN} \cong \overline{CA}$

3. On the other side of ∠G, construct $\overline{GP} \cong \overline{CB}$. Draw $\overline{NP}$.

Figure 6.44

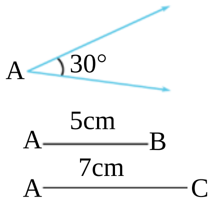
Group work 6.4

Work with a partner.

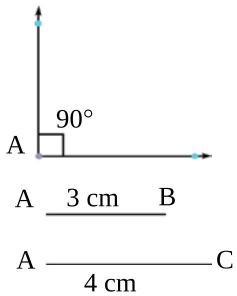
Materials: ruler, pencil, paper, protractor and a pair of compasses.

1. **Construct a triangle given the length of two sides and the measure of included angle between them. Measure $\overline{BC}$ in each case.**

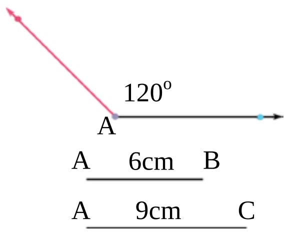
a)



c)



b)



Compare

(i) AB + BC ________ AC

(ii) AB + AC ________ BC

(iii) BC + AC ________ AB

Figure 6.45

2. **Construct a triangle given the measure of two angles and the length of one side. Measure the third angle and the other sides in each case.**

   a) $\angle A = 30^o$, $\angle B = 70^o$ and AB = 6cm

   b) $\angle A = 60^o$, $\angle B = 80^o$ and AC = 10cm

Compare

(i) AB + BC ________________ AC

(ii) AB + AC ________________ BC

(iii) BC + AC ________________ AB

Discuss

- Which angle measure is the largest? Which side is the longest? Which angle is opposite to this longest side?
- Which angle measure is the smallest? Which side is the shortest? Which angle is opposite to the shortest side?

3. Use a ruler and a protractor to measure the three sides and the three angles of ΔABC (Figure 6.46).

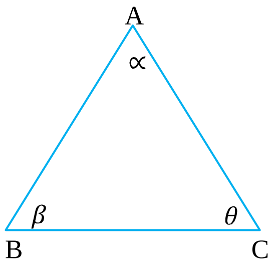


Figure 6.46

a) Which side is the longest?

b) Which side is the shortest?

c) Which angle measure is the largest? Which side is opposite to this largest angle?

d) Which angle measure is the smallest? Which side is opposite to this smallest angle?

e) Compare (i) AB + BC ___ AC

(ii) AB + AC ___ BC

(iii) BC + AC ___ AB

Note: The answers to the above Group work will help us to conclude the following about the relationship between sides and angles of a triangle.

1. **If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the shorter side.**

Example 13

In Figure. 6.47, it is given that AC >AB, therefore M(∠ABC) >m(∠ACB).

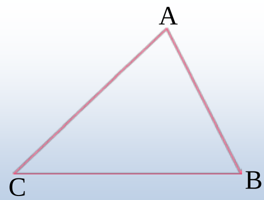


Figure 6.47

2. If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Example 14

In ΔDEF (Figure 6.48), it is known that $m(<E) > m(<F)$, therefore $DF>DE$

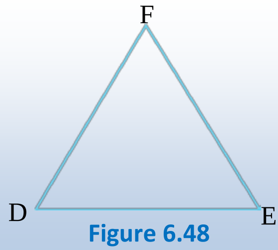


Figure 6.48

3. Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example 15

In ΔABC (Figure 6.49), each of the following holds true.

(i) $AB + BC > AC$

(ii) $AB + AC > BC$

(iii) $BC + AC > AB$

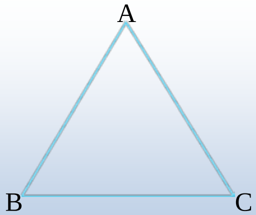


Figure 6.49

Can you give examples of measures of three sides which enables to construct a triangle? Can you construct a triangle with measures of the three sides 3cm, 4cm and 7cm? why?

Example 16

Which of the following three numbers can represent measures of three sides of a triangle?

a) 3, 4, 5 b) 6, 6, 4 c) 1, 4, 5

Solution: a) $3 + 4 > 5$, $4 + 5 > 3$ and $3 + 5 > 4$.

Yes. The three numbers can represent measures of sides of a triangle.

a) $\left.\begin{array}{l} 6 + 4 > 6 \\ 6 + 6 > 4 \\ 4 + 6 > 6 \end{array}\right\}$ yes. The three numbers can represent measures of the sides of a triangle.

c) $1 + 4 = 5$. It does not fulfill the condition of triangle inequality. And it cannot represent measures of sides of a triangle.

Exercise 6.C

1. Is it possible to construct a triangle with the lengths of sides indicated?
   a) 2cm, 3cm, 3cm
   b) 10cm, 10cm, 10cm
   c) 4cm, 3cm, 7cm
   d) 0.4.cm, 0.5cm, 0.8cm

2. Name the largest angle and the smallest angle of the triangle (Figure 6.50)

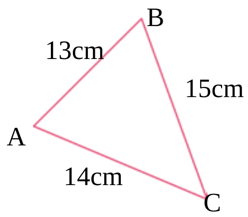


Figure 6.50

3. Name the longest side and the shortest side of the triangle (Figure 6.51)

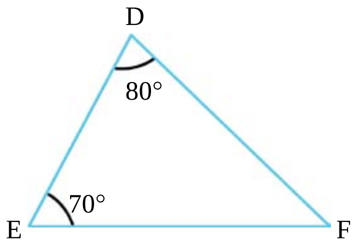


Figure 6.51

4. In ΔPQR, $m(\angle P) = 60^o$, and $m(\angle Q) = 75^o$, then which side is the longest side? Which side is the shortest side?

6.3. Congruent Triangles

6.3.1. Congruency

Consider the designs shown below.

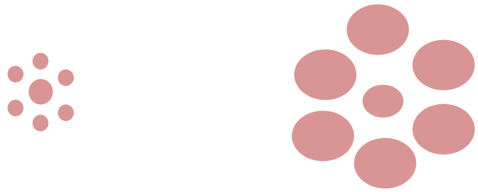


Figure 6.52

Do you think the three figures have the same size and shape? If you were to trace them, you would find that the first and third figures have the same size and shape, but the one in the middle is slightly larger.

Whenever two figures have the same size and shape they are called **congruent**. You are already familiar with congruent segments and congruent angles. Here you will learn about congruent triangles.

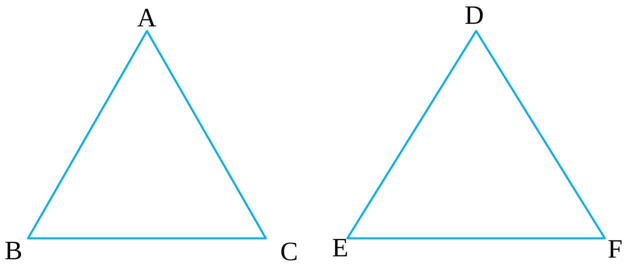


Figure 6.53

Triangles ABC and DEF are congruent. If you mentally slide ΔABC to the right, you can fit it exactly over ΔDEF by matching up the vertices like this: $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$.

The sides and angles will then match up like this:

Corresponding angles	corresponding sides
$\angle A \leftrightarrow \angle D$	$\overline{AB} \leftrightarrow \overline{DE}$
$\angle B \leftrightarrow \angle E$	$\overline{BC} \leftrightarrow \overline{EF}$
$\angle C \leftrightarrow \angle F$	$\overline{AC} \leftrightarrow \overline{DF}$

We have the following definition:

Definition 6.8: **Two triangles are congruent** if and only if their vertices can be matched up so that the corresponding parts (angles and sides) of the triangles are congruent.

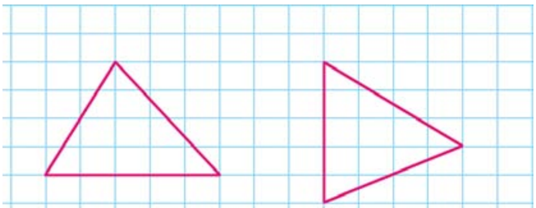
Example 17

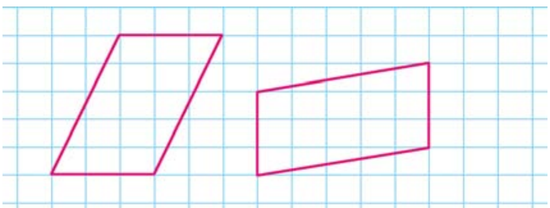
Figure 6.54

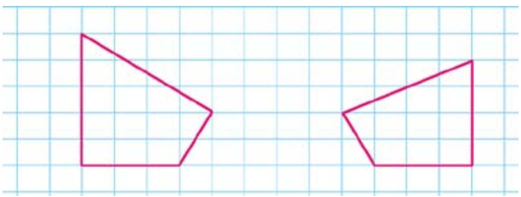
$\Delta ABC \cong \Delta PQR$,

Activity 6.5

Tell whether the figures are congruent.

1. 

2. 

3. 

4. Explain how you know whether two figures are congruent.

Note

1. **When referring to congruent triangles, we name their corresponding vertices in the same order. For the triangles above (Example 17), the following statements are also correct:**

   $\Delta ABC \cong \Delta PQR$, $\Delta BCA \cong \Delta QRP$, $\Delta CAB \cong \Delta RPQ$

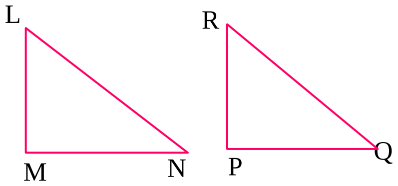
$\Delta ABC \cong \Delta PQR$

Corresponding angles	Corresponding sides
$\angle A \cong \angle P$	$\overline{BC} \cong \overline{QR}$
$\angle B \cong \angle Q$	$\overline{CA} \cong \overline{RP}$
$\angle C \cong \angle R$	$\overline{AB} \cong \overline{PQ}$

2. **You may check congruency of triangles by tracing, cutting and overlapping one over the other.**

Exercise 6.D

1. Identify whether each of the following statements is true or false.
   a) If $\Delta ABC \cong \Delta DEF$, then $\overline{BC} \cong \overline{EF}$.
   b) If $\Delta PQR \cong \Delta STU$, then $\angle Q \cong \angle U$.
   c) If $\Delta GHI \cong \Delta KLM$, then $\Delta HGI \cong \Delta LKM$.
2. Complete each congruence statement.
   a) $\Delta LMN \cong \Delta RPQ$



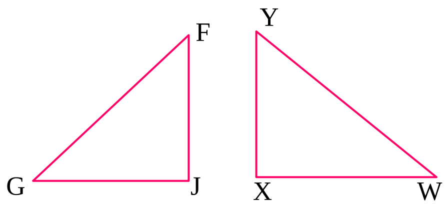
$\overline{MN} \cong \overline{PQ}$ $\angle M \cong \angle P$

$\overline{NL} \cong$ ______ $\angle L \cong$ ___

   b) $\Delta FGJ \cong \Delta YWX$

$\overline{JF} \cong \overline{XY}$ $\angle G \cong \angle W$

$\overline{FG} \cong$ ___ $\angle J \cong$ _____



c) $\Delta ABC \cong \Delta DEF$ $\quad \angle A \cong$ _____ $\quad \angle B \cong$ _____ $\quad \angle C \cong$ _____

$\overline{AC} \cong$ _____ $\quad \overline{BC} \cong$ _____ $\quad \overline{AB} \cong$ ___

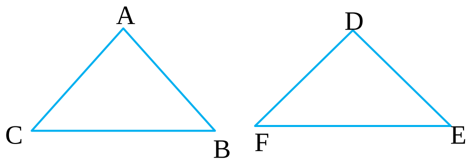


Figure 6.55

3. Are the figures below congruent or not congruent?

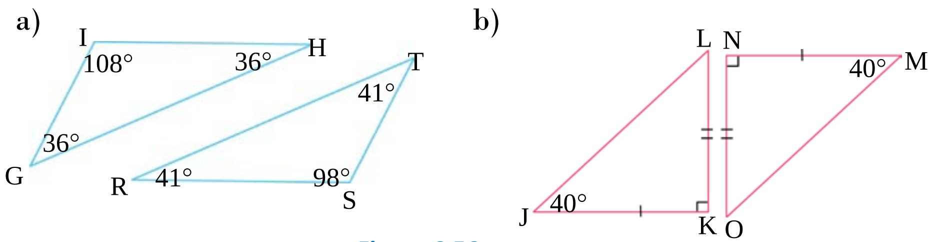


Figure 6.56

4. Use the diagram shown below to complete each of the following.

a. $\angle ABC \cong$ __________ d. $\Delta ABC \cong$ _________

b. $\overline{AB} \cong$ _______ e. $\Delta BAC \cong$ _________

c. $\angle F \cong$ _______ f. $\Delta CAB \cong$ _________

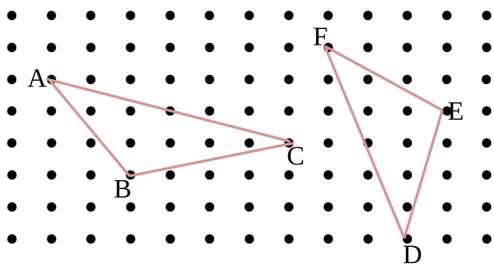


Figure 6.57

6.3.2. Tests for Congruency of Triangles

Here you will learn three different ways to show that two triangles are congruent.

Activity 6.6

Consider the two triangles given (Figure 6.58). Observe that $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{GF}$ and $\overline{AC} \cong \overline{EG}$. Now, measure the angles of each triangle. Did you observe that $\angle A \cong \angle E$, $\angle B \cong \angle F$ and $\angle C \cong \angle G$? Can you state any congruency statement between the two triangles?

What does this imply to you?

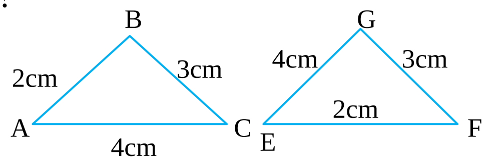


Figure 6.58

The following postulate will give you a way to show that two triangles are congruent by comparing three pairs of corresponding parts.

SSS (side, side, side) postulate: if three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Example18

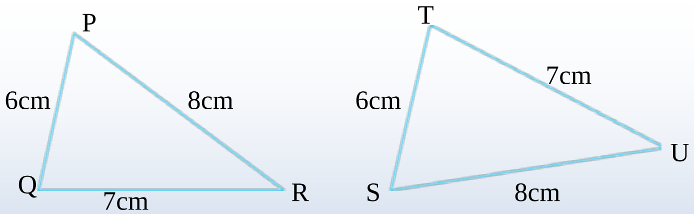


Figure 6.59

Observe that $\overline{QP} \cong \overline{ST}$, $\overline{QR} \cong \overline{TU}$ and $\overline{PR} \cong \overline{SU}$. Thus, by SSS postulate we can conclude that $\Delta PQR \cong \Delta STU$.

Sometimes it is helpful to describe the parts of a triangles interms of their relative position.

In Figure. 6.60, $\overline{AB}$ is opposite to angle C.

$\overline{AB}$ is included between $\angle A$ and $\angle B$.

$\angle A$ is opposite $\overline{BC}$.

$\angle A$ is included between $\overline{AB}$ and $\overline{AC}$.

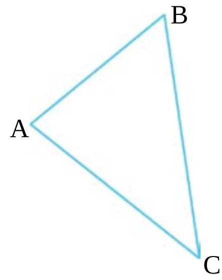


Figure 6.60

Activity 6.7

Consider ΔABC and ΔDEF in Figure 6.61. Observe that $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

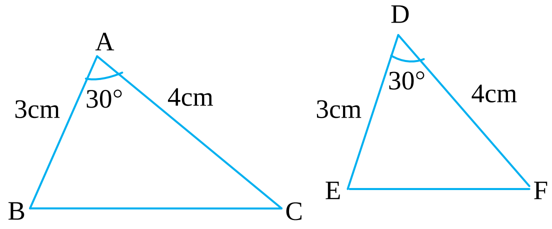


Figure 6.61

Now, measure sides $\overline{BC}$ and $\overline{EF}$.

Did you find that $\overline{BC} \cong \overline{EF}$?

Can you state any congruency between the two triangles by SSS postulate?

Let us state the second postulate on congruency of triangles as follows:

SAS (Side, Angle, Side) Postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Example 19

In Figure. 6.62, $\Delta ABC \cong \Delta DFE$ by SAS.

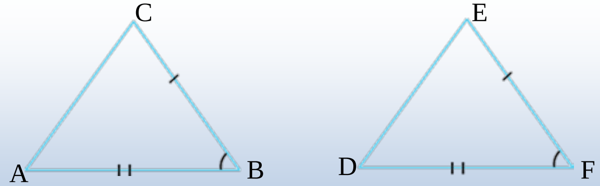


Figure 6.62

Activity 6.8

Consider the two triangles given (Figure. 6.63). Observe that $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$ and $\angle B \cong \angle E$.

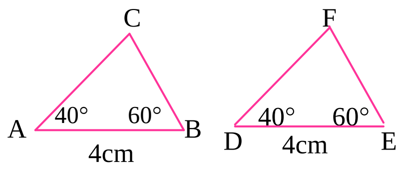


Figure 6.63

Measure the lengths of sides $\overline{AC}$ and $\overline{DF}$ (or $\overline{BC}$ and $\overline{EF}$). What do you observe? Did you find that $\overline{AC} \cong \overline{DF}$? Can you apply SAS postulate to state congruency between the two triangles?

Let us state the third postulate on congruency of triangles. This postulate will generalize the idea you have observed in the above Activity.

ASA(Angle, side, Angle) Postulate: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example 20

In Figure. 6.64,
$\Delta PQR \cong \Delta FGH$ by ASA.

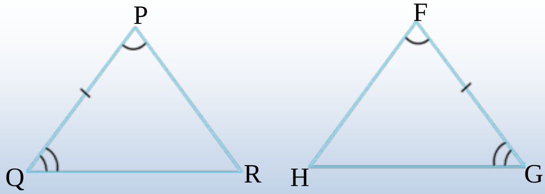


Figure 6.64

Exercise 6.E

Decide whether there is a triangle congruent to ΔABC. If so, write the congruence and name the postulate used. If not, write no congruence can be deduced.

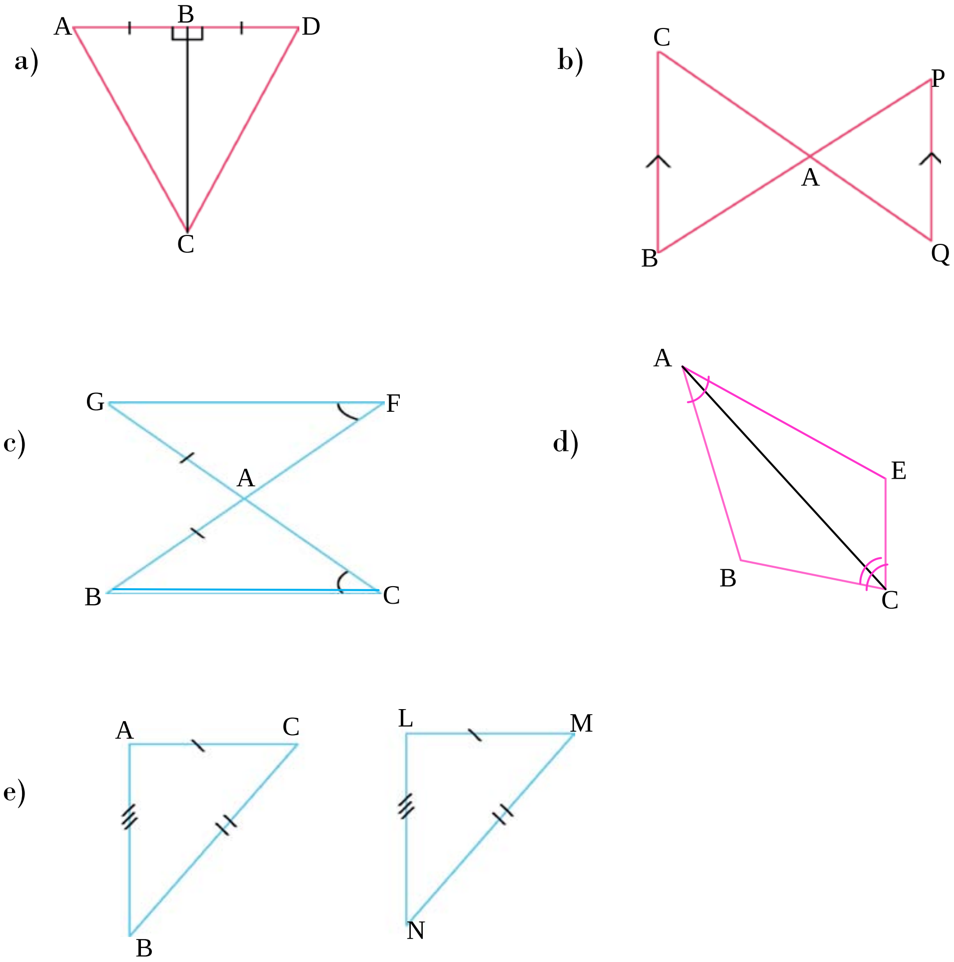


Figure 6.65

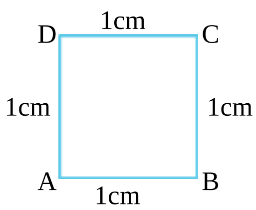
6.4 Measurement

6.4.1 Areas of Right Angled Triangles and Perimeter of Triangles

A. Perimeter of triangles

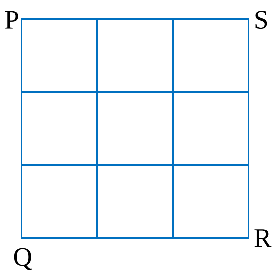
In your earlier studies you have learnt how to find the perimeters and areas of squares and rectangles. Moreover you are familiar with the following definitions and properties.

- For measuring areas of plane figures, we define **a square unit** by considering a small square whose each side equals 1 unit.



ABCD is a square of side 1cm

$\therefore$ Area of square ABCD = 1sq.cm or $1cm^2$



The region enclosed by the figure contains 9 small squares. Each small square has the area $1cm^2$.

$\therefore$ Area of square PQRS $= 9 \times 1cm^2$

$= 9cm^2$

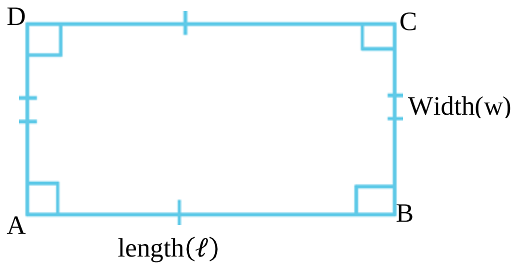


Figure 6.67

Let ABCD be a rectangle

(a) Area of rectangle ABCD= a(ABCD) = AB × BC thus, a(ABCD) = ℓw

(b) Perimeter of rectangle ABCD = P(ABCD) = AB + BC + CD + AD

= AB + BC + AB + BC(since AB = CD and BC = AD)

= 2AB + 2BC

P(ABCD) = 2(ℓ + w)

In order to help you revise the knowledge on perimeter and areas of squares and rectangles, do the following Activity.

Activity 6.9

Find the area and perimeter of each of the following figures.

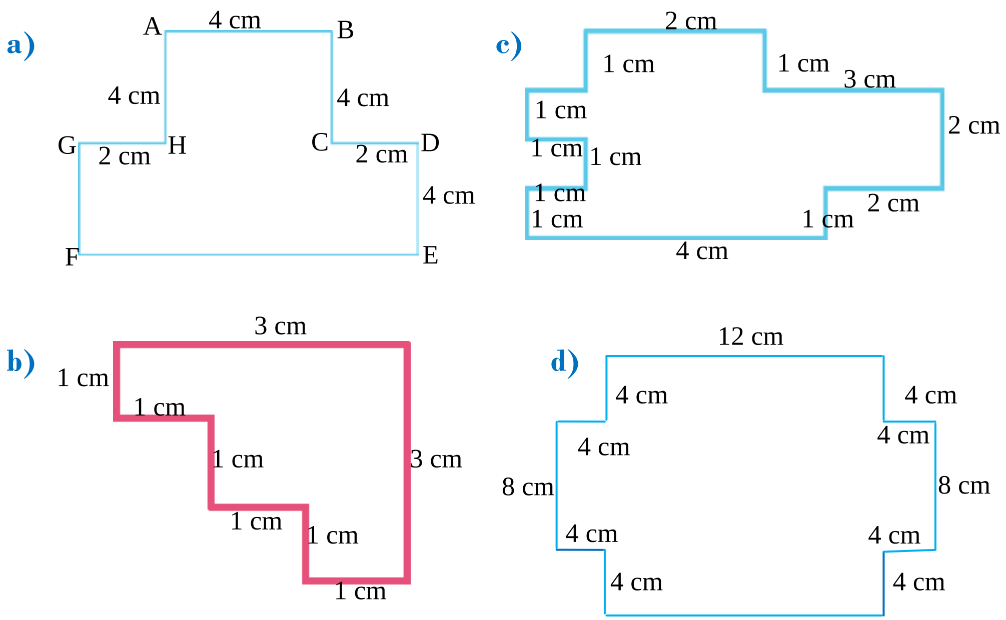


Figure 6.68

B. Areas of Right Angled Triangles

Group work 6.5

Work with a partner.

Materials: Graph paper, scissors

- Draw a parallelogram of any shape or size on a piece of graph paper.
- Draw a diagonal in the rectangle.
- Cut along the diagonal.

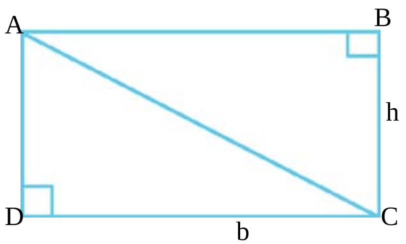


Figure 6.69

Discuss

a. What two shapes are formed?

b. How do the two shapes compare?

c. What is the area of the original rectangle?

d. What is the area of each triangle?

Area of a right angled triangle

The area of a right angled triangle is equal to half the product of the length of its legs (base and height).

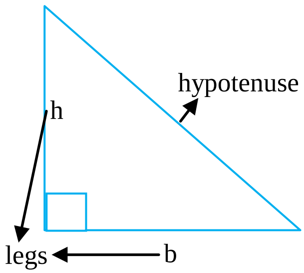


Figure 6.70

That is, if a right angled triangle has a base of b units and a height of h units, then the area, a square units, is $a = \frac{1}{2}bh$

Note

In a right angled triangle, one leg is the base and the other leg is the height.

Example 21

Find the area of the triangle at the right. (Figure 6.71)

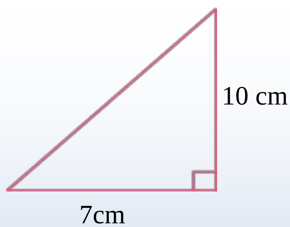


Figure 6.71

Solution: $a = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 10$

replace b with 7 and h with 10

$= \frac{1}{2} \times 70$

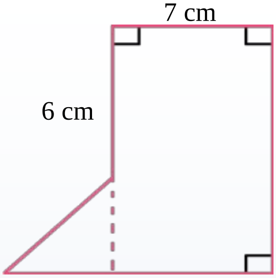
$= 35$

The area of the triangle is 35 square centimeters.

Notice that not all geometric figures are shapes with which you are familiar. Some of them, however, can be divided in to familiar shapes.

Example 22

Find the area of the figure shown right.



Solution: Use the area formulas to find the areas of the triangle and the rectangle as follows:

Area of a right angled triangle $= \frac{1}{2}$ bh$= \frac{1}{2}(2)(4)$

$= \frac{1}{2}(8)$

$= 4\text{cm}^2$

Area of a rectangle = bh

$= 7 \times 10$

$= 70\text{cm}^2$

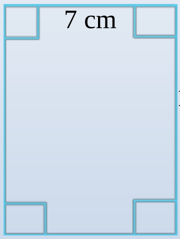


Figure 6.72

We may find the total area by adding the area of each figure.

Total area = area of triangle + area of rectangle

$= 4 + 70$

$= 74 \text{ cm}^2$

The total area is 74cm^2.

In your previous mathematics lessons you have learnt the definition of perimeter and how to measure the perimeters of simple closed figures which do not intersect themselves (For example rectangles or squares). Here once again you are going to deal with perimeter of triangles.

Note

The sum of the lengths of all the sides of a triangles is called its **perimeter**.

The perimeter of a triangle is the sum of the measures of the sides.

$P = a + b + c$, Where $a = BC$,

$b = AC$ and

$c = AB$

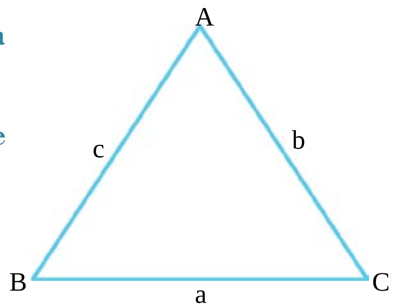


Figure 6.73

Example 23

Compare the perimeter of the following triangles.

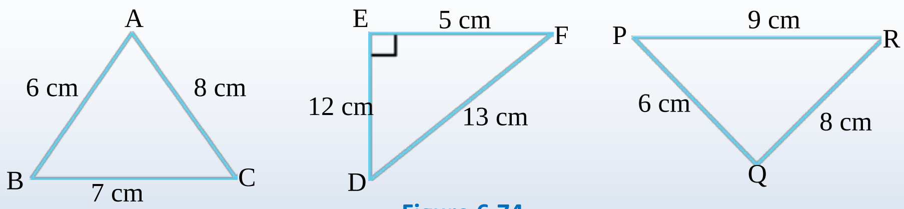


Figure 6.74

Solution:

$$P_{\Delta ABC} = 6cm + 7cm + 8cm = 21cm$$

$$P_{\Delta DEF} = 5cm + 12cm + 13cm = 30cm$$

$$P_{\Delta PQR} = 6cm + 8cm + 9cm = 23cm$$

Thus, we see that $P_{\Delta DEF} > P_{\Delta PQR} > P_{\Delta ABC}$

Note: When you find area and perimeter of a rectangle or square or a triangle, be sure that base and height are in the same unit. If they are in different units, first convert them in to the same unit.

Example 24

Length and width of a rectangle are 2m and 30cm find its perimeter.

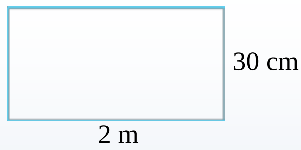


Figure 6.75

Solution: Let the figure be as shown (Figure 6.75)

$$\ell = 2\text{m} = 2 \times 100 \text{ cm} = 200\text{cm}$$

$$w = 30\text{cm}$$

$$\begin{aligned} \text{Perimeter} = p &= 2(\ell + w) \\ &= 2(200 + 30) \\ &= 2(230) \\ P &= 460\text{cm} \end{aligned}$$

Units of Measurement of length and area: we use various units of measurement of length depending on the length of the object, such as meter, centimeter, millimeter, decimeter, etc.

Conversion

Units of length	Units of Area
1m = 100cm	$1\text{m}^2 = 1\text{m} \times 1\text{m} = 100\text{cm} \times 100\text{cm} = 10{,}000$ sq.cm
	$\therefore 1\text{m}^2 = 10{,}000 \text{ cm}^2$
1cm = 0.01m	$1\text{cm}^2 = 1\text{cm} \times 1\text{cm} = 0.01\text{m} \times 0.01\text{m} = 0.0001\text{m}^2$
	$\therefore 1\text{cm}^2 = 0.0001\text{m}^2$
	1 hectare = $100\text{m} \times 100\text{m} = 10{,}000\text{m}^2$
	Or $1\text{m}^2 = 0.0001$ hectare
1cm = 10mm	$1\text{cm}^2 = 10\text{mm} \times 10\text{mm} = 100$sq mm
	$\therefore 1\text{cm}^2 = 100\text{mm}^2$
	Or $1\text{mm}^2 = 0.01\text{cm}^2$

Activity 6.11

Convert 1 hectare to cm^2.

Example 25

Find the area of the rectangle whose length and width are 25cm and 110 mm respectively.

Solution: $\ell = 25cm$

$w = 110mm = 11cm$ (why?)

$a = \ell \times w = 25cm \times 11cm = 275\ cm^2$

Example 26

Convert a) $20m^2$ to cm^2

b) $60cm^2$ to m^2

c) 10 hectare to m^2

d) $3000m^2$ to hectare

e) $400cm^2$ to mm^2

f) $90mm^2$ to cm^2

Solution: a) $1m^2 = 10,000\ cm^2$

Thus, $20\ m^2 = 20 \times 10,000\ cm^2 = 200,000\ cm^2$.

b) $1cm^2 = 0.0001\ m^2$

It implies that $60\ cm^2 = 60 \times 0.0001\ m^2 = 0.006\ m^2$.

c) 1 hectare $= 10,000\ m^2$

Thus 10 hectare $= 10 \times 10,000\ m^2 = 100,000m^2$.

d) $1m^2 = 0.0001$ hectare

Thus $3000\ m^2 = 3000 \times 0.0001$ hectare $= 0.3$ hectare.

e) $1cm^2 = 100\ mm^2$

Thus, $400\ cm^2 = 400 \times 100\ mm^2 = 4000\ mm^2$.

f) $1mm^2 = 0.01\ cm^2$

Thus, $90\ mm^2 = 90 \times 0.01\ cm^2 = 0.9\ cm^2$.

Exercise 6.F

1. Convert:

   a) $50m^2$ to cm^2

   b) $100cm^2$ to m^2

   c) 0.4 hectare to m^2

   d) $1000m^2$ to hectare

   e) 7.5 cm^2 to mm^2

   f) 800 mm^2 to cm^2

   g) 0.09 hectare to cm^2

2. Find the area.

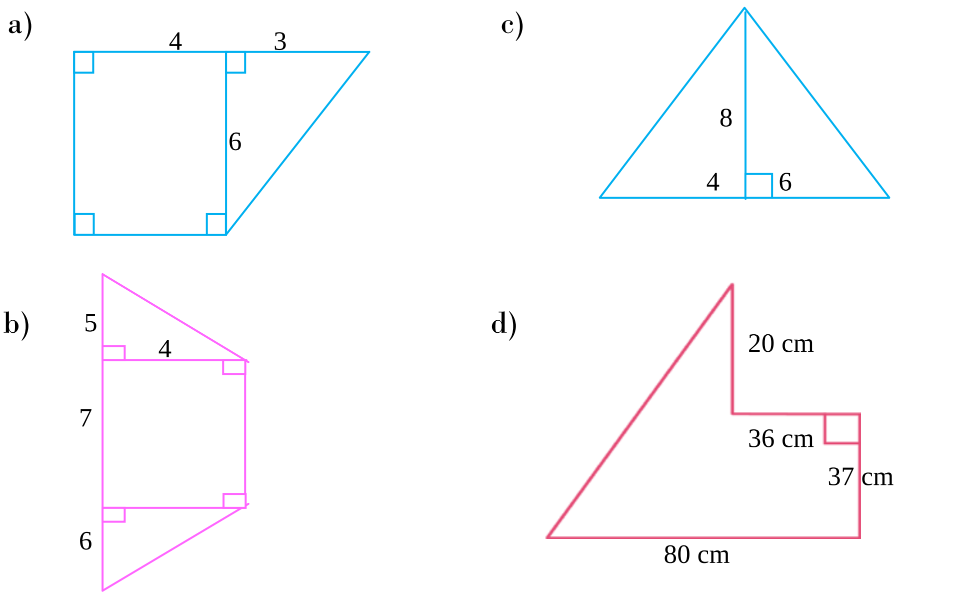


Figure 6.76

3. The area of a right angled triangle is 48 sq cm. If the height of the triangle is 12cm, then find the base of the triangle.

4. Compare perimeters of the following triangles.

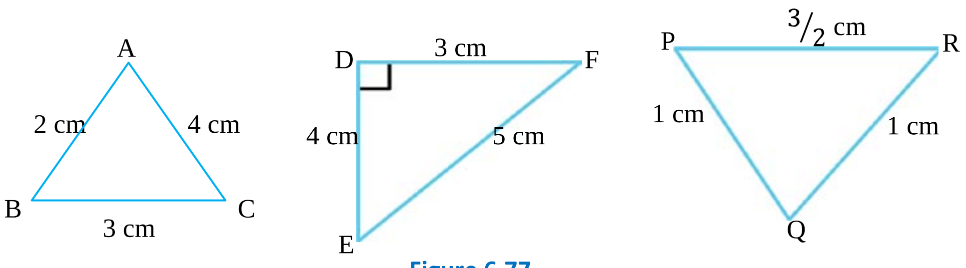


Figure 6.77

5. A carpet is in the shape of a right triangle and has area 160 sq.m. If the base of the carpet is 40m, then find its height.
6. Find the area of the shaded region shown below.

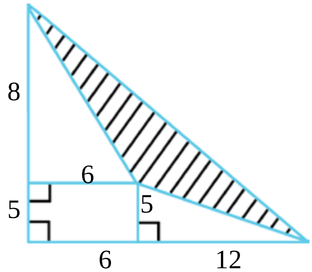
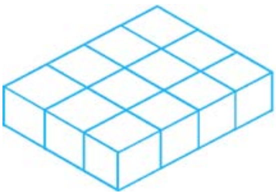


Figure 6.78

6.4.2. Volume of Rectangular Prism

Activity 6.12

Count the number of unit cubes to find the volume of the boxes.

a) 

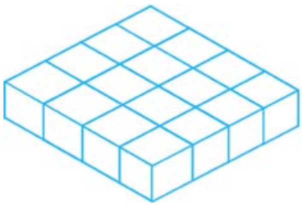
b) 

Figure 6.79

In this lesson, you will learn how to find the amount of space inside a prism. Can you discuss with the students what a prism is? Can you give an example of a rectangular prism?

Any three – dimensional figure can be filled completely with congruent cubes and parts of cubes. The **volume** of a three – dimensional figure is the number of cubes it can hold. Each cube represents a unit of measure called a cubic unit.

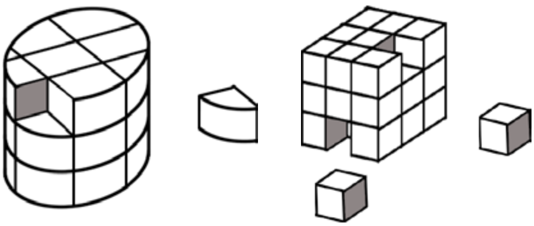
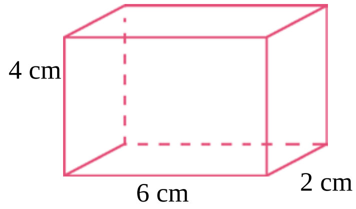


Figure 6.80

Remember that **volume** is the measure of the space occupied by a solid figure. It is measured in cubic units. A **cubic unit** is a cube whose edges are 1 unit long. You can use cubes to make models of solid figures.



The container at the right has a length of 6cm, and width of 2cm, and a height of 4cm.

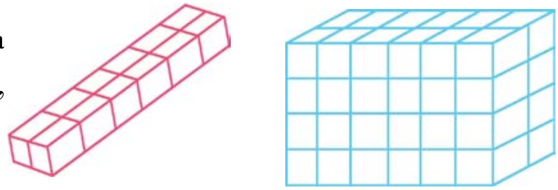


Figure 6.81

The model is made of 4 layers. Each layer has 12 cubes. The area of the base is 12 square cm, the product of the length and width. Since the container is 4 layers high and has a base of 12 one – cm cubes, it will take 4 × 12 or 48 one-cm cubes to fill the container. The volume of the container is 48 cubic cm.

Volume of a rectangular prism: The volume (V) of a rectangular prism is found by multiplying the length (ℓ), the width(w), and the height(h).

In symbols: $V = \ell wh$

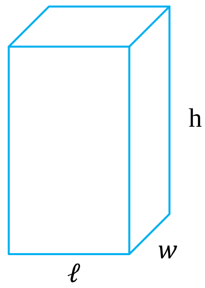


Figure 6.82

Example 27

Draw and label a rectangular prism whose length is 5cm, width is 2cm, and height is 8cm. Find its volume.

Solution:

$V = \ell wh = 5 \times 2 \times 8$ .... Replace ℓ with 5, w with 2, and h with 8

$\therefore\ V = 80$

The prism has a volume of 80cm^3.

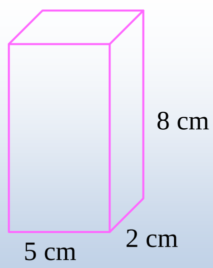


Figure 6.83

Example 28

Find the volume of the rectangular prism shown right.

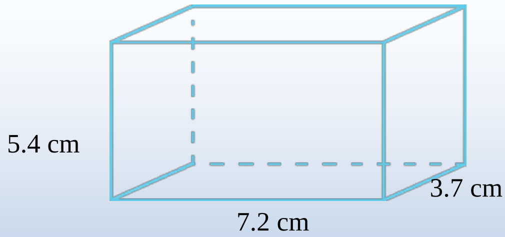


Figure 6.84

Solution:

$$V = \ell wh$$
$$= 7.2 \times 3.7 \times 5.4$$
$$= 143.856$$

The prism has a volume of 143.856 cubic cm.

Note: When you find volume of a rectangular prism, be sure that length, width and height are in the same unit. If they are in different units, first convert them in to the same unit.

Example 29

Length, width and height of a rectangular prism are 2m, 30cm and 70mm, find its volume.

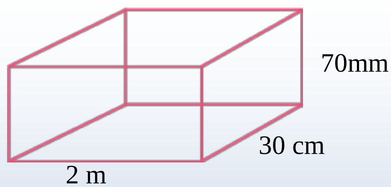


Figure 6.85

Solution: ℓ = 2m=200 cm (why?)

w = 30cm

And h = 70mm = 7cm (why?)

$V = \ell wh = 200 \times 30 \times 7$

$\therefore$ v = 42,000 cm^3

Units of Measurement of Volume: By common agreement, the usual choice to measure volume of a solid is cubic unit such as mm^3, cm^3, m^3, etc.

Conversion

$1m^3 = 1m \times 1m \times 1m = 100cm \times 100cm \times 100cm = 1{,}000{,}000\ cm^3$

$\therefore 1m^3 = 1{,}000{,}000\ cm^3$

Or $1cm^3 = 0.000001\ m^3$

$1cm^3 = 1cm \times 1cm \times 1\ cm = 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$

$\therefore 1cm^3 = 1000\ mm^3$

Or $1mm^3 = 0.001cm^3$

1 Litre = 1000 mℓ = 1000 cm^3

Or $1cm^3 = 0.001\ell$

Example 30

Convert a) $0.3m^3$ in to cm^3 d) 2.5L in to cm^3
b) 2000 cm^3 in to m^3 e) 500 cm^3 in to L
c) $5cm^3$ in to mm^3

Solution: a) $1m^3 = 1{,}000{,}000\ cm^3$, thus $0.3m^3$
$= 300{,}000\ cm^3$

b) $1cm^3 = 0.000001m^3$, thus $2000cm^3 = 0.002m^3$

c) $1cm^3 = 1000mm^3$, thus $5cm^3 = 5000mm^3$

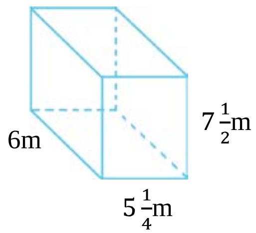
d) $1L = 1000\ cm^3$, thus $2.5L = 2500\ cm^3$

e) $1cm^3 = 0.001L$, thus $500cm^3 = 0.5L$

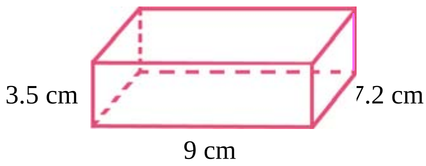
Exercise 6.G

1. Find the volume of each rectangular prism.

a.



b.



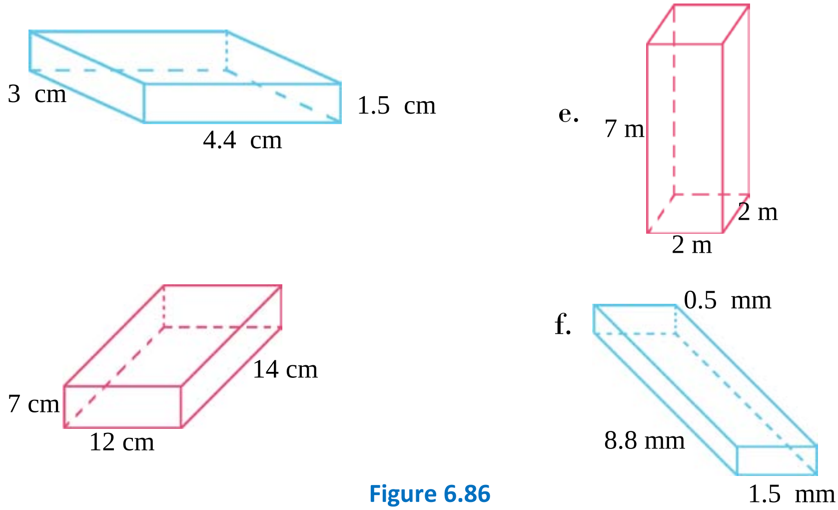


Figure 6.86

g. Length = 5 mm, width = 7 mm, height= 10 cm

h. Length = 12 m, width = 9m , height = 7 cm

i. Length = 12.1 cm , width = 8.2 cm, height = 10.6 mm

2. A cube has sides that are 7 cm long.

   a. What is the volume of the cube?

   b. Write a formula for finding the volume of a cube.

3. Find the height of each rectangular prism given the volume, length, and width.

a) $V = 122{,}500\ cm^3$	b) $V = 22.05\ m^3$	c) $V = 3{,}375 mm^3$
$\ell = 50cm$	$\ell = 3.5\ m$	$\ell = 15mm$
$w = 35cm$	$w = 4.2m$	$w = 15mm$

4. Convert

   a) $20\ m^3$ to cm^3

   b) $100\ cm^3$ to m^3

   c) $0.5\ m^3$ to litres

   d) $5000\ m^3$ to cm^3

   e) 3 litres to cm^3

   f) $2000\ cm^3$ to litres

   g) $100\ cm^3$ to mm^3

UNIT SUMMARY

Important facts you should know:

- **When a transversal crosses two parallel lines different angles are formed.**

  ∠1 and ∠5, ∠ 3 and ∠7,
  ∠2 and ∠6, ∠4 and ∠8 ......
  corresponding angles
  ∠3 and ∠6, ∠4 and ∠5 .....
  alternate interior angles
  ∠1 and ∠8, ∠2 and ∠7 .....
  alternate exterior angles
  ∠1 and ∠4 ..... vertically opposite angles
  ∠1 and ∠2 ..... adjacent angles

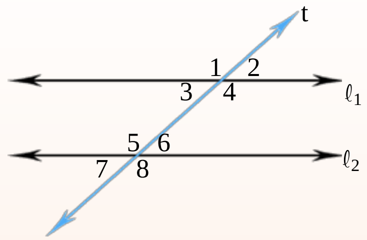


Figure 6.87

- **Important relationship between sides and angles of a triangle.**
  1. **If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.**
  2. **If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.**
  3. **Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.**
- **Congruent Triangles: Two triangles are congruent if and only if their vertices can be matched up so that the corresponding parts (angles and sides) of the triangles are congruent.**

- **Ways to show two triangles are congruent:**

1. **SSS postulate:** **If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.**
2. **SAS postulate:** **If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.**
3. **ASA postulate:** **If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.**

- **Area of a right angled triangle:** **The area of a right angled triangle with legs b units and h units is given by** $A = \frac{1}{2}bh$.

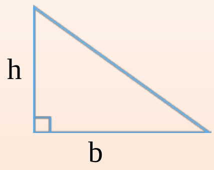


Figure 6.88

- **Perimeter of triangles:** **If BC =a, AC = b and AB = c, then** $P = a + b + c$

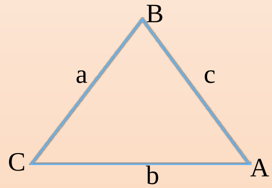


Figure 6.89

- **Volume of rectangular prism:** **The volume of a rectangular prism with length(ℓ), width (w) and height (h) is given by** $V = \ell wh$

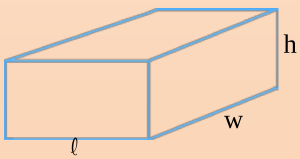
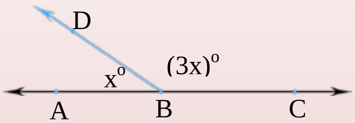


Figure 6.90

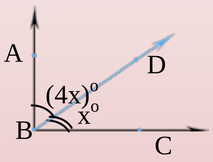
Review Exercise

1. Identify whether each of the following statements is true or false.
   a. An obtuse angled triangle is always scalene.
   b. The sum of the measures of any two sides of a triangle is always greater than the measure of the third side.
   c. Two rays that have the same end point form an angle.
   d. A theorem is a mathematical statement that can be proved.
   e. An equilateral triangle is equiangular.
   f. If two angles are both supplementary and adjacent, then they are congruent.
2. In the figures shown below, find the value of x.
   a.



$\overleftrightarrow{AC}$ is a straight line

   b.



m(∠ABC) = 90°

   c.

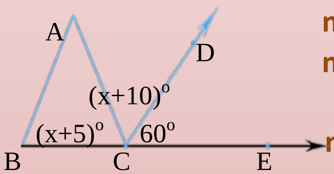


Figure 6.91

Given that m(∠ABC) = (x + 5)°
m(∠ACD) = (x + 10)°
m(∠ECD) = 60° and
m(∠DCE)= m(∠ACD). Here, find m(∠B) also.

3. If in the figure shown at the right, lines ℓ, m, and n intersect at A where ℓ ⊥ m, Find the measures of angles x, y and z if y = 2x .

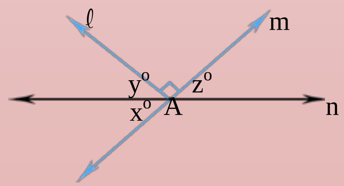


Figure 6.92

4. If, in the figure shown at the right, m(∠DOB) = 70°, m(∠COA) = 80°, and m(∠DOA) = 110°, then what is m(∠COB)?

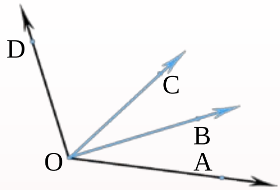


Figure 6.93

5. In which one of the following cases is that the lines marked parallel?

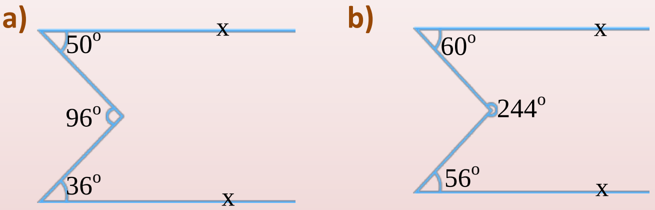


Figure 6.94

6. Find the degree measures of angles marked x and y

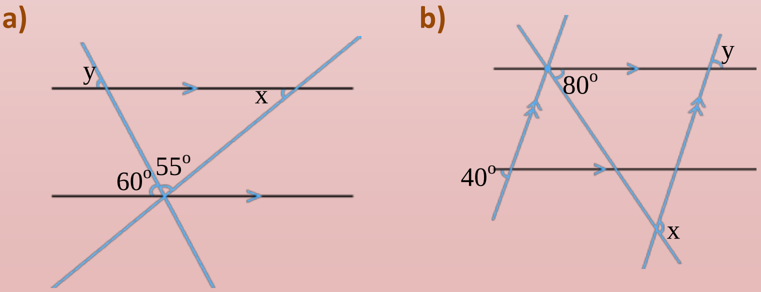
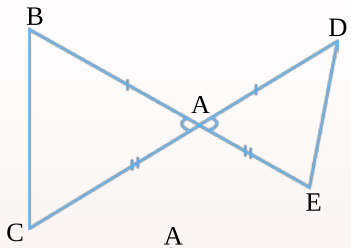


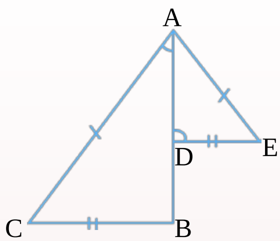
Figure 6.95

7. Name the postulate (SSS, SAS, or ASA), if any, that will prove the triangles to be congruent.

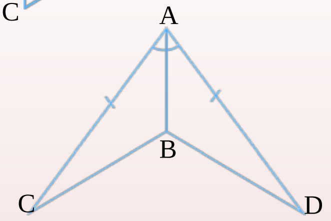
a)



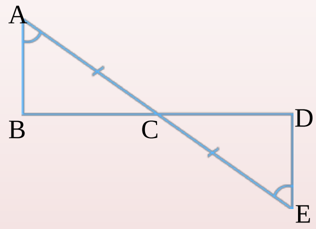
d)



b)



e)



c)

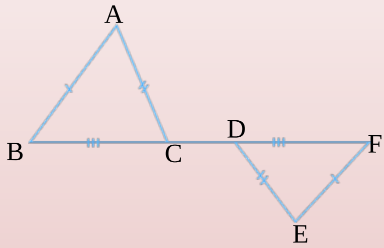


Figure 6.96

8. **The average of the lengths of the sides of ΔABC is 14. How much longer than the average is the longest side?**

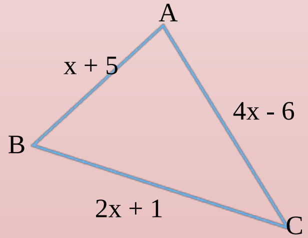


Figure 6.97

9. **Let AB = x + 3**
AC = 3x + 2
BC = 2x + 3
And perimeter of ΔABC = 20.
Show that ΔABC is scalene.

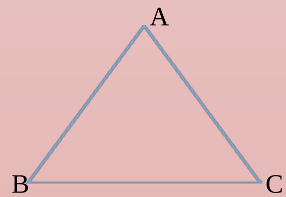


Figure 6.98

10. How many different isosceles triangles can you find that have sides that are whole – number lengths and that have a perimeter of 18?

11. Find the area of the shaded region.

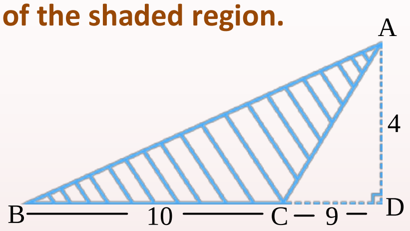


Figure 6.99

12. Find the volume of the prism shown below.

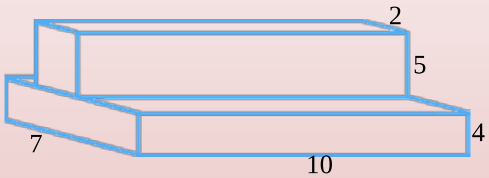


Figure 6.100

Hint: you may think of the prism as two boxes as shown below (one box resting on the other)

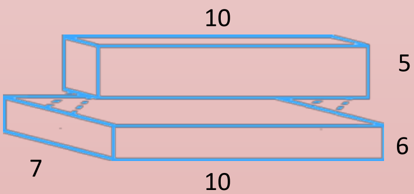


Figure 6.101

13. Suppose that a cube has base area equal to $16cm^2$, then determine the volume of this cube.
14. A rectangular tank has a height of 9 metres, a width of 5 metres, and a length of 12 metres. What is the volume of the tank?
15. The volume of a cube is $125m^3$. What is the base area of this cube?
16. If, in figure 6.102, the volume of the smaller is $27\ cm^3$, then what is the volume of the larger box?

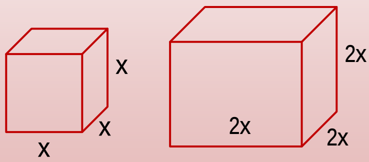


Figure 6.102